Problem Set 22
"Time Series: Forecasting"

(22.1) [1-step-ahead forecast interval] We are trying to forecast tomorrow's ice-cream sales y_t using lagged ice-cream sales y_{t-1} and lagged temperature forecasts x_{t-1} from the Weather Service. Today's values are $y_T = 125$ and $x_T = 83$. To simply calculations, we have transformed the data and estimated our model as follows. Standard errors are in parentheses.

$$y_t = \underset{(1.50)}{140} + \underset{(0.072)}{0.16}\,(y_{t-1}-125) + \underset{(0.61)}{2.4}\,(x_{t-1}-83)$$

The sample size is 200. The sum of squared residuals is 1329.75. Let $\hat{f}_T$ denote our point forecast of y_{T+1} at time T, let $e_{T+1} = (y_{T+1} - \hat{f}_T)$ denote the forecast error, and let σ^2 denote the variance of the error term.

a. Give the value of the point forecast $\hat{f}_T$.
b. Compute the unbiased estimate of the variance of the error term $\hat{\sigma}^2$.
c. Give the standard error of the forecast $SE(\hat{f}_T)$.
d. Compute the standard error of the forecast error $SE(e_{T+1})$.
e. Assume the error term is normally distributed and serially uncorrelated. Compute a 95% forecast interval. Also compute a 99% forecast interval.

(22.2) [1-step-ahead forecast interval] We are trying to forecast natural-gas demand y_t using lagged natural-gas demand y_{t-1} and lagged temperature forecasts from the Weather Service x_{t-1}. Today's values are $y_T = 478$ and $x_T = 34$. To simply calculations, we have transformed the data and estimated our model as follows. Standard errors are in parentheses.

$$y_t = \underset{(2.20)}{520} + \underset{(0.022)}{0.08}\,(y_{t-1}-478) - \underset{(0.93)}{3.4}\,(x_{t-1}-34)$$

The sample size is 300. The sum of squared residuals is 5987.52. Let $\hat{f}_T$ denote our point forecast of y_{T+1} at time T, let $e_{T+1} = (y_{T+1} - \hat{f}_T)$ denote the forecast error, and let σ^2 denote the variance of the error term.

a. Give the value of the point forecast $\hat{f}_T$.
b. Compute the unbiased estimate of the variance of the error term $\hat{\sigma}^2$.
c. Give the standard error of the forecast $SE(\hat{f}_T)$.
d. Compute the standard error of the forecast error $SE(e_{T+1})$.
e. Assume the error term is normally distributed and serially uncorrelated. Compute a 95% forecast interval. Also compute a 99% forecast interval.

(22.3) [Forecasting AR(2) model] Suppose we have the AR(2) model

$$y_t = 7.0 + 0.3\ y_{t-1} + 0.1\ y_{t-2} + \varepsilon_t\ .$$

Assume the error term is independent identically distributed with $E(\varepsilon_t) = 0$. In our sample, $y_{T-1} = 15.0$ and $y_T = 12.0$.

a. Compute point forecasts of y_{T+1}, y_{T+2}, and y_{T+3} .
b. Compute the limit of the forecast y_{T+h}, as $h \to \infty$. [Hint: This is the unconditional mean of the process.]

(22.4) [Forecasting VAR model] Suppose we have the vector autoregression (VAR) model

$$y_t = 9.0 + 0.7\ y_{t-1} + 0.1\ z_{t-1} + \varepsilon_t$$
$$z_t = 12.0 + 0.4\ z_{t-1} + 0.2\ y_{t-1} + \nu_t\ .$$

Assume the error terms are each independent identically distributed with mean zero, and the error terms are independent of each other. The last observations in our sample are $y_T = 14.0$ and $z_T = 6.0$.

a. Compute one-step-ahead point forecasts of y_{T+1} and z_{T+1} .
b. Compute two-steps-ahead point forecasts of y_{T+2} and z_{T+2} .
c. Compute the limits of the forecasts y_{T+h} and z_{T+h} as $h \to \infty$. [Hint: Set $y = y_t = y_{t-1}$ and $z = z_t = z_{t-1}$. Then solve for y and z.]

(22.5) [Forecasting VAR model] Suppose we have the vector autoregression (VAR) model

$$y_t = 3.0 + 0.5\ y_{t-1} + 0.1\ z_{t-1} + \varepsilon_t$$
$$z_t = 6.0 + 0.6\ z_{t-1} + 0.2\ y_{t-1} + \nu_t\ .$$

Assume the error terms are each independent identically distributed with mean zero, and the error terms are independent of each other. The last observations in our sample are $y_T = 5.0$ and $z_T = 10.0$.

a. Compute one-step-ahead point forecasts of y_{T+1} and z_{T+1} .
b. Compute two-steps-ahead point forecasts of y_{T+2} and z_{T+2} .
c. Compute the limits of the forecasts y_{T+h} and z_{T+h} as $h \to \infty$. [Hint: Set $y = y_t = y_{t-1}$ and $z = z_t = z_{t-1}$. Then solve for y and z.]

(22.6) [Forecasting random walk with drift] Suppose we are analyzing a random walk with drift with the formula

$$\text{(i)} \quad y_t = 2.0 + y_{t-1} + \varepsilon_t \ .$$

Assume the error term is independent identically distributed with $E(\varepsilon_t) = 0$.

a. Let s denote an early time period and t denote a later time period. Use formula (i) to show that

$$\text{(ii)} \quad y_t = 2.0\,(t-s) + y_s + \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \varepsilon_{t-2} + \ldots + \varepsilon_{s+1}$$

The first and last observations in our sample are $y_0 = 3.0$ and $y_T = 145.0$. The sample size is T=50. We want to forecast y using the information available at time period T, looking h periods into the future. Two forecast formulas are proposed:

$$\text{(iii)} \quad f_{T,h} = 2.0\,(50+h) + 3.0 \qquad \text{and} \qquad \text{(iv)} \quad g_{T,h} = 2.0\,h + 145.0$$

b. Use formula (ii) to show that forecast (iii) equals $E(y_{T+h}|y_0)$, the expected value of y_{T+h} given y_0.
c. Use formula (ii) Show that forecast (iv) equals $E(y_{T+h}|y_T)$, the expected value of y_{T+h} given y_T.
d Which forecast formula is likely to be more accurate? Why?
e. Use the more accurate forecast formula to compute forecasts of y_{T+1}, y_{T+2}, and y_{T+3} .

(22.7) [Granger causality] Suppose the following vector autoregression (VAR) model has been estimated (standard errors in parentheses) using a large sample.

y_t	=	7.6	+	0.22	y_{t-1}	+	0.05	z_{t-1}
		(1.7)		(0.06)			(0.02)	
z_t	=	2.3	+	0.16	y_{t-1}	+	0.45	z_{t-1}
		(1.2)		(0.09)			(0.13)	

a. Test whether z Granger-causes y, at 5 percent significance. Give
   - the *value* of the test statistic,
   - the *critical point* from the appropriate table at the back of your textbook (or compute the *p-value* using a spreadsheet program),
   - your *conclusion*: whether you can reject the null hypothesis of no Granger causality at 5% significance.
b. Test whether y Granger-causes z, at 5 percent significance. Give
   - the *value* of the test statistic,
   - the *critical point* from the appropriate table at the back of your textbook (or compute the *p-value* using a spreadsheet program),
   - your *conclusion*: whether you can reject the null hypothesis of no Granger causality at 5% significance.

[end of problem set]