Problem Set 21
"Time Series: Testing for Unit Roots"

(21.1) [Dickey-Fuller test] Consider the two regression models

$$\text{(i)} \quad y_t = \beta_1 + \beta_2\, y_{t-1} + \text{error term}$$
$$\text{(ii)} \quad \Delta y_t = \alpha_1 + \alpha_2\, y_{t-1} + \text{error term} .$$

a. Show that if $\alpha_1 = \beta_1$ and $\alpha_2 = \beta_2 - 1$, then the residuals from model (i) must be identical to the residuals from model (ii).
b. If the coefficients of both models are estimated by least-squares will the estimates necessarily satisfy $\hat{\alpha}_1 = \hat{\beta}_1$ and $\hat{\alpha}_2 = \hat{\beta}_2 - 1$? Explain your reasoning.

(21.2) [Dickey-Fuller test] To test whether y_t has a unit root, the following equation was estimated. Standard errors are in parentheses.

$$\Delta y_t = \underset{(0.921)}{3.722} - \underset{(0.085)}{0.238}\; y_{t-1}$$

a. Compute the Dickey-Fuller test statistic for these data.
b. In a Dickey-Fuller test, which is the null hypothesis: that the series has a unit root, or that the series does not have a unit root?
c. Find the critical points at 10%, 5%, and 1% significance. [Hint: Don't use a t table! See slideshow.]
d. Can we reject the null hypothesis at 10%? At 5%? At 1%?

(21.3) [Augmented Dickey-Fuller test] To test whether y_t has a unit root as well as first-order serial correlation in Δy_t, the following equation was estimated. Standard errors are in parentheses.

$$\Delta y_t = \underset{(0.562)}{1.431} - \underset{(0.055)}{0.176}\; y_{t-1} + \underset{(0.098)}{0.122}\; \Delta y_{t-1}$$

a. Compute the Dickey-Fuller test statistic for these data.
b. In a Dickey-Fuller test, which is the null hypothesis: that the series has a unit root, or that the series does not have a unit root?
c. Find the critical points at 10%, 5%, and 1% significance. [Hint: Don't use a t table! See slideshow.]
d. Can we reject the null hypothesis at 10%? At 5%? At 1%?

(21.4) [Augmented Dickey-Fuller test with time trend] To test whether y_t has a unit root in the presence of a time trend as well as first-order serial correlation in Δy_t, the following equation was estimated. Standard errors are in parentheses.

$$\Delta y_t = \underset{(0.572)}{0.915} - \underset{(0.075)}{0.240}\, y_{t-1} + \underset{(0.001)}{0.003}\, \text{time} + \underset{(0.081)}{0.107}\, \Delta y_{t-1}$$

a. Compute the Dickey-Fuller test statistic for these data.
b. In a Dickey-Fuller test, which is the null hypothesis: that the series has a unit root, or that the series does not have a unit root?
c. Find the critical points at 10%, 5%, and 1% significance. [Hint: Don't use a t table! See slidshow.]
d. Can we reject the null hypothesis at 10%? At 5%? At 1%?

(21.5) [Spurious regression] Find two coins.

a. Flip the first coin ten times (t=1,…,10) and compute x_t as the cumulative number of heads after t flips. Here is an example.

t	1	2	3	4	5	6	7	8	9	10
Heads or tails?	H	T	T	H	T	H	H	T	T	H
x_t	1	1	1	2	2	3	4	4	4	5

Report your values for $x_1, \ldots, x_{10}$.
b. Flip the second coin ten times and compute y_t as the cumulative number of heads after t flips. Report your values for $y_1, \ldots, y_{10}$.
c. Are x_t and y_t stationary or integrated processes? Why?
d. Using a spreadsheet program or other software, estimate the two-variable regression $y_t = \beta_1 + \beta_2 x_t$. Report the R^2 value and the value of the t-statistic for β_2.
e. Is there a relationship between x_t and y_t? Explain.

(21.6) [Cointegration] Economic reasoning suggests that prices of assets subject to arbitrage (such as the price of gold in New York and London) or prices of goods which consumers view as close substitutes (such as Rome apples and Macintosh apples) are likely to be *cointegrated.* If y_t and x_t are cointegrated, then each variable y_t and x_t might be an I(1) process—perhaps a random walk—and yet there is a linear combination of them (y_t-βx_t) that is an I(0) process. Thus the equation $y_t = \beta x_t$ expresses the *long-run relationship* between y_t and x_t.

a. Suggest three pairs of prices in the real world that economic reasoning suggests might be cointegrated.
b. For each pair, explain why you think the pair might be cointegrated.
c. For each pair, define y_t and x_t, and give the likely value of β. That is, give an equation expressing the long-run relationship between y_t and x_t. Note that this equation should *not* contain an error term.

(21.7) [Cointegration] Suppose y_t and x_t are both I(1) processes, but they are cointegrated with parameter β=2. Thus $s_t = (y_t-2x_t)$ is an I(0) process. Now consider an alternative value of β, namely 5 and define $s_t' = (y_t-5x_t)$.

a. Show that $s_t' = (y_t-5x_t)$ can be written as the sum of an I(0) process and an I(1) process.
b. Is $s_t' = (y_t-5x_t)$ an I(0) process or an I(1) process? Explain your reasoning.

(21.8) [Error correction model] Suppose we have the error correction model

$$\Delta y_t = 20 + 2\,\Delta x_t - 0.2\,(y_{t-1}-5x_{t-1}) + \varepsilon_t\ .$$

Assume $x_1 = x_2 = x_3 = 30$ and assume $y_1 = 500$.

a. Compute the error correction term $[-\,0.2\,(y_{t-1}-5x_{t-1})]$ when t=2.
b. Assume $\varepsilon_2 = \varepsilon_3 = 0$ and compute y_2 and y_3 .
c. What is the long-run value of y if $x = 30$? [Hint: Set $\Delta y_t = \Delta x_t = \varepsilon_t = 0$.]

[end of problem set]