Problem Set 20
"Time Series: Models of Highly Persistent Series"

(20.1) [Random walk]

a. Flip a coin thirty times (t=1,…,30) and compute x_t as follows. Begin by setting $x_0 = 0$. Thereafter, add one if the coin comes up "heads" and subtract one if the coin comes up "tails." Report your values for $x_1, \ldots, x_{30}$. Here is an example.

t	0	1	2	3	4	5	6	7	etc.
Heads or tails?		H	T	T	H	H	H	T	etc.
x_t	0	1	0	-1	0	1	2	1	etc.

b. Compute the sample mean for the first ten coin flips $\hat{\mu}_{10} = \frac{1}{10}\sum_{t=1}^{10} x_t$, the first twenty coin flips $\hat{\mu}_{20} = \frac{1}{20}\sum_{t=1}^{20} x_t$, and the first thirty coin flips $\hat{\mu}_{30} = \frac{1}{30}\sum_{t=1}^{30} x_t$. [Hint: Use the Excel function AVERAGE(range).]

c. Does x_t have a fixed, constant population mean? If so, compute it. If not, explain why not.

d. Similarly, compute the sample variance for the first ten coin flips $\hat{\sigma}_{10}^2 = \frac{1}{10-1}\sum_{t=1}^{10}\left(x_t - \hat{\mu}_{10}\right)^2$, the first twenty coin flips $\hat{\sigma}_{20}^2 = \frac{1}{20-1}\sum_{t=1}^{20}\left(x_t - \hat{\mu}_{20}\right)^2$, and the first thirty coin flips $\hat{\sigma}_{30}^2 = \frac{1}{30-1}\sum_{t=1}^{30}\left(x_t - \hat{\mu}_{30}\right)^2$. [Hint: Use the Excel function VARA(range).]

e. Does x_t have a fixed, constant population variance? If so, compute it. If not, explain why not.

(20.2) [Random walk, differencing] Consider the following random walk process with drift:

$$u_t = 3.6 + u_{t-1} + \varepsilon_t$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 16$. Assume $u_0 = 0$.

a. Show that $u_t = u_0 + 3.6\, t + \varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_t$.
b. Find a formula in terms of *t* for $E(u_t)$.
c. Find a formula in terms of *t* for $Var(u_t)$.
d. Given your answers to parts (b) and (c), is u_t stationary, trend stationary, or nonstationary? Why?
e. Show that $\Delta u_t = 3.6 + \varepsilon_t$.
f. Compute $E(\Delta u_t)$.
g. Compute $Var(\Delta u_t)$.
h. Given your answers to parts (f) and (g), is Δu_t stationary, trend stationary, or nonstationary? Why?

(20.3) [Random walk, differencing] Consider the following random walk process with drift:

$$u_t = 5.8 + u_{t-1} + \varepsilon_t$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 25$. Assume $u_0 = 0$.

a. Show that $u_t = u_0 + 5.8\, t + \varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_t$.
b. Find a formula in terms of *t* for $E(u_t)$.
c. Find a formula in terms of *t* for $Var(u_t)$.
d. Given your answers to parts (b) and (c), is u_t stationary, trend stationary, or nonstationary? Why?
e. Show that $\Delta u_t = 5.8 + \varepsilon_t$.
f. Compute $E(\Delta u_t)$.
g. Compute $Var(\Delta u_t)$.
h. Given your answers to parts (f) and (g), is Δu_t stationary, trend stationary, or nonstationary? Why?

(20.4) [Trend-stationary series]

a. Flip a coin thirty times (t=1,…,30) and compute x_t as follows. If the coin comes up "heads," set $x_t = t+1$. If the coin comes up "tails," set $x_t = t-1$. Report your values for $x_1, \ldots, x_{30}$. Here is an example.

t	1	2	3	4	5	6	7	etc.
Heads or tails?	H	T	T	H	H	H	T	etc.
x_t	2	1	2	5	6	7	6	etc.

b. Compute the sample means after subtracting the time trend for the first ten coin flips $\hat{\mu}_{10} = \frac{1}{10}\sum_{t=1}^{10}(x_t - t)$, the first twenty coin flips $\hat{\mu}_{20} = \frac{1}{20}\sum_{t=1}^{20}(x_t - t)$, and the first thirty coin flips $\hat{\mu}_{30} = \frac{1}{30}\sum_{t=1}^{30}(x_t - t)$.

c. Does $(x_t - t)$ have a population mean? If so, compute it. If not, explain why not.

d. Similarly, compute the sample variance after subtracting the time trend for the first ten coin flips $\hat{\sigma}^2_{10} = \frac{1}{10-1}\sum_{t=1}^{10}(x_t - \hat{\mu}_{10})^2$, the first twenty coin flips $\hat{\sigma}^2_{20} = \frac{1}{20-1}\sum_{t=1}^{20}(x_t - t)^2$, and the first thirty coin flips $\hat{\sigma}^2_{30} = \frac{1}{30-1}\sum_{t=1}^{30}(x_t - t)^2$.

e. Does $(x_t - t)$ have a population variance? If so, compute it. If not, explain why not.

(20.5) [Trended series, differencing] Consider the trended series:

$$u_t = 2.1 + 0.7\,t + \varepsilon_t$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 4$.

a. Find a formula in terms of *t* for $E(u_t)$.

b. Find a formula in terms of *t* for $Var(u_t)$.

c. Given your answers to parts (a) and (b), is u_t stationary, trend stationary, or nonstationary? Why?

d. Show that $\Delta u_t = 0.7 + \varepsilon_t - \varepsilon_{t-1}$.

e. Find an expression for $E(\Delta u_t)$.

f. Find an expression for $Var(\Delta u_t)$.

g. Given your answers to parts (e) and (f), is Δu_t stationary, trend stationary, or nonstationary? Why?

(20.6) [Trended series, differencing] Consider the trended series:

$$u_t = 6.7 + 0.4\ t + \varepsilon_t$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 9$.

a. Find a formula in terms of *t* for $E(u_t)$.
b. Find a formula in terms of *t* for $Var(u_t)$.
c. Given your answers to parts (a) and (b), is u_t stationary, trend stationary, or nonstationary? Why?
d. Show that $\Delta u_t = + 0.4 + \varepsilon_t - \varepsilon_{t-1}$.
e. Find an expression for $E(\Delta u_t)$.
f. Find an expression for $Var(\Delta u_t)$.
g. Given your answers to parts (e) and (f), is Δu_t stationary, trend stationary, or nonstationary? Why?

[end of problem set]