Problem Set 19
"Time Series: Testing and Correcting for Serial Correlation"

(19.1) [Durbin-Watson test] The time-series regression

$$\text{water consumption}_t = \beta_1 + \beta_2 \text{ population}_t + \varepsilon_t$$

was estimated with 30 annual observations. To check for serial correlation, a Durbin Watson statistic was calculated. It turned out to have a value of 0.8. Assume that *population* is strictly exogenous and the error term is normally distributed.

a. Does this value of the Durbin-Watson statistic suggest possible *positive* serial correlation or possible *negative* serial correlation? Explain your answer.
b. Use a table of the Durbin-Watson statistic at 5 percent significance to find the lower and upper confidence points for this regression. [A link to the table is given next to the link to this problem set. Notice that k' in the table denotes the number of regressors excluding the intercept, so in our notation, k' = K – 1.]
c. Test the null hypothesis of no serial correlation against the alternative hypothesis of positive serial correlation at 5% significance. Do you accept the null hypothesis, reject the null hypothesis, or is the test inconclusive?
d. Assuming serial correlation is present, use the value of the Durbin-Watson statistic to compute an estimate of the autocorrelation parameter ρ .

(19.2) [Durbin-Watson test] The time-series regression

$$\text{gas consumption}_t = \beta_1 + \beta_2 \text{ gas price}_{t2} + \beta_3 \text{ income}_t + \varepsilon_t$$

was estimated with 40 annual observations. To check for serial correlation, a Durbin Watson statistic was calculated. It turned out to have a value of 1.4. Assume that *gas price* and *income* are strictly exogenous and the error term is normally distributed.

a. Does this value of the Durbin-Watson statistic suggest possible *positive* serial correlation or possible *negative* serial correlation? Explain your answer.
b. Use a table of the Durbin-Watson statistic at 5 percent significance to find the lower and upper confidence points for this regression. [A link to the table is given next to the link to this problem set. Notice that k' in the table denotes the number of regressors excluding the intercept, so in our notation, k' = K – 1.]
c. Test the null hypothesis of no serial correlation against the alternative hypothesis of positive serial correlation at 5% significance. Do you accept the null hypothesis, reject the null hypothesis, or is the test inconclusive?
d. Assuming serial correlation is present, use the value of the Durbin-Watson statistic to compute an estimate of the autocorrelation parameter ρ .

(19.3) [Durbin's alternative test] Suppose we have estimated the regression

$$\text{energy}_t = \beta_1 + \beta_2 \text{ price}_t + \beta_3 \text{ energy}_{t-1} + \varepsilon_t$$

on a large sample, but we fear that ε_t may be serially correlated. Accordingly, we have used the residuals to estimate the following auxiliary regression (standard errors in parentheses).

$\hat{\varepsilon}_t$	=	0.035	–	0.012	price$_t$	+	0.025	energy$_{t-1}$	+	0.455	$\hat{\varepsilon}_{t-1}$
		(0.023)		(0.12)			(0.006)			(0.182)	

a. Compute the Durbin's alternative test statistic for testing the null hypothesis of no serial correlation against the alternative hypothesis of positive serial correlation.
b. Give the critical point at 5% significance from the appropriate table at the back of your textbook (or compute the p-value using a spreadsheet program). Can you reject the null hypothesis at 5% significance?
c. Give the critical point at 1% significance from the appropriate table at the back of your textbook (or compute the p-value using a spreadsheet program). Can you reject the null hypothesis at 1% significance?

(19.4) [Durbin's alternative test] Suppose we have estimated the regression

$$\text{consumption}_t = \beta_1 + \beta_2 \text{ income}_t + \beta_3 \text{ interest-rate}_t + \varepsilon_t$$

on a large sample, but we fear that ε_t may be serially correlated. Accordingly, we have used the residuals to estimate the following auxiliary regression (standard errors in parentheses).

$\hat{\varepsilon}_t$	=	0.12	+	0.27	$\hat{\varepsilon}_{t-1}$	–	0.05	income$_t$	+	0.05	Interest rate$_t$
		(0.05)		(0.18)			(0.16)			(0.02)	

a. Compute the Durbin's alternative test statistic for testing the null hypothesis of no serial correlation against the alternative hypothesis of positive serial correlation.
b. Give the critical point at 5% significance from the appropriate table at the back of your textbook (or compute the p-value using a spreadsheet program). Can you reject the null hypothesis at 5% significance?
c. Give the critical point at 1% significance from the appropriate table at the back of your textbook (or compute the p-value using a spreadsheet program). Can you reject the null hypothesis at 1% significance?

(19.5) [Breusch-Godfrey test] Suppose we have estimated the regression

$$\text{house price}_t = \beta_1 + \beta_2 \text{ GDP}_t + \beta_3 \text{ house price}_{t-1} + \beta_4 \text{ house price}_{t-2} + \varepsilon_t$$

using 101 annual time-series observations, but we fear that ε_t may be serially correlated. Accordingly, we have used the residuals to estimate the auxiliary regression

$$\hat{\varepsilon}_t = \alpha_1 + \alpha_2 GDP_t + \alpha_3 house\,price_{t-1} + \alpha_4 house\,price_{t-2} + \alpha_5 \hat{\varepsilon}_{t-1} + \nu_t$$

where ν_t denotes the error term in the auxiliary regression. (Note that this auxiliary regression must be estimated on observations 2 through 101 of the original data.) The R^2 value from this auxiliary regression is 0.058 .

a. Compute the Breusch-Godfrey LM statistic for testing the null hypothesis of no serial correlation, against the alternative of first-order serial correlation.
b. What are the degrees of freedom for this statistic?
c. Give the critical point at 5% significance from the appropriate table at the back of your textbook (or compute the p-value using a spreadsheet program). Can you reject the null hypothesis at 5% significance?
d. Give the critical point at 1% significance from the appropriate table at the back of your textbook (or compute the p-value using a spreadsheet program). Can you reject the null hypothesis at 1% significance?

(19.6) [Breusch-Godfrey test] Suppose we have estimated the regression

$$\text{inflation}_t = \beta_1 + \beta_2 \text{ money}_t + \beta_3 \text{ inflation}_{t-1} + \varepsilon_t$$

where *money* denotes the growth rate of the money supply, using 154 quarterly time-series observations. We fear that ε_t may be serially correlated. Accordingly, we have used the residuals to estimate the auxiliary regression

$$\hat{\varepsilon}_t = \alpha_1 + \alpha_2 \text{money}_t + \alpha_3 \text{inflation}_{t-1} + \alpha_4 \hat{\varepsilon}_{t-1} + \alpha_5 \hat{\varepsilon}_{t-2} + \alpha_6 \hat{\varepsilon}_{t-3} + \alpha_7 \hat{\varepsilon}_{t-4} + \nu_t$$

where ν_t denotes the error term in the auxiliary regression. (Note that this auxiliary regression must be estimated on observations 5 through 154 of the original data.) The R^2 value from this auxiliary regression is 0.064 .

a. Compute the Breusch-Godfrey LM statistic for testing the null hypothesis of no serial correlation, against the alternative of fourth-order serial correlation.
b. What are the degrees of freedom for this statistic?
c. Give the critical point at 5% significance from the appropriate table at the back of your textbook (or compute the p-value using a spreadsheet program). Can you reject the null hypothesis at 5% significance?
d. Give the critical point at 1% significance from the appropriate table at the back of your textbook (or compute the p-value using a spreadsheet program). Can you reject the null hypothesis at 1% significance?

(19.7) [Breusch-Godfrey test] Suppose the following time-series model is estimated: $y_t = 2.5 + 0.7x_{2t} + 0.5x_{3t} + \varepsilon_t$. To check for serial correlation using Durbin's alternative test, an auxiliary regression equation must be estimated. By mistake the following auxiliary equation is estimated: $\hat{\varepsilon}_t = \alpha_1 + \alpha_2 x_{2t} + \alpha_3 x_{3t} + \alpha_4 y_t + v_t$.

a. What is wrong with this auxiliary equation?
b. What R^2 value will be obtained if this equation is estimated?
c. What values of α_1, α_2, α_3, and α_4 will be obtained if this equation is estimated?

(19.8) [Quasi-differencing] Suppose we wish to estimate the time series regression

$$\text{consumption}_t = \beta_1 + \beta_2 \text{ income}_t + u_t.$$

However, we believe the error term is serially correlated, following the AR(1) process $u_t = 0.8\ u_{t-1} + \varepsilon_t$. We must transform the data to eliminate the serial correlation. The table below shows the first three observations on income_t and consumption_t. Compute transformed values of all three observations using the Prais-Winsten method.

Obs.	Raw data		Transformed data		
t	consumption_t	income_t	consumption_t	Replacement for intercept	income_t
1	25	15			
2	40	30			
3	50	35			

(19.9) [Quasi-differencing] Suppose we wish to estimate the time series regression

$$\text{investment}_t = \beta_1 + \beta_2 \text{ interest rate}_t + u_t.$$

However, we believe the error term is serially correlated, following the AR(1) processs $u_t = 0.6\ u_{t-1} + \varepsilon_t$. We must transform the data to eliminate the serial correlation. The table below shows the first three observations on *interest rate*$_t$ and *investment*$_t$. Compute transformed values of all three observations using the Prais-Winsten method.

Obs.	Raw data		Transformed data		
t	investment_t	interest rate_t	investment_t	Replacement for intercept	interest rate_t
1	15	5			
2	35	10			
3	40	15			

[end of problem set]