Econometrics Problem Sets
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Problem Set 18
"Time Series: Models of Serial Correlation"

(18.1) [MA(1) process] Consider the first-order moving average process:

$$u_t = \varepsilon_t + 0.4\ \varepsilon_{t-1}$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 4$.

a. Use the rules of the expectation operator $E(\ )$ to find $E(u_t)$.
b. Use the rules of the variance operator $Var(\ )$ to find $Var(u_t)$.
c. Find $Cov(u_t , u_{t-1})$. [Hint: In view of your answer to part (a), $Cov(u_t , u_{t-1}) = E(u_t\ u_{t-1})$. So multiply $(\varepsilon_t + 0.4\ \varepsilon_{t-1})$ times $(\varepsilon_{t-1} + 0.4\ \varepsilon_{t-2})$, and find the expected value term-by-term.]
d. Find $Cov(u_t, u_{t-2})$. [Hint: In view of your answer to part (a), $Cov(u_t , u_{t-2}) = E(u_t\ u_{t-2})$. So multiply $(\varepsilon_t + 0.4\ \varepsilon_{t-1})$ times $(\varepsilon_{t-2} + 0.4\ \varepsilon_{t-3})$, and find the expected value term-by-term.]
e. Explain why u_t is "weakly dependent."

(18.2) [MA(1) process] Consider the first-order moving average process:

$$u_t = \varepsilon_t + 0.2\ \varepsilon_{t-1}$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 4$.

a. Use the rules of the expectation operator $E(\ )$ to find $E(u_t)$.
b. Use the rules of the variance operator $Var(\ )$ to find $Var(u_t)$.
c. Find $Cov(u_t , u_{t-1})$. [Hint: In view of your answer to part (a), $Cov(u_t , u_{t-1}) = E(u_t\ u_{t-1})$. So multiply $(\varepsilon_t + 0.2\ \varepsilon_{t-1})$ times $(\varepsilon_{t-1} + 0.2\ \varepsilon_{t-2})$, and find the expected value term-by-term.]
d. Find $Cov(u_t, u_{t-2})$. [Hint: In view of your answer to part (a), $Cov(u_t , u_{t-2}) = E(u_t\ u_{t-2})$. So multiply $(\varepsilon_t + 0.2\ \varepsilon_{t-1})$ times $(\varepsilon_{t-2} + 0.2\ \varepsilon_{t-3})$, and find the expected value term-by-term.]
e. Explain why u_t is "weakly dependent."

(18.3) [MA(2) process] Consider the second-order moving average process:

$$u_t = \varepsilon_t + 0.7\,\varepsilon_{t-1} + 0.1\,\varepsilon_{t-2}$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 4$.

a. Use the rules of the expectation operator E( ) to find $E(u_t)$.
b. Use the rules of the variance operator Var( ) to find $Var(u_t)$.
c. Find $Cov(u_t , u_{t-1})$. [Hint: In view of your answer to part (a), $Cov(u_t , u_{t-1}) = E(u_t\, u_{t-1})$. So multiply $(\varepsilon_t + 0.7\,\varepsilon_{t-1} + 0.1\,\varepsilon_{t-2})$ times $(\varepsilon_{t-1} + 0.7\,\varepsilon_{t-2} + 0.1\,\varepsilon_{t-3})$, and find the expected value term-by-term.]
d. Find $Cov(u_t, u_{t-2})$. [Hint: In view of your answer to part (a), $Cov(u_t , u_{t-2}) = E(u_t\, u_{t-2})$. So multiply $(\varepsilon_t + 0.7\,\varepsilon_{t-1} + 0.1\,\varepsilon_{t-2})$ times $(\varepsilon_{t-2} + 0.7\,\varepsilon_{t-3} + 0.1\,\varepsilon_{t-4})$, and find the expected value term-by-term.]
e. Find $Cov(u_t, u_{t-3})$. [Hint: In view of your answer to part (a), $Cov(u_t , u_{t-3}) = E(u_t\, u_{t-3})$. So multiply $(\varepsilon_t + 0.7\,\varepsilon_{t-1} + 0.1\,\varepsilon_{t-2})$ times $(\varepsilon_{t-3} + 0.7\,\varepsilon_{t-4} + 0.1\,\varepsilon_{t-5})$, and find the expected value term-by-term.]
f. Explain why u_t is "weakly dependent."

(18.4) [MA(2) process] Consider the second-order moving average process:

$$u_t = \varepsilon_t + 0.5\,\varepsilon_{t-1} - 0.1\,\varepsilon_{t-2}$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 4$.

a. Use the rules of the expectation operator E( ) to find $E(u_t)$.
b. Use the rules of the variance operator Var( ) to find $Var(u_t)$.
c. Find $Cov(u_t , u_{t-1})$. [Hint: In view of your answer to part (a), $Cov(u_t , u_{t-1}) = E(u_t\, u_{t-1})$. So multiply $(\varepsilon_t + 0.5\,\varepsilon_{t-1} - 0.1\,\varepsilon_{t-2})$ times $(\varepsilon_{t-1} + 0.5\,\varepsilon_{t-2} - 0.1\,\varepsilon_{t-3})$, and find the expected value term-by-term.]
d. Find $Cov(u_t, u_{t-2})$. [Hint: In view of your answer to part (a), $Cov(u_t , u_{t-2}) = E(u_t\, u_{t-2})$. So multiply $(\varepsilon_t + 0.5\,\varepsilon_{t-1} - 0.1\,\varepsilon_{t-2})$ times $(\varepsilon_{t-2} + 0.5\,\varepsilon_{t-3} - 0.1\,\varepsilon_{t-4})$, and find the expected value term-by-term.]
e. Find $Cov(u_t, u_{t-3})$. [Hint: In view of your answer to part (a), $Cov(u_t , u_{t-3}) = E(u_t\, u_{t-3})$. So multiply $(\varepsilon_t + 0.5\,\varepsilon_{t-1} - 0.1\,\varepsilon_{t-2})$ times $(\varepsilon_{t-3} + 0.5\,\varepsilon_{t-4} - 0.1\,\varepsilon_{t-5})$, and find the expected value term-by-term.]
f. Explain why u_t is "weakly dependent."

(18.5) [AR(1) process] Consider the first-order autoregressive process:

$$u_t = 0.4\, u_{t-1} + \varepsilon_t$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 6$. Assume u_t is stationary and $Cov(u_{t-1},\varepsilon_t) = 0$.

a. Use the rules of the expectation operator E( ) (or the formulas in the slideshows or the textbook) to find $E(u_t)$.
b. Use the rules of the variance operator Var( ) (or the formulas in the slideshows or the textbook) to find $Var(u_t)$.
c. Find $Cov(u_t , u_{t-1})$ and $Corr(u_t , u_{t-1})$.
d. Find $Cov(u_t, u_{t-2})$ and $Corr(u_t , u_{t-2})$.
e. Find $Cov(u_t, u_{t-10})$ and $Corr(u_t , u_{t-10})$.
f. Explain why u_t is "weakly dependent."

(18.6) [AR(1) process] Consider the first-order autoregressive process:

$$u_t = 0.6\, u_{t-1} + \varepsilon_t$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 6$. Assume u_t is stationary and $Cov(u_{t-1},\varepsilon_t) = 0$.

a. Use the rules of the expectation operator E( ) (or the formulas in the slideshows or the textbook) to find $E(u_t)$.
b. Use the rules of the variance operator Var( ) (or the formulas in the slideshows or the textbook) to find $Var(u_t)$.
c. Find $Cov(u_t , u_{t-1})$ and $Corr(u_t , u_{t-1})$.
d. Find $Cov(u_t, u_{t-2})$ and $Corr(u_t , u_{t-2})$.
e. Find $Cov(u_t, u_{t-10})$ and $Corr(u_t , u_{t-10})$.
f. Explain why u_t is "weakly dependent."

(18.7) [AR(1) process] Consider the first-order autoregressive process:

$$u_t = 0.2\, u_{t-1} + \varepsilon_t$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 6$. Assume u_t is stationary and $Cov(u_{t-1},\varepsilon_t) = 0$.

a. Use the rules of the expectation operator E( ) (or the formulas in the slideshows or the textbook) to find $E(u_t)$.
b. Use the rules of the variance operator Var( ) (or the formulas in the slideshows or the textbook) to find $Var(u_t)$.
c. Find $Cov(u_t , u_{t-1})$ and $Corr(u_t , u_{t-1})$.
d. Find $Cov(u_t, u_{t-2})$ and $Corr(u_t , u_{t-2})$.
e. Find $Cov(u_t, u_{t-10})$ and $Corr(u_t , u_{t-10})$.
f. Explain why u_t is "weakly dependent."

(18.8) [AR, MA, and ARMA processes] Let ε_t denote an independent, identically-distributed series with $E(\varepsilon_t) = 0$. For each process below, indicate whether the first autocorrelation $Corr(u_t, u_{t-1})$ is nonzero, whether the second autocorrelation $Corr(u_t, u_{t-2})$ is nonzero, and whether the tenth autocorrelation $Corr(u_t, u_{t-10})$ is nonzero.

a. $u_t = \varepsilon_t + 0.3\ \varepsilon_{t-1}$
b. $u_t = \varepsilon_t + 0.2\ \varepsilon_{t-1} + 0.1\ \varepsilon_{t-2}$
c. $u_t = 0.5\ u_{t-1} + \varepsilon_t$
d. $u_t = 0.4\ u_{t-1} + 0.2\ u_{t-2} + \varepsilon_t$
e. $u_t = 0.4\ u_{t-1} + 0.2\ u_{t-2} + \varepsilon_t + 0.3\ \varepsilon_{t-1}$

[end of problem set]