# Problem Set 17
# "Time Series: Functional Forms, Trends, and Seasonality"

GENERAL INSTRUCTIONS: Assume for this problem set that all error terms are homoskedastic, not serially-correlated, and normally-distributed. Assume all regressors are strictly exogenous.

(17.1) [Time trend] Suppose the following equation is estimated using annual data

$$\ln(q) = 19.1 - 0.32 \ln(p) + 0.021 \text{ time}$$

Here, *q* denotes annual gasoline consumption, *p* denotes the real price of gasoline, and *time* denotes a time trend equal to 1 in the first observation, 2 in the second observation, etc.

a. According to this specification, is the price elasticity of demand for gasoline constant or not constant? If it is not constant, explain why not. If it is constant, give its value.
b. Does this specification include a linear time trend, an exponential time trend, or neither?
c. According to this specification, if the real price of gasoline were held constant, does gasoline consumption rise by a *constant amount* or a *constant percentage*, from one year to the next? How much?

(17.2) [Quadratic time trend] Suppose we have estimated the following model with a quadratic time trend:

$$GDP_t = 5.7 + 0.9 \text{ employment}_t - 0.03 \text{ t} + 0.0002 \text{ t}^2 + \varepsilon_t$$

a. Find an expression for $dy_t/dt$ and show that it depends on t.
b. For what values of t is the trend positive?
c. For what values of t is the trend negative?

(17.3) [Time trend] Suppose we try to estimate the following equation with lagged regressors:

$$y_t = \beta_1 + \beta_2 x_t + \beta_3 x_{t-1} + \beta_4 t + \beta_5 s + \varepsilon_t ,$$

where $s = t-1$ is the lagged time trend.

a. Some smart computer programs will refuse to estimate the equation. Why?
b. Other, cruder computer programs (such as Excel) will produce estimates but with some strange standard errors. What will be strange about the standard errors? Which coefficients will be affected: $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$ or $\beta_5$ ?
c. What should be done to fix this problem and estimate the equation?

(17.4) [Seasonality] Suppose we try to estimate the following time-series equation for movie admissions, using quarterly data:

$$\text{admissions} = \beta_1 + \beta_2\ t + \beta_3\ q1 + \beta_4\ q2 + \beta_5\ q3 + \beta_6\ q4 + \varepsilon_t\ .$$

Here, *admissions* denotes the number of admissions each quarter in thousands, *t* is a quarterly time trend, *q1* is a dummy variable equal to one in the first quarter and zero otherwise, *q2* is a dummy variable equal to one in the second quarter and zero otherwise, and *q3* is a dummy variable equal to one in the third quarter and zero otherwise.

a. Some Smart computer programs will refuse to estimate the equation. Why?
b. Other, cruder computer programs (such as Excel) will produce estimates but with some strange standard errors. What will be strange about the standard errors? Which coefficients will be affected: $\beta_1$ , $\beta_2$ , $\beta_3$ , $\beta_4$ , $\beta_5$ or $\beta_6$ ?
c. What should be done to fix this problem and estimate the equation?

(17.5) [Seasonality] Suppose we have estimated the following time-series equation for Postal Service letter volume, using quarterly data:

$$\ln(\text{letters}) = 5.3 - 0.03\ q1 + 0.01\ q2 - 0.04\ q3$$

Here, *letters* denotes the volume of letters delivered each quarter in millions, *q1* is a dummy variable equal to one in the first quarter and zero otherwise, *q2* is a dummy variable equal to one in the second quarter and zero otherwise, and *q3* is a dummy variable equal to one in the third quarter and zero otherwise.

a. Which is the reference quarter?
b. Is volume higher in the first quarter or the second quarter? By about how much? Is your answer in millions or percent?
c. Is volume higher in the second quarter or the third quarter? By about how much?
d. Is volume higher in the third quarter or the fourth quarter? By about how much?
e. Is volume higher in the fourth quarter or the first quarter? By about how much?

(17.6) [Testing for seasonality] Suppose we have estimated a model using 95 observations using quarterly data with and without seasonal dummy variables. The coefficient estimates and sums of squared residuals for the two specifications are given below.

$$\text{sales} = 2.7 + 0.41\ \text{advertising} - 0.31\ \text{q1} - 0.12\ \text{q2} - 0.44\ \text{q3} \qquad \text{SSR} = 360$$

$$\text{sales} = 2.5 + 0.37\ \text{advertising} \qquad \text{SSR} = 402.$$

Here, *adv* denotes spending on advertising, *q1* is a dummy variable taking the value one in the first quarter and zero in other quarters. Similarly, *q2* is a dummy variable for the second quarter, and *q3* is a dummy variable for the third quarter. We wish to test the null hypothesis that no seasonality is present in the data.

a. Which is the reference quarter?
b. Compute the relevant F-statistic for testing the null hypothesis that there is no seasonality.
c. Give the degrees of freedom in the numerator and the denominator.
d. Give the critical points at 5 percent significance and 1 percent significance from the appropriate tables at the back of your textbook (or compute the p-value using a spreadsheet program).
e. Can you reject the null hypothesis at 5 percent significance?
f. Can you reject the null hypothesis at 1 percent significance?

(17.7) [Forecasting with time trend and seasonality] Suppose we have estimated the following time-series equation for traffic volume on a particular highway, using quarterly data:

$$\text{vehicles} = 4.7 + 0.02\,\text{t} - 0.1\ \text{q2} + 0.2\ \text{q3} + 0.1\ \text{q4} + \varepsilon_t\,.$$

$$\hat{\sigma}_\varepsilon^2 = 0.1$$

Here, *vehicles* denotes the number of vehicles each quarter in millions, *t* is a time trend that increases by 1 each quarter, *q1* is a dummy variable equal to one in the first quarter and zero otherwise, *q2* is a dummy variable equal to one in the second quarter and zero otherwise, and *q3* is a dummy variable equal to one in the third quarter and zero otherwise. The last observation in our sample is for the fourth quarter of 2023, when $t = 40$.

a. Which is the reference quarter?
b. Predict traffic volume for the first quarter of 2024.
c. Compute a 95% prediction interval for traffic volume in the first quarter of 2024.
d. Predict traffic volume for the second quarter of 2024.
e. Compute a 95% prediction interval for traffic volume in the second quarter of 2024.

[end of problem set]