Problem Set 16
"Time Series: Regression With Strictly Exogenous Errors"

(16.1) [Static model versus lagged model] For each of the following economic relationships, determine which would be more sensible: a *static model* or a *finite distributed lag*. Assume we have annual data. Justify your choices by referring to the likely time required for economic agents to adjust to price changes.

a. Demand for apples.
b. Demand for natural gas.
c. Supply of new houses.

(16.2) [Finite distributed lag] Suppose we estimate the following model with a finite distributed lag:

$$UR_t = \beta_1 + \beta_2 \text{ GDP GR}_t + \beta_3 \text{ GDP GR}_{t-1} + \varepsilon_t$$

Here, "UR" is the unemployment rate and "GDP GR" is the growth rate of real GDP. We wish to test the hypothesis that the long-run multiplier (also called the "long-run propensity") equals zero. We need to rearrange the regression equation so that the required t-statistic can be produced automatically even by a simple computer regression program.

a. Let θ denote the long-run multiplier. Give an equation for θ in terms of the βs.
b. Solve the equation you found in part (a) for β_2 in terms of θ and β_3.
c. Substitute your answer to part (b) into the regression equation and rearrange that equation so that θ is the coefficient of a transformed regressor. [Hint: See the slideshow on "t-Tests Involving More Than One Coefficient" for examples of how to do this.]

(16.3) [Finite distributed lag] Suppose we estimate the following model with a finite distributed lag:

$$gas_t = \beta_1 + \beta_2 \text{ gas price}_t + \beta_3 \text{ gas price}_{t-1} + \beta_4 \text{ gas price}_{t-2} + \varepsilon_t$$

Here, "gas" is gasoline consumption and "gas price" is the price of gasoline. We wish to test the hypothesis that the long-run multiplier (also called the "long-run propensity") equals zero. We need to rearrange the regression equation so that the required t-statistic can be produced automatically even by a simple computer regression program.

a. Let θ denote the long-run multiplier. Give an equation for θ in terms of the βs.
b. Solve the equation you found in part (a) for β_2 in terms of θ, β_3, and β_4.
c. Substitute your answer to part (b) into the regression equation and rearrange that equation so that θ is the coefficient of a transformed regressor. [Hint: See the slideshow on "t-Tests Involving More Than One Coefficient" for examples of how to do this.]

(16.4) [Test static model versus distributed lag] Suppose we wished to test a static model versus a finite distributed lag for a given model and dataset. Assume we have 64 observations on y_t and all the observations we need, including lags, on x_t. The two models are shown below.

$$\text{(i) } INF_t = \beta_1 + \beta_2 M_t + \beta_3 M_{t-1} + \beta_4 M_{t-2} + \varepsilon_t$$

$$\text{(ii) } INF_t = \beta_1 + \beta_2 M_t + \varepsilon_t$$

Here, "INF" denotes the inflation rate and "M" denotes the growth rate of the money supply.

a. Which model is the restricted model? Which is the unrestricted model?
b. What are the restrictions to be tested? (Give equations involving the βs.)
c. Suppose both models have been estimated on all 64 observations. The sum of squared residuals for model (i) was 195 and the sum of squared residuals for model (ii) was 221. Test model (i) against (ii) using an F test at 5% significance. Give
   - the degrees of freedom in the numerator
   - the degrees of freedom in the denominator
   - the value of the relevant F-statistic
   - the *critical point* from the appropriate table at the back of your textbook (or compute the *p-value* using a spreadsheet program)
   - your conclusion: whether you reject the null hypothesis at 5% significance.

(16.5) [Test static model versus distributed lag] Suppose we wished to test a static model versus a finite distributed lag for a given model and dataset. Assume we have 50 observations on y_t and all the observations we need, including lags, on x_t. The two models are shown below.

$$\text{(i) } ENR_t = \beta_1 + \beta_2 WGAP_t + \beta_3 WGAP_{t-1} + \beta_4 WGAP_{t-2} + \beta_5 WGAP_{t-3} + \varepsilon_t$$

$$\text{(ii) } ENR_t = \beta_1 + \beta_2 WGAP_t + \varepsilon_t$$

Here, "ENR" denotes enrollment in college and "WGAP" denotes the gap in wages between workers with college degrees and workers without college degrees..

a. Which model is the restricted model? Which is the unrestricted model?
b. What are the restrictions to be tested? (Give equations involving the βs.)
c. Suppose both models have been estimated on all 50 observations. The sum of squared residuals for model (i) was 135 and the sum of squared residuals for model (ii) was 153. Test model (i) against (ii) using test at 5% significance. Give
   - the degrees of freedom in the numerator
   - the degrees of freedom in the denominator
   - the value of the relevant F-statistic
   - the *critical point* from the appropriate table at the back of your textbook (or compute the *p-value* using a spreadsheet program)
   - your conclusion: whether you reject the null hypothesis at 5% significance.

[end of problem set]