Econometrics Problem Sets
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Problem Set 11
"Two-Variable Regression: Practical Issues"

(11.1) [Functional forms] Consider the following five equations. Assume x and y are positive.

(i)	$y = 3 + 2\,x$
(ii)	$y = 3 + 2\,(1/x)$
(iii)	$y = 3 + 2\,\ln(x)$
(iv)	$\ln(y) = 3 + 0.02\,x$
(v)	$\ln(y) = 3 + 2\,\ln(x)$

For which equation is each of the following statements true?

a. A one percent increase in x causes a two percent increase in y.
b. The equation relating y and x is a straight line.
c. As x approaches infinity, y approaches 3.
d. A one-unit increase in x causes a two percent increase in y.
e. An increase in x causes a decrease in y.
f. The elasticity of y with respect to x is constant and equal to 2.

(11.2) [Functional forms] Consider the following five equations. Assume x and y are positive.

(i)	$y = 5 + 3\,\ln(x)$
(ii)	$\ln(y) = 5 + 0.03\,x$
(iii)	$\ln(y) = 5 + 3\,\ln(x)$
(iv)	$y = 5 + 3\,x$
(v)	$y = 5 + 3\,(1/x)$

For which equation is each of the following statements true?

a. An increase in x causes a decrease in y.
b. The elasticity of y with respect to x is constant and equal to 3.
c. A one percent increase in x causes a three percent increase in y.
d. The equation relating y and x is a straight line.
e. As x approaches infinity, y approaches 5.
f. A one-unit increase in x causes a three percent increase in y.

(11.3) [Confidence intervals, t-tests, functional forms] Suppose the relationship between a worker's weekly earnings and educational background is modeled as a linear relationship $y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$, where y denotes the natural log of weekly earnings and x denotes years of education. This equation is estimated by least-squares on a sample of several hundred workers, with the following results. (The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors.)

Natural log of weekly earnings	=	4.75 (0.50)	+	0.15 (0.06)	Years of education

a. According to these results, if a person acquires one more year of education, then by how much does the *natural logarithm of earnings* increase? How much do *earnings* increase?
b. Compute a 95% confidence interval for the intercept.
c. Test the null hypothesis that education has no effect on earnings, against the alternative hypothesis that education has a positive effect (a one-tailed test) at 5% significance. Give
   - the *value* of the test statistic
   - the *critical point* from the appropriate table at the back of your textbook (or compute the *p-value* using a spreadsheet program)
   - your conclusion: whether you reject the null hypothesis at 5% significance.

(11.4) [Elasticity, units of measure] Suppose the relationship between household income and electricity usage is modeled as a linear relationship $y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$, where y denotes annual electricity use (in kilowatt hours) and x denotes annual household income in dollars. This equation is estimated by least-squares on a sample of several thousand households, with the following results. (The numbers on top are the least-squares estimates of the intercept and slope, and the numbers in parentheses are standard errors.)

Electricity usage in kilowatt-hours	=	11200.0 (637.0)	+	0.49 (0.06)	Household income in dollars

a. According to these results, if household income increases by $1000, then by how much does electricity usage increase?
b. In this sample, the mean household income in the sample was $40,000 and the mean electricity usage was 28,000 kilowatt hours. Compute an estimate of the income elasticity of demand for electricity at the sample mean. [Hint: Recall that the income elasticity of demand is the percent change in quantity demanded divided by the percent change in income.]
c. Suppose that electricity usage data had been reported in *megawatt-hours* instead of kilowatt-hours. (One megawatt-hour equals 1000 kilowatt-hours.) Compute the resulting least-squares estimate of the slope β_2 . Also compute the resulting value of the income elasticity of demand.

(11.5) [Exact confidence intervals, t-tests, functional forms] Suppose the demand for water is estimated by least squares using a sample of 20 communities. Assume the error term is normally-distributed. (The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors.)

Natural log of water use per capita	=	7.00 (1.40)	-	0.22 (0.10)	Natural log of price of water

a. According to these results, if the price of water increases by ten percent, does water use per capita increase or decrease? By how much (in percent)?
b. Compute a 95% confidence interval for the intercept.
c. Test the null hypothesis that water demand is perfectly inelastic, against the alternative hypothesis that water demand is negatively related to price (a one-tailed test) at 5% significance. Give
   - the *value* of the test statistic
   - the *critical point* from the appropriate table at the back of your textbook (or compute the *p-value* using a spreadsheet program)
   - your conclusion: whether you reject the null hypothesis at 5% significance.

(11.6) [Units of measure] Suppose the relationship between the price of a television and the size of its screen is estimated by least squares using a large sample, with the following results. (The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors.)

Price of television	=	-5.61 (1.40)	+	6.05 (0.85)	Size of screen in inches

a. According to these results, if the size of a television screen increases by two inches, by how much does the price increase?
b. Suppose the size of the screen were measured in centimeters instead of inches. (Assume 2.5 centimeters ≈ 1 inch.) Compute the new least-squares estimates of the intercept and slope.
c. Suppose the size of the screen were still measured in inches, but the price were measured in cents instead of dollars. (That is, if the price of model i were \$150, then $y_i = 15000$.) Compute the new least-squares estimates of the intercept and slope.

[end of problem set]