Problem Set 5
"Probability: Important Distributions"

(5.1) [Bernoulli random variable] Suppose the variable X is a Bernoulli random variable, equal to 1 with probability P, and equal to 0 with probability (1-P). Here, P is a number between zero and one.

a. Prove that E(X) = P. Justify each step of your proof.
b. Prove that Var(X) = P(1-P). Justify each step of your proof.
c. Use calculus to find the value of P that maximizes the Var(X).
d. Compute the maximum value of Var(X).

(5.2) [Distributions related to the normal] Which of the following distributions have *symmetric* distributions covering the entire real line? Which of the following distributions have *asymmetric* density functions covering only the positive half of the real line?

a. Normal distribution.
b. Chi-square distribution.
c. "t" distribution.
d. "F" distribution.

(5.3) [Standard normal statistical table] Use the tables of critical values in the back of your textbook to answer the following questions. Suppose Z is a standard normal random variable. [Hint: The last row of the table of the t-distribution, with degrees of freedom = ∞, shows the standard normal distribution.]

a. If Prob{Z>z}=0.05, then what number is z?
b. If Prob{|Z|>z}=0.05, then what number is z? [Hint: Remember that the distribution is symmetric around zero.]
c. Why is your answer to (b) greater than your answer to (a)?
d. If Prob{|Z|>z}=0.01, then what number is z?

(5.4) [Chi-square statistical table] Use the tables of critical values in the back of your textbook to answer the following questions. Suppose Y is a random variable with an "chi-square" distribution with 12 degrees of freedom.

a. If Prob{Y>y)=0.05, then what number is y?
b. If Prob{Y>y)=0.01, then what number is y?
c. Why is your answer to (b) greater than your answer to (a)?

(5.5) [“t” distribution statistical table] Use the tables of critical values in the back of your textbook to answer the following questions. Suppose W is a random variable with a “t” distribution with 15 degrees of freedom.

a. If Prob{W>w)=0.05, then what number is w?
b. If Prob{|W|>w)=0.01, then what number is w? [Hint: Remember that this distribution is symmetric around zero.]

(5.6) [“F” distribution statistical table] Use the tables of critical values in the back of your textbook to answer the following questions. Suppose V is a random variable with an “F” distribution with 5 degrees of freedom in the numerator and 25 degrees of freedom in the denominator.

a. If Prob{V>v)=0.05, then what number is v?
b. If Prob{V>v)=0.01, then what number is v?

(5.7) [Functions of normal random variables] Fill in the blanks below.

a. If $X_1 \sim N(2,3)$ then $X_2 = (3X_1+5)$ has a __________ distribution with mean $E(X_2)$ = _______ and variance $Var(X_2)$ = _______.
b. If $X_1 \sim N(2,3)$ and $X_2 \sim N(6,4)$, and X_1 and X_2 are independent, then $X_3 = (X_1+X_2)$ has a __________ distribution with mean $E(X_3)$ = _______ and variance $Var(X_3)$ = _______.
c. If X_1, X_2, and X_3 are independent normal random variables, each with mean equal to zero and variance equal to one, then $X_4 = (X_1^2+X_2^2+X_3^2)$ has a ____________ distribution with _______ degrees of freedom.

(5.8) [Functions of chi-square random variables] Fill in the blanks below.

a. If X_1 has a chi-square distribution with 2 degrees of freedom, and X_2 has a chi-square distribution with 4 degrees of freedom, and X_1 and X_2 are independent, then $X_3 = (X_1+X_2)$ has a __________ distribution with _______ degrees of freedom.
b. If X_1 has a chi-square distribution with 3 degrees of freedom, and X_2 has a chi-square distribution with 12 degrees of freedom, and X_1 and X_2 are independent, then $X_3 = (4X_1/X_2)$ has a __________ distribution with _______ and _______ degrees of freedom.

[end of problem set]