Econometrics Problem Sets
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Problem Set 4
"Probability: Moments"

(4.1) [Mean, variance, standard deviation] Suppose the random variable X takes just three values, with the following probabilities.

Value of X	Probability
1	0.5
3	0.25
11	0.25

a. Compute the mean of X, E(X).
b. Compute the variance of X, Var(X).
c. Compute the standard deviation of X, SD(X).

(4.2) [Mean, variance, standard deviation] Suppose the random variable X takes four values, with the following probabilities.

Value of X	Probability
3	0.2
5	0.4
10	0.3
14	0.1

a. Compute the mean of X, E(X).
b. Compute the variance of X, Var(X).
c. Compute the standard deviation of X, SD(X).

(4.3) [Moments of linear function] Suppose a random variable X has mean 5 and variance 4. Let the random variable Y = 3X + 3.

a. Compute the mean of Y, that is, E(Y).
b. Compute the variance of Y, that is Var(Y).
c. Compute the standard deviation of Y, that is, SD(Y).

(4.4) [Moments of linear function] Suppose a random variable X has mean 2 and variance 25. Let the random variable Y = 7X + 4.

a. Compute the mean of Y, that is, E(Y).
b. Compute the variance of Y, that is Var(Y).
c. Compute the standard deviation of Y, that is, SD(Y).

(4.5) [Mean, variance, covariance] Consider the following table, which shows the joint probabilities of discrete random variables X and Y.

		Y	
		1	6
X	4	P=0.2	P=0.4
	9	P =0.2	P=0.2

a. Compute the expectation E(X).
b. Compute the expectation E(Y).
c. Compute the variance Var(X).
d. Compute the variance Var(Y).
e. Compute the covariance Cov(X,Y).

(4.6) [Conditional expectation] Consider the following table, which shows the joint probabilities of discrete random variables X and Y.

			Y	
		1	2	3
X	1	P=0.1	P=0.2	P=0.1
	2	P =0.3	P=0.1	P=0.2

a. Compute the conditional expectation E(Y|X=1).
b. Compute the conditional expectation E(Y|X=2).
c. Compute the unconditional expectation E(Y).
d. Explain why your answers to (a), (b), and (c) are different.

(4.7) [Covariance versus dependence] Consider the following probability table for the random variables X and Y.

			Y	
		1	2	3
X	1	P=0	P=0.5	P=0
	2	P =0.25	P=0	P=0.25

a. Are X and Y independent? Why or why not?
b. Compute the unconditional expectations E(X), and E(Y) and the covariance of X and Y, Cov(X,Y).
c. Are random variables with zero covariance necessarily independent? Comment in a few sentences based on this example.

(4.8) [Joint distribution, moments] We are given the following information about the moments of three random variables, X_1, X_2, and X_3.

$E(X_1) = 12$	$Var(X_1) = 4$	$Cov(X_1, X_2) = 3$
$E(X_2) = 20$	$Var(X_2) = 9$	

a. Compute $E(X_1 + X_2)$.
b. Compute $Var(X_1 + X_2)$.
c. Compute $SD(X_1 + X_2)$.
d. Compute $Corr(X_1, X_2)$.

(4.9) [Joint distribution, moments] We are given the following information about the moments of three random variables, X_1, X_2, and X_3.

$E(X_1) = 12$	$Var(X_1) = 16$	$Cov(X_1, X_2) = -2$
$E(X_2) = 20$	$Var(X_2) = 9$	$Cov(X_2, X_3) = 3$
$E(X_3) = 5$	$Var(X_3) = 12$	$Cov(X_3, X_1) = 1$

a. Compute $E(X_1 + X_2 + X_3)$.
b. Compute $Var(X_1 + X_2 + X_3)$.
c. Compute $Corr(X_1, X_2)$.

(4.10) [Moments of sum] Suppose $X_1, \ldots, X_{36}$ are random variables with identical mean μ and variance σ^2. Let $Y = \sum_{i=1}^{36} X_i$.

a. Use the properties of expectation to prove that $E(Y) = 36\mu$. Justify each step of your proof.
b. Use the properties of variance to prove, step-by-step, that, if $Cov(X_i,X_j) = 0$ for all $i \neq j$, then $Var(Y) = 36\sigma^2$. Justify each step of your proof.
c. Use your answer to (b) to find a formula for the standard deviation of Y, SD(Y).

(4.11) [Moments of sum] Suppose $X_1, \ldots, X_{25}$ are random variables with identical mean 7 and variance 4. Assume $Cov(X_i,X_j) = 0$ for all $i \neq j$. Let $Y = \sum_{i=1}^{25} X_i$.

a. Compute the mean of Y, that is, E(Y).
b. Compute the variance of Y, that is, Var(Y).
c. Compute the standard deviation of Y, that is, SD(Y).

(4.12) [Moments of sum] Suppose $X_1, \ldots, X_{400}$ are random variables with identical mean 5 and variance 9. Assume $Cov(X_i,X_j) = 0$ for all $i \neq j$. Let $Y = \sum_{i=1}^{400} X_i$.

a. Compute the mean of Y, that is, E(Y).
b. Compute the variance of Y, that is, Var(Y).
c. Compute the standard deviation of Y, that is, SD(Y).

(4.13) [Moments of sample mean] Suppose $X_1, \ldots, X_{225}$ are random variables with identical mean μ and variance σ^2 . Let $\overline{X} = \frac{1}{225}\sum_{i=1}^{225} X_i$.

a. Use the properties of expectation to prove that $E(\overline{X}) = \mu$. Justify each step of your proof.
b. Use the properties of variance to prove that, if $Cov(X_i,X_j) = 0$ for all $i \neq j$, then $Var(\overline{X}) = (\sigma^2 / 225)$. Justify each step of your proof.
c. Use your answer to (b) to find a formula for $SD(\overline{X})$.

(4.14) [Moments of sample mean] Suppose $X_1, \ldots, X_{50}$ are random variables with identical mean 11 and variance 12.5. Assume $Cov(X_i,X_j) = 0$ for all $i \neq j$. Let $\overline{X} = \frac{1}{50}\sum_{i=1}^{50} X_i$.

a. Compute $E(\overline{X})$.
b. Compute $Var(\overline{X})$.
c. Compute $SD(\overline{X})$.

(4.15) [Moments of sample mean] Suppose $X_1, \ldots, X_{900}$ are random variables with identical mean 13 and variance 36. Assume $Cov(X_i,X_j) = 0$ for all $i \neq j$. Let $\overline{X} = \frac{1}{900}\sum_{i=1}^{900} X_i$.

a. Compute $E(\overline{X})$.
b. Compute $Var(\overline{X})$.
c. Compute $SD(\overline{X})$.

[end of problem set]