Econometrics Problem Sets
William M. Boal

Problem Set 3
"Probability: Random Variables"

(3.1) [Random variables] Which of the following is a random variable in the classical sense? Justify each answer.

a. Your age.
b. The age of your professor.
c. The age of someone chosen by throwing a dart at a page in the telephone book.
d. The average age of five persons chosen by throwing five darts at a telephone book.

(3.2) [Random variables] Which of the following is a random variable in the classical sense? Justify each answer.

a. Your income last year.
b. The income last year of someone chosen by putting the names of all the students in this class into a hat and drawing one name.
c. The average incomes last year of three students chosen by putting the names of all the students in this class into a hat and drawing three names.
d. The President's income last year.

(3.3) [Discrete versus continuous random variables] Suppose we select a household at random. Should the following characteristics be modeled as *discrete or continuous* random variables? Why?

a. Number of people in the household.
b. Age of the head of household.
c. Household income.
d. Number of cars owned.
e. Whether the household has high-speed internet service.

(3.4) [Discrete versus continuous random variables] Suppose we select a firm at random. Should the following characteristics be modeled as *discrete or continuous* random variables? Why?

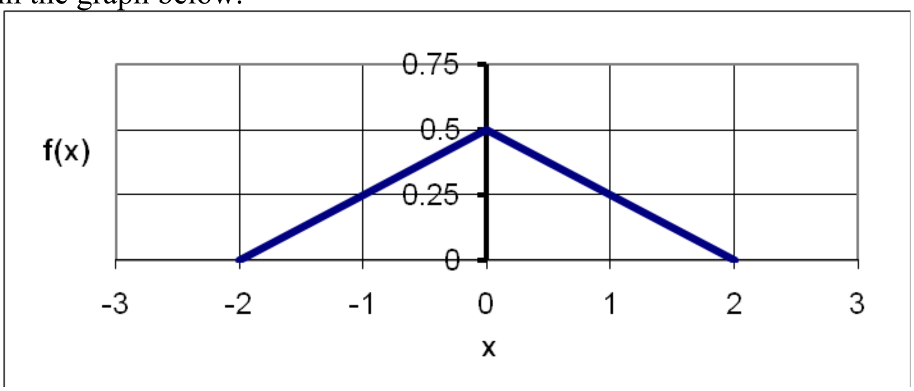
a. Number of employees in the firm.
b. Whether the firm is unionized.
c. The firm's revenue.
d. Number of profitable years out of the last five years.
e. Number of outside (non-management) members of the Board of Directors.

(3.5) [Probability distributions] Suppose the random variable X has the probability function shown in the following table.

x	-3	-2	-1	0	1	2	3
Prob{X=x}	0.1	0.1	0.2	0.2	0.2	0.1	0.1

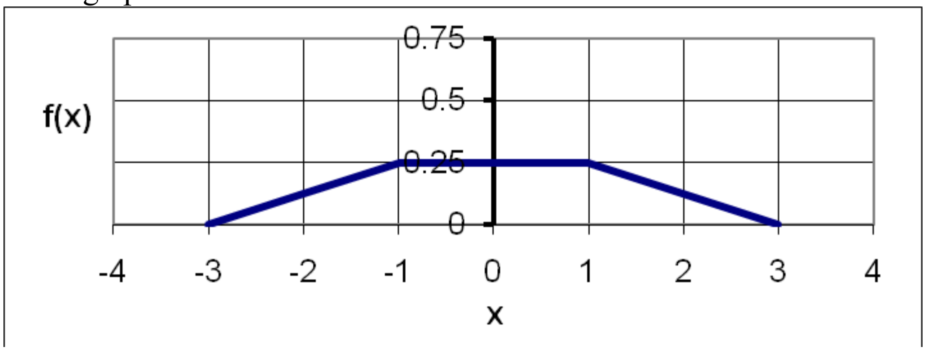
a. Is the random variable X discrete or continuous? Why?
b. Verify that the total probability of all possible values of X sums to one. Show your work.
c. Compute Prob{X < 0}.
d. Compute Prob{-2 ≤ X ≤ 2}.
e. Compute Prob{X ≥ 2}.
f. Compute Prob{|X| ≥ 2}.

(3.6) [Probability distributions] Suppose the random variable X has the density function depicted in the graph below.



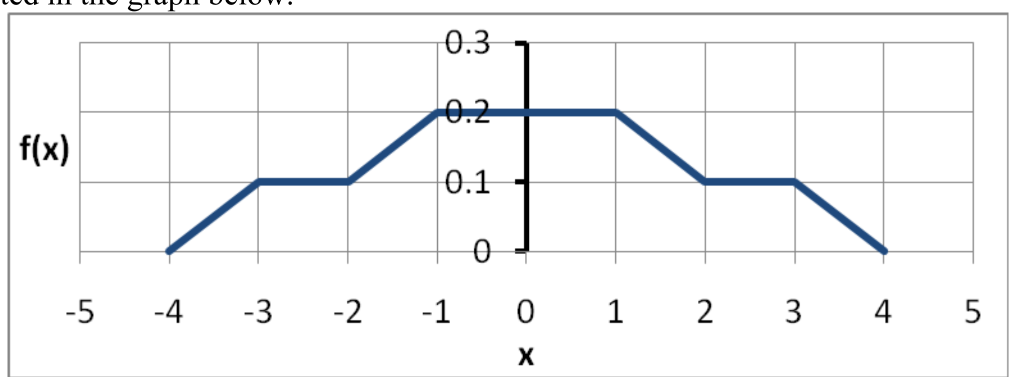
a. Is the random variable X discrete or continuous? Why?
b. Verify that the area under the density function sums to one. Show your work.
c. Compute Prob{X < 0}.
d. Compute Prob{-1 < X < 1}.
e. Compute Prob{X > 1}.
f. Compute Prob{|X| > 1}.

(3.7) [Probability distributions] Suppose the random variable X has the density function depicted in the graph below.



a. Is the random variable X discrete or continuous? Why?
b. Verify that the area under the density function sums to one. Show your work.
c. Compute Prob{X < 0}.
d. Compute Prob{-1 < X < 1}.
e. Compute Prob{X > 1}.
f. Compute Prob{|X| > 1}.

(3.8) [Probability distributions] Suppose the random variable X has the density function depicted in the graph below.



a. Is the random variable X discrete or continuous? Why?
b. Verify that the area under the density function sums to one. Show your work.
c. Compute Prob{X < 0}.
d. Compute Prob{-2 < X < 2}.
e. Compute Prob{X > 2}.
f. Compute Prob{|X| > 2}.

(3.9) [Joint distributions] Suppose two fair dice are rolled. The dice may be assumed to be independent from each other.

a. What is the probability of rolling "snake eyes" (1,1)?
b. What is the probability of rolling "box cars" (6,6)?
c. What is the probability that the sum of the numbers on both dice will equal three?
d. What is the probability that the sum of the numbers on both dice will equal seven?
e. What is the probability that the number on the second die is a 5, given that the number on the first die is a 5?

The next four problems refer to the following table, which shows the joint probabilities of discrete random variables X and Y.

		Y		
		1	2	3
X	1	P=0.1	P=0.2	P=0.1
	2	P =0.3	P=0.1	P=0.2

(3.10) [Joint distributions] Refer to the probability table above.

a. Compute the joint probability that X and Y are *both* equal to two: Prob{X=2,Y=2}.
b. Compute the probability that X and Y sum to three: Prob{X+Y=3}.
c. Compute the probability that (X+Y) is an even number.
d. Compute the probability that X = Y: Prob{X=Y}.
e. Compute the probability that Y > X: Prob{Y>X}.

(3.11) [Joint distributions] Refer to the probability table above.

a. Compute the marginal probability Prob{X=1}.
b. Compute the marginal probability Prob{Y=2}.
c. Are X and Y independent? Justify your answer.

(3.12) [Conditional probabilities] Refer to the probability table above.

a. Compute the conditional probability Prob{Y=2|X=1}.
b. Compute the conditional probability Prob{X=1|Y=2}.
c. Explain in words why your answers to (a) and (b) are different. That is, explain the conceptual difference between Prob{Y=2|X=1} and Prob{X=1|Y=2}.

(3.13) [Conditional probabilities] Refer to the probability table above.

a. Compute the conditional probability Prob{Y=3|X=2}.
b. Compute the conditional probability Prob{X=2|Y=3}.
c. Explain in words why your answers to (a) and (b) are different. That is, explain the conceptual difference between Prob{Y=3|X=2} and Prob{X=2|Y=3}.

(3.14)[1] [Probability, random variables] Much is made of the fact that certain mutual funds outperform the market year after year (that is, the return from holding shares in the mutual fund is higher than the return from holding a market portfolio such as the S&P 500). For concreteness, consider a ten-year period and let the population be the 4,170 mutual funds reported in the *Wall Street Journal* on January 1, 1995. Suppose that performance relative to the market were random, meaning that each fund had a 50-50 chance of outperforming the market in any year and that performance was independent from year to year.

a. Compute the probability that any particular fund would outperform the market in all 10 years.
b. Compute the probability that any particular fund would *not* outperform the market in all 10 years (though it might outperform the market in some years).
c. Compute the probability than *none* of the 4,170 funds outperform the market in all 10 years (though they might outperform the market in some years).
d. Compute the probability that *at least one* fund out of 4,170 funds would outperform the market in all 10 years. Comment on the implications of your answer.

[end of problem set]

[1] Taken from Wooldridge, *Introductory Econometrics, 2nd edition*, 2003, problem B.3, page 729