

EXAMINATION 2 ANSWER KEY

“Two-Variable Regression”

Version A

I. Multiple choice

(1)b. (2)d. (3)b. (4)a. (5)b. (6)c. (7)c. (8)b. (9)a.

II. Short answer

- | | | | | |
|-----|---------------------|--------------|---------------------|--------------------------|
| (1) | a. true | b. false | c. false | d. true |
| (2) | a. 12 | b. 0.48 | | |
| (3) | a. 5 | b. 0 | c. 0 | |
| (4) | a. not req'd | b. not req'd | c. not req'd | d. required e. not req'd |
| (5) | a. not req'd | b. not req'd | c. required | d. required e. required |
| (6) | a. not req'd | b. required | c. required | d. required e. required |
| (7) | a. false | b. true | c. false | d. true |
| (8) | a. true | b. false | c. false | d. true |
| (9) | a. 1392 (no change) | b. -3700 | c. 0.25 (no change) | |
| | d. -7.4 (no change) | | | |

III. Problems

- (1)
- $65 + 22 \times 70 = \$1605$.
 - $\Delta y = 10 \times 22 = \220 .
 - 95% confidence interval for intercept = $65 \pm 1.96 \times 20 = 65 \pm 39.2 = (25.8, 140.2)$.
 - test statistic = 4.4, critical point = 1.645, can reject null hypothesis at 5%.
- (2)
- DOF = $22 - 2 = 20$.
 - 90% confidence interval for slope = $18 \pm 1.725 \times 0.05 = 0.18 \pm 0.08625 = (0.0938, 0.2663)$.
 - Should transform x variable (income) by subtracting \$400.
 - Predicted food expenditures = intercept of transformed equation = \$85.9.
 - Standard error of prediction error = $\sqrt{9.5^2 + 682} = 27.79$.
 - 95% prediction interval = $85.9 \pm 2.086 \times 27.79 = 85.9 \pm 57.97 = (27.9, 143.9)$.

IV. Critical thinking

(1) The two most important reasons why we should care whether the error term is distributed as normal are as follows. First, if the error term is normal, one can compute standard errors, confidence intervals, and t-tests even *when the sample is small*, using the t-distribution. Second, if the error term is normal, then one can compute *prediction intervals*. A less important reason is that least squares estimators become *maximum likelihood* estimators if the error terms are

normal, which implies that they are best (lowest-variance) among all unbiased estimators (“BUE”).

(2) One should *disagree* with this statement. Ordinary least squares is not limited to fitting linear relationships ($y = \beta_1 + \beta_2 x$) if one first *transforms the variables*. For example, least squares can easily fit these nonlinear relationships:

$$\begin{aligned} \log(y) &= \beta_1 + \beta_2 x \\ y &= \beta_1 + \beta_2 (1/x) \end{aligned}$$

Version B

I. Multiple choice

(1)e. (2)b. (3)d. (4)b. (5)a. (6)c. (7)d. (8)d. (9)b.

II. Short answer

- | | | | | | |
|-----|---------------------|-------------|---------------------|--------------|--------------|
| (1) | a. false | b. true | c. true | d. true | |
| (2) | a. 18 | b. 0.6 | | | |
| (3) | a. 0 | b. 1 | c. 1 | | |
| (4) | a. not req'd | b. required | c. not req'd | d. not req'd | e. not req'd |
| (5) | a. not req'd | b. required | c. required | d. not req'd | e. required |
| (6) | a. not req'd | b. required | c. required | d. required | e. required |
| (7) | a. false | b. false | c. true | d. false | |
| (8) | a. false | b. true | c. true | d. false | |
| (9) | a. 16704 | b. -444 | c. 0.25 (no change) | | |
| | d. -7.4 (no change) | | | | |

III. Problems

- (1)
- a. $65 + 22 \times 80 = \$1825$.
 - b. $\Delta y = 5 \times 22 = \110 .
 - c. 90% confidence interval for intercept = $65 \pm 1.645 \times 20 = 65 \pm 32.9 = (32.1, 97.9)$.
 - d. test statistic = 4.4, critical point = 1.282, can reject null hypothesis at 5%.
- (2)
- a. $\text{DOF} = 17 - 2 = 15$.
 - b. 95% confidence interval for slope = $18 \pm 2.131 \times 0.05 = 18 \pm 0.106555 = (0.0735, 0.2866)$.
 - c. Should transform x variable (income) by subtracting \$600.
 - d. Predicted food expenditures = intercept of transformed equation = \$122.3.
 - e. Standard error of prediction error = $\sqrt{6.5^2 + 682} = 26.91$.
 - f. 95% prediction interval = $122.3 \pm 2.131 \times 26.91 = 122.3 \pm 57.35 = (64.95, 179.65)$.

IV. Critical thinking

Same as Version A.

[end of answer key]