STAT 170 – Regression and Time Series
Drake University, Fall 2024
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EXAMINATION 1 VERSION A
"Introduction and Review"
September 17, 2024

INSTRUCTIONS: This exam is closed-book, closed-notes. You may use a calculator on this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. A table of the t-distribution is attached.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [2 pts each–12 pts total]

(1) A data set that includes many observations at the same point in time is called

a. a cross-section.
b. a time-series.
c. a pooled data set.
d. a panel data set.

(2) $\frac{\partial}{\partial \alpha} \sum_{i=1}^{n} (x_i - 5\alpha)^2 =$

a. $-10 \sum_{i=1}^{n} (x_i - 5\alpha)$.
b. $2 \sum_{i=1}^{n} x_i$.
c. $-2 \sum_{i=1}^{n} 5$.
d. $\sum_{i=1}^{n} (x_i - 5\alpha)^2$.
e. $\sum_{i=1}^{n} -5\alpha x_i$.

(3) Suppose we wish to fit the equation $y = \beta_1 + \beta_2 x$ to data by the method of *least squares*. This method minimizes which function of the data?

a. $f(\beta_1, \beta_2) = \sum (y_i - \beta_1 - \beta_2 x_i)$.
b. $f(\beta_1, \beta_2) = \sum (y_i^2 - (\beta_1 + \beta_2 x_i)^2)$.
c. $f(\beta_1, \beta_2) = \sum (y_i - \beta_1 - \beta_2 x_i)^2$.
d. $f(\beta_1, \beta_2) = \sum |y_i - \beta_1 - \beta_2 x_i|$.
e. $f(\beta_1, \beta_2) = \sum (\beta_1 + \beta_2 x_i)^2$.

(4) An estimator $\hat{\theta}$ of an unknown population parameter θ is said to be *asymptotically unbiased* if

a. $E(\hat{\theta}) = \theta$.
b. $E(\hat{\theta}) = 0$.
c. $\lim_{n \to \infty} E(\hat{\theta}) = 0$.
d. $\lim_{n \to \infty} E(\hat{\theta}) = \theta$.
e. $\lim_{n \to \infty} \text{Prob}(|\hat{\theta} - \theta| > \delta) = 0$, for all $\delta > 0$.

(5) A symmetric 90 percent confidence interval for an unknown parameter is necessarily ________________ a symmetric 95 percent confidence interval for the same parameter, computed from the same data.

a. narrower than
b. wider than
c. identical to
d. wider or narrower, depending on the true value of the parameter, than

(6) Suppose the p-value for a test statistic is 0.037. If the size of the test is 5 percent, we

a. can reject the null hypothesis.
b. cannot reject the null hypothesis.
c. cannot compute the test statistic.
d. answer cannot be determined from the information given.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [Summation operator: 6 pts] Let $\bar{x} = \frac{1}{n}\sum x_i$ and consider the following equations. Write "TRUE" for equations that are necessarily true for any values of x_i and y_i. Write "FALSE" for equations that are not necessarily true. All sums run from i = 1, ..., n.

a. $\sum(x_i - \bar{x}) = 0$	
b. $\sum x_i^2 = (\sum x_i)^2$	
c. $\sum(\bar{x}\, x_i) = n\, \bar{x}^2$	

(2) [Bernoulli random variable: 4 pts] Suppose X is a Bernoulli random variable, with Prob{X=1} = 0.3 and Prob{X=0} = 0.7.

a. Compute the mean of X, that is, E(X).	
b. Compute the variance of X, that is, Var(X).	

(3) [Conditional mean: 6 pts] Consider the following table, which shows the joint probabilities of discrete random variables X and Y .

		X: 1	X: 2	X: 3
Y	1	Prob = 0.2	Prob = 0.1	Prob = 0.1
	2	Prob = 0.2	Prob = 0.3	Prob = 0.1

a. Compute the conditional mean E(Y|X=1).

b. Compute the conditional mean E(Y|X=2).

c .Compute the unconditional mean E(Y).

(4) [Mean and variance of linear function: 4 pts] Suppose X is a random variable with mean E(X) = 3 and variance Var(X) = 2 . Suppose Y = 5 + 3 X .

a. Compute the mean of Y, that is, E(Y).

b. Compute the variance of Y, that is, Var(Y).

(5) [Mean and variance of sample mean: 4 pts] Suppose $X_1, \ldots, X_{10}$ are random variables each with identical mean 3 and variance 20 . Assume $Cov(X_i,X_j) = 0$ for all $i \neq j$. Let $\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$.

a. Compute the mean of $\bar{X}$, that is, E($\bar{X}$).

b. Compute the variance of $\bar{X}$, that is, Var($\bar{X}$).

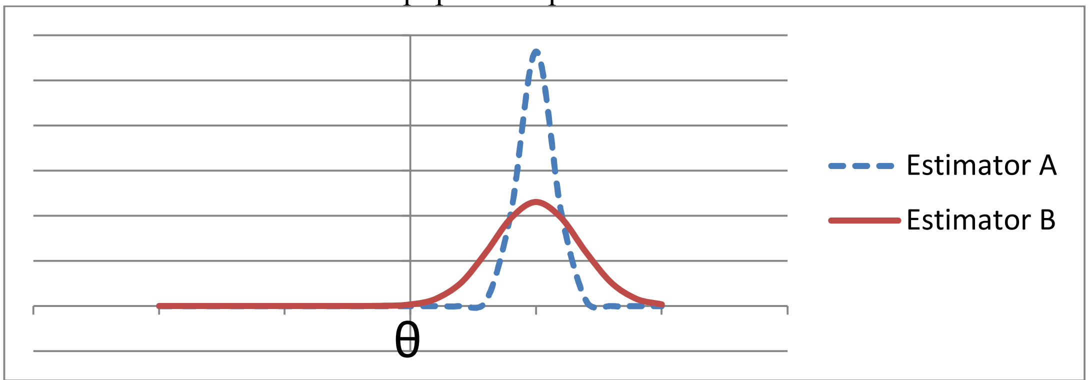
(6) [Important distributions: 6 pts] Consider the following probability distributions. Some of them have density functions which are symmetric about zero. Write "SYMMETRIC" for these distributions. Write "ASYMMETRIC" for the others.

a. Standard normal distribution.

b . *t* distribution with 9 degrees of freedom.

c. Chi-square (χ^2) distribution with 6 degrees of freedom.

(7) [Properties of estimators: 4 pts] The graph below shows the density functions for two alternative estimators of an unknown population parameter θ.

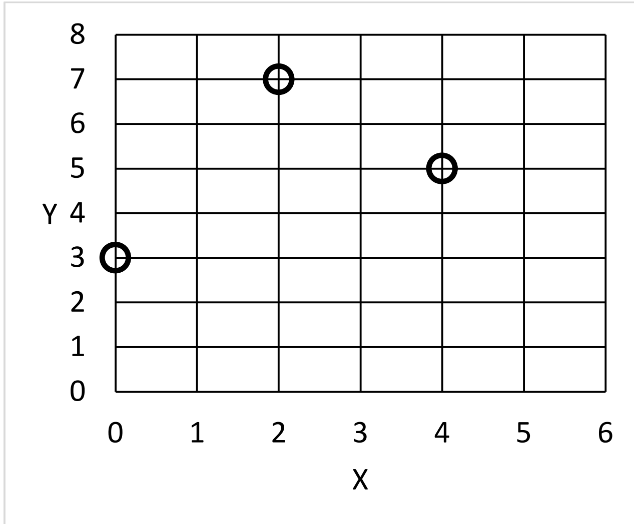


a. Which estimator has greater **bias**? Answer "A," "B," or "EQUAL."

b. Which estimator has greater **variance**? Answer "A," "B," or "EQUAL."

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Definition of least-squares: 15 pts] Suppose we have three observations on X and Y shown in the graph below



It can be shown that the least-squares estimate for the intercept is $\beta_1 = 4$ and the least squares estimate for the slope is $\beta_2 = 0.5$.

a. Compute the three fitted values $\hat{y}_i$ of this least-squares estimated regression line.

b. Sketch the least-squares fitted line in the graph above.

c. Compute the sample means $(\bar{x}, \bar{y})$ and verify numerically that the fitted line passes through the sample means.

d. Compute the three residuals $\hat{\varepsilon}_i$ of this estimated least-squares regression line.

e. Compute the sum of squared residuals.

(2) [Estimation: 12 pts] Suppose we wish to estimate the mean of a population using the following (peculiar) estimator applied to a random sample of 10 observations.

$$\hat{\mu} = 1 + \frac{1}{8}\sum_{i=1}^{10} x_i$$

Compute the following properties of the estimator under the assumption that the true population mean is $E(X_i) = 12$ and the true population variance is $Var(X_i) = 32$. Circle your final answers.

a. Compute $E(\hat{\mu})$.

b. Compute $Bias(\hat{\mu})$.

c. Compute $Var(\hat{\mu})$.

d. Compute $MSE(\hat{\mu})$.

(3) [Inference for arbitrary distribution, large sample: 18 pts] Suppose we wish to analyze the distribution of the number of computers per household in a population. Let μ denote the unknown true population mean number of computers per household. Observations X_i have been collected on **500** households, with the following summary values. Here, $\overline{X}$ is the sample mean.

$$\sum_{i=1}^{500} X_i = 1340 \qquad \sum_{i=1}^{500} \left(X_i - \overline{X}\right)^2 = 2055$$

a. [3 pts] Is the population distribution discrete or continuous? Explain your answer.

b. [3 pts] Compute an unbiased estimate of μ.

c. [3 pts] Compute the standard error of your estimate of μ.

d. [3 pts] Compute a **90%** asymptotic confidence interval for μ.

e. [6 pts] Test the null hypothesis that $\mu = 3$ against the two-sided alternative hypothesis that $\mu \neq 3$, at **5%** significance using an asymptotic test. Give the value of the test statistic, the critical point from a table, and your conclusion (whether you can reject null hypothesis).

Value of test statistic = ________________. Critical point(s) = __________________.

Can you reject null hypothesis? ________________________________.

IV. CRITICAL THINKING: Answer just *one* of the questions below (your choice). [4 pts]

(1) Suppose you want to estimate the effect of education level on health, using a random sample of people in the U.S. population.

a. Describe a situation where the purpose of the regression would be *prediction*. [Hint: Imagine you are employed by an insurance company.]
b. Describe a situation where the purpose of the regression would be *causal inference*. [Hint: Imagine you are writing a report to a committee of the state legislature.]
c. If the purpose is *causal inference*, what perils might you face? [Hint: Do you think eighteen-year-olds who choose to go on to college are as healthy as eighteen-year-olds who do not, on average?]

(2) Consider the random outcome when a coin is tossed. Let $X_i = 1$ denote "heads" and $X_i = 0$ denote "tails." The method of moments principle proposes to estimate the population mean—which equals the probability of "heads"—by the sample mean. Suppose a particular coin is tossed three times and comes up "heads" each time.

a. What is the method-of-moments estimate of the mean of X? (Give a number.)
b. Do you find this estimate reasonable? Why or why not?

Circle the question you are answering and write your answer below. Full credit requires good grammar, legible writing, accurate spelling, and correct reasoning.

[end of exam]