Introduction to Econometrics (Econ 107)
Drake University, Spring 2010
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MIDTERM EXAMINATION #2 VERSION C
"Two-Variable Regression"
February 26, 2010

INSTRUCTIONS: This exam is closed-book, closed-notes. You may use a calculator on this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. A table of the t-distribution is attached.

NOTATION: In this exam, $\hat{\beta}_1$ and $\hat{\beta}_2$ denote the least-squares estimators of the intercept and slope of the line $y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$, $\hat{y}_i$ denotes a least-squares fitted value, $\hat{\varepsilon}_i$ denotes a least-squares residual, and the sample size is denoted n. The true or population value of the variance of the unobserved error term ε_i is denoted σ^2. The (unbiased) least-squares estimate of σ^2 is denoted $\hat{\sigma}^2$. The sample means of x and y are denoted $\bar{x}$ and $\bar{y}$ respectively. The natural logarithm function is denoted ln(.).

I. MULTIPLE CHOICE: Circle the one best answer to each question. Feel free to use margins for scratch work [2 pts each—10 pts total]

(1) Suppose we wish to fit the equation $y = \beta_1 + \beta_2 x$ to data by the method of least squares. This method minimizes which function of the data?

a. $f(\beta_1, \beta_2) = \sum (y_i - \beta_1 - \beta_2 x_i)^2$.

b. $f(\beta_1, \beta_2) = \sum |y_i - \beta_1 - \beta_2 x_i|$.

c. $f(\beta_1, \beta_2) = \sum (\beta_1 + \beta_2 x_i)^2$.

d. $f(\beta_1, \beta_2) = \sum (y_i - \beta_1 - \beta_2 x_i)$.

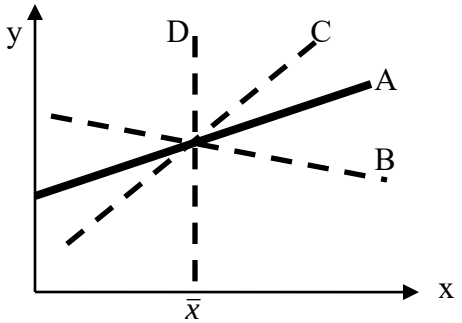
e. $f(\beta_1, \beta_2) = \sum (y_i^2 - (\beta_1 + \beta_2 x_i)^2)$.

(2) In the model $y_i = 6.7 + 3.4 x_i + \varepsilon_i$, assuming $E(\varepsilon_i | x_i)=0$, the conditional mean of y (that is, $E(y_i|x_i)$) is

a. zero.
b. 6.7 .
c. 3.4 .
d. $3.4 x_i$.
e. $6.7 + 3.4 x_i$.

(3) In the graph below, the solid line denoted "A" is the true population regression line. If the error term has mean zero but is *positively correlated* with x, then the least-squares estimated line will tend to resemble

a. line A.
b. line B.
c. line C.
d. line D.
e. cannot be determined from the information given.



(4) In a time-series dataset, any two trended variables must be
a. correlated with each other.
b. uncorrelated with each other.
c. normally-distributed.
d. caused by a third variable.
e. related through cause and effect.

(5) In the equation
$$\ln(y) = 4 + 0.05\,x,$$
which of the following is correct?
a. If x increases by 1 unit, then y increases by 0.05 units.
b. If x increases by 1%, then y increases by 0.05 units.
c. If x increases by 1 unit then y increases by 5%.
d. The elasticity of y with respect to x is 0.05.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [Algebraic properties: 3 pts] Which equations hold necessarily, regardless of the data or the model? Write "TRUE" or "FALSE" in the boxes below.

a. $\sum \hat{y}_i \hat{\varepsilon}_i = 0$

b. $\sum x_i y_i = 0$

b. $\sum (\hat{y}_i - \bar{y})^2 = \sum (y_i - \bar{y})^2 + \sum \hat{\varepsilon}_i^2$

(2) [Algebraic properties: 2 pts] Suppose least-squares estimation of $y = \beta_1 + \beta_2 x$ yields a sum of squared residuals $\sum \hat{\varepsilon}_i^2 = 24$ while the total sum of squares is $\sum (y_i - \bar{y})^2 = 30$.

a. Compute the explained or regression sum of squares $\sum (\hat{y}_i - \bar{y})^2$ for this regression equation.

b. Compute the value of r^2 for this regression equation.

(3) [Algebraic properties: 3 pts] Suppose the equation $x = \beta_1 + \beta_2 x$ were estimated by least-squares by mistake. Note that the regressor and the dependent variable are the same. Give numerical answers to the following questions.

a. What would be the value of the sum of squared residuals, $\sum \hat{\varepsilon}_i^2$?

b. What would be the least-squares estimate of the slope, $\hat{\beta}_2$?

c. What would be the least-squares estimate of the intercept, $\hat{\beta}_1$?

(4) [Properties: 5 pts] Which assumptions are required for the least-squares estimators to be *"method-of-moments"* estimators? Write "REQUIRED" or "NOT REQUIRED" in the boxes below.

a. Variance of error term is zero: $Var(\varepsilon_i) = 0$.	
b. Error term is normally-distributed: $\varepsilon_i \sim N(0, \sigma^2)$	
c. Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant.	
d. Conditional mean of error term is zero: $E(\varepsilon_i \mid x_i) = 0$.	
e. No autocorrelation: $Cov(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$.	

(5) [Properties: 5 pts] Which assumptions are required for the usual formulas for t tests to be correct for *large samples*? Write "REQUIRED" or "NOT REQUIRED" in the boxes below.

a. Variance of error term is zero: $Var(\varepsilon_i) = 0$.	
b. Error term is normally-distributed: $\varepsilon_i \sim N(0, \sigma^2)$	
c. Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant.	
d. Conditional mean of error term is zero: $E(\varepsilon_i \mid x_i) = 0$.	
e. No autocorrelation: $Cov(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$.	

(6) [Properties: 5 pts] Which assumptions are required for the usual formulas for t tests to be correct for *small samples*? Write "REQUIRED" or "NOT REQUIRED" in the boxes below.

a. Variance of error term is zero: $Var(\varepsilon_i) = 0$.	
b. Error term is normally-distributed: $\varepsilon_i \sim N(0, \sigma^2)$	
c. Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant.	
d. Conditional mean of error term is zero: $E(\varepsilon_i \mid x_i) = 0$.	
e. No autocorrelation: $Cov(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$.	

(7) [Variance of LS estimators: 4 pts] Write "TRUE" or "FALSE" in the boxes below. The variance of the least-squares slope estimator $\hat{\beta}_2$ is *smaller*, and thus the true value of β_2 is estimated *more precisely*,

a. the larger the variance of the error term σ^2.

b. the larger the sample size n.

c. the larger the sample variance of x: $\frac{1}{n}\sum(x_i - \bar{x})^2$.

d. the larger the sample mean of x, that is, $\bar{x}$.

(8) [Variance of LS predictor: 4 pts] Write "TRUE" or "FALSE" in the boxes below. Suppose we use the least-squares predictor ($\hat{y}_{n+1} = \hat{\beta}_1 + \hat{\beta}_2 x_{n+1}$) to predict y_{n+1}. The variance of the prediction error ($y_{n+1} - \hat{y}_{n+1}$) is *larger*, and thus prediction is *less precise*,

a. the smaller the variance of the error term σ^2.

b. the closer x_{n+1} is to $\bar{x}$.

c. the smaller the sample variance of x: $\frac{1}{n}\sum(x_i - \bar{x})^2$.

d. the smaller the sample size n.

(9) [Units of measure: 4 pts] Suppose the relationship between house size and family income is estimated by least squares as follows. The r^2 value is 0.52 .

House size in square feet	=	261 (54.2)	+	0.12 (0.05)	Family income in dollars

Now suppose the data on family income were converted from dollars to thousands of dollars, and the equation were re-estimated by least squares. For example, an observation that was formerly x = \$85,000 would now be converted to x = \$85. Compute the new values of the following items.

a. New value of least-squares intercept ($\hat{\beta}_1$).

b. New value of least-squares slope ($\hat{\beta}_2$).

c. The new r^2 value.

d. New t-statistic for slope ($\hat{\beta}_2$), for testing H_0: true slope = 0.

III. PROBLEMS: Write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [LS confidence intervals, tests, elasticity: 24 pts] The relationship between weekly income and weekly spending on food has been estimated for a sample of n=400 families. For each family i, let y_i denote spending on food and x_i denote income. The model $y_i = \beta_1 + \beta_2 x_i$ is estimated with the following results. The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors.

Spending on food	=	104.0 (20)	+	0.07 (0.02) Family income

a. [3 pts] Suppose a family has a weekly income of $500. According to these results, how much would this family spend on food?

b. [3 pts] Suppose a family's income increased by $200. By how much would its spending on food increase? That is, what is the predicted change Δy when $\Delta x = 200$?

c. [3 pts] Suppose the sample mean income is $800 and the sample mean spending on food is $160. (That is, $\bar{y} = 160.$ and $\bar{x} = 800.$) Compute the estimated elasticity of spending on food with respect to family income at the sample means.

d. [6 pts] Compute a **95%** confidence interval for the **intercept,** β_1 .

e. [9 pts] Test the hypothesis that income has a positive effect on spending on food, against the null hypothesis that income has no effect (a **one-tailed test**) at **5%** significance. Give the value of the test statistic, the critical point(s) from a table, and your conclusion (whether you can reject null hypothesis).

Value of test statistic = _____. Critical point(s) = _____.

Can you reject null hypothesis? _____.

(2) [LS confidence intervals, prediction: 24 pts] The effect of the price of high-speed internet access on penetration is measured using a sample of n=13 cities. For each city i, let y_i denote the penetration rate (which varies from zero to 100) and let x_i denote the monthly price of internet access in the same city. The model $y_i = \beta_1 + \beta_2 x_i$ is estimated with the following results. The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors. Assume the error term is **normally distributed**.

y_i	=	80	– 0.50	x_i
		(16.5)	(0.20)	

a. [3 pts] What are the "degrees of freedom" for these estimates? Give a numerical answer.

b. [6 pts] Compute a **95%** confidence interval for the **slope,** β_2 .

We wish to predict penetration rate (y_{n+1}) when the price (x_{n+1}) is \$70. So we first transform the data to simplify calculations.

c. [3 pts] Which variable (x_i , y_i, or both) should be transformed? How?

Suppose the following equation has been estimated on the *transformed data* with the following results. (Numbers on top are the least-squares intercept and slope. Numbers at the bottom in parentheses are standard errors.) The estimated variance of the error term is $\hat{\sigma}^2 = 5.0$.

new y_i	=	45.0	– 0.50	new x_i
		(2.0)	(0.20)	

d. [3 pts] Predict the penetration rate (y_{n+1}) when the price (x_{n+1}) is \$70.

e. [3 pts] Compute the standard error of prediction error.

f. [6 pts] Compute a 95% prediction interval for the penetration rate (y_{n+1}) when the price (x_{n+1}) is \$70.

IV. CRITICAL THINKING: [7 pts] To investigate the possible effect of libraries on violent crime, the following regression was estimated using data on U.S. states[1]:

y_i	=	-1091.	+ 11.85	x_i
		(535)	(1.30)	

where y_i denotes the number of violent crimes committed in state i in 2007 and x_i denotes the number of libraries in state i in the same year. The estimate for the slope coefficient is significantly greater than zero at 0.1 percent significance. So the number of violent crimes is clearly *positively correlated* with the number of number of libraries across states. Is this evidence that libraries *cause* violent crime? If yes, explain why. If no, explain why not and suggest a better way to estimate the regression equation using these state-level data.

[end of exam]

[1] These are actual least-squares estimates using data from the *Statistical Abstract of the United States*, 2010. n=50.