Introduction to Econometrics (Econ 107)
Drake University, Spring 2008
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MIDTERM EXAMINATION #4 VERSION B
"Multiple Regression With Time-Series Data"
April 24, 2008

INSTRUCTIONS: This exam is closed-book, closed-notes. You may use a calculator for this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. Tables of the t distribution, F distribution, and chi-square distribution are attached.

NOTATION: In this exam, $\hat{\beta}_j$ denotes the least-squares coefficient estimators of the linear model $y_t = \beta_1 + \beta_2 x_{t2}, +\ldots+ \beta_K x_{tK} + \varepsilon_t$. The least-squares fitted value is denoted $\hat{y}_t$. The least-squares residual is denoted $\hat{\varepsilon}_t$. The array denoted X includes all regressors in all time periods in the sample. The sample size is denoted *T*. The true or population value of the variance of the unobserved error term ε_t is denoted σ^2. The (unbiased) least-squares estimator of σ^2 is denoted $\hat{\sigma}^2$. The sample mean of y is denoted $\bar{y}$.

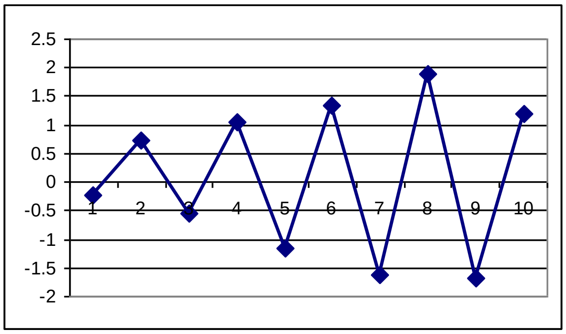
I. MULTIPLE CHOICE: Circle the one best answer to each question. Feel free to use margins for scratch work [2 pts each—18 pts total]

(1) Which is *not* a static model?
a. $q_t = 4.9 – 1.2\, p_{t-1} + \varepsilon_t$.
b. $q_t = 3.2 – 0.8\, p_t – 0.8\, i_t + \varepsilon_t$.
c. $q_t = 4.5 – 2.4\, p_t + \varepsilon_t$.
d. $p_t = 0.4 – 0.5\, q_t + \varepsilon_t$.

(2) In the time-series model
$$y_t = \beta_1 + \beta_2 x_{t,2} + \beta_3 x_{t,3} + \varepsilon_t ,$$
the assumption $E(\varepsilon_t|X) = 0$ means
a. "strict exogeneity of regressors."
b. "contemporaneous exogeneity of regressors."
c. "homoskedasticity of the error term."
d. "no serial correlation in the error term."
e. "normality of the error term."

(3) Which model below contains an exponential time trend?
a. $y_t = 8.3 + 0.02\, t + 5.1\, x_t$.
b. $y_t^2 = 5.1 + 0.15\, t + 0.27\, x_t + 0.02\, x_t^2$.
c. $\ln(y_t) = 7.2 + 0.03\, t + 3.2\, x_t$.
d. $\exp(y_t) = 10.3 + 0.43\, t + 7.6\, x_t$.
e. $y_t = 12.1 + 0.13\, t + 0.01 \exp(x_t)$.

(4) The time series ε_t graphed below appears to be
a. positively serially-correlated.
b. negatively serially-correlated.
c. serially uncorrelated.
d. Cannot be determined from the information given.



(5) Assume that ε_t is an IID (independent identically-distributed) random process. Which of the following is a first-order moving average process [“MA(1)”]?
a. $u_t = 0.4\, u_{t-1} + 0.2\, u_{t-2} + \varepsilon_t$.
b. $u_t = \varepsilon_t + 0.5\, \varepsilon_{t-1} + 0.2\, \varepsilon_{t-2}$.
c. $u_t = 0.2 + u_{t-1} + \varepsilon_t$.
d. $u_t = \varepsilon_t + 0.5\, \varepsilon_{t-1}$.
e. $u_t = 0.3\, u_{t-1} + \varepsilon_t$.
f. $u_t = u_{t-1} + \varepsilon_t$.

(6) If there is positive serial correlation, the Durbin-Watson test statistic is most likely
a. less than zero.
b. approximately 0.
c. between 0 and 1.
d. approximately 1.
e. between 0 and 2.
f. approximately 2.
g. between 2 and 4.
h. approximately 4.

(7) If the random process u_t has a unit root, then
a. the square root of $u_t = 1$.
b. u_t does not tend to revert back to its mean.
c. $u_t = 1$.
d. $E(u_t) = 1$.
e. $Var(u_t) = 1$.

(8) If y_t and x_t are two independent random walks, then a regression of y_t on x_t will typically produce
a. a valid t statistic for the coefficient of x_t .
b. an R-square value close to zero.
c. an excessively small (in absolute value) t-statistic for the coefficient of x_t.
d. an excessively large (in absolute value) t-statistic for the coefficient of x_t.

(9) Suppose we estimate the following vector autoregression model:

$$wage_t = \alpha_1 + \alpha_2\, wage_{t-1} + \alpha_3\, cpi_{t-1} + \varepsilon_t$$
$$cpi_t = \beta_1 + \beta_2\, cpi_{t-1} + \beta_3\, wage_{t-1} + \upsilon_t$$

If we reject the hypothesis that $\beta_3 = 0$, then we conclude that
a. wage "Granger-causes" cpi.
b. cpi "Granger-causes" wage.
c. both of the above.
d. none of the above.

II. MULTIPLE ANSWER: The questions below may have more than one correct answer. Write "YES" next to all correct answers and "NO" next to all incorrect answers.

(1) [5 pts] For the model $y_t = \beta_1 + \beta_2 x_t + \varepsilon_t$, ordinary least squares yields consistent estimators of β_1 and β_2 if

a. ε_t is serially-correlated, but stationary and weakly dependent, and x_t is weakly exogenous.
b. ε_t is an independent, identically-distributed process, and x_t is strictly exogenous.
c. x_t and y_t are integrated processes but are not co-integrated.
d. x_t and y_t are co-integrated processes.
e. ε_t is a random walk.

(2) [5 pts] Which is a test for serial correlation of the error term in a regression equation?

a. Breusch-Pagan test.
b. Breusch-Godfrey test.
c. Goldfeld-Quandt test.
d. Durbin-Watson test.
e. Dubin's alternative test.

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Finite distributed lag: 6 pts] Suppose we have estimated the time-series model

$$y_t = 2.7 + 1.5\, x_t - 0.2\, x_{t-1} + 0.4\, x_{t-2} + \varepsilon_t\ .$$

a. Is this a *static* model or a *dynamic* model?

b. Compute the impact propensity (also called the "impact multiplier" or "short-run effect").

c. Compute the long-run propensity (also called the "long-run multiplier" or "long-run effect").

(2) [Breusch-Godfrey test: 12 pts] Suppose we have estimated the regression model

$$y_t = \beta_1 + \beta_2\, x_t + \beta_3\, y_{t-1} + \varepsilon_t$$

using 121 observations, but we fear that ε_t might have first-order serial correlation. Accordingly, we must estimate an auxiliary regression.

a. What should be the dependent variable of the auxiliary regression?

b. What should be the regressors of the auxiliary regression?

(Note that this auxiliary regression must be estimated on observations 2 through 121 of the original data.) The R^2 value from this auxiliary regression is 0.024 . Test the null hypothesis of no serial correlation at 5% significance. Give the value of the test statistic, its degrees of freedom, the critical point from the appropriate table, and your conclusion (whether you can reject the null hypothesis).

c. Degrees of freedom =

d. Value of test statistic =

e. Critical point =

f. Can you reject the null hypothesis?

(3) [Quasi-differencing: 12 pts] Suppose we wish to estimate the time-series regression model

$$y_t = \beta_1 + \beta_2 x_t + u_t .$$

However, we believe the error term is serially correlated, following the process

$$u_t = 0.4\, u_{t-1} + \varepsilon_t ,$$

where ε_t is an independent, identically-distributed random error term. We need to transform the data to eliminate the serial correlation. The table below shows the first three observations on y_t, x_{2t}, and x_{3t}. Compute transformed values of the second and third observations using the Cochrane-Orcutt method.

Obs.	Raw data		Transformed data		
t	y_t	x_t	y_t	Replacement for intercept	x_t
1	10	5			
2	15	10			
3	22	13			

(4) [Random walk: 12 pts] Consider the following process:

$$u_t = 0.4 + u_{t-1} + \varepsilon_t .$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 5.3$. Assume $u_0 = 0$.

a. Find a formula in terms of *t* for the unconditional mean $E(u_t)$.

b. Find a formula in terms of *t* for the unconditional variance $Var(u_t)$.

c. Is u_t a *stationary* process or a *nonstationary* process?

d. Let $\Delta u_t = u_t - u_{t-1}$. Compute $E(\Delta u_t)$.

e. Compute $Var(\Delta u_t)$.

f. Suppose $u_{20} = 13.1$. Compute the forecast of u_{21}.

(5) Dickey-Fuller test: 6 pts] We wish to test whether the GDP has a unit root using an augmented Dickey-Fuller test. We have estimated the following by ordinary least squares, using a large sample, standard errors in parentheses.

$$\Delta GDP_t = \underset{(2.6)}{14.5} - \underset{(0.12)}{0.42}\ GDP_{t-1} + \underset{(0.19)}{0.34}\ \Delta GDP_{t-1}$$

a. Which is the null hypothesis of the Dickey-Fuller test: that GDP *does not have* a unit root, or that GDP *has* a unit root?
b. Compute the Dickey-Fuller test statistic.
c. The critical point at 5% significance for the augmented Dickey-Fuller test statistic is –2.86. Can you reject the null hypothesis at 5% significance?

(6) [Forecasting, trends and seasonality: 6 pts] Suppose we have estimated the following model for output, using 80 quarterly observations from the first quarter of 1986 to the fourth quarter of 2005.

$$output_t = 56.3 + 0.3\ trend - 0.5\ q1_t - 0.3\ q2_t - 0.1\ q3_t + \varepsilon_t\ .$$

The regressor "trend" equals 1 in the first quarter of 1986, equals 2 in the second quarter of 1986, and so forth, and equals 80 in the fourth quarter of 2005. The regressors "q1," "q2," and "q3," are quarterly dummy variables for the first, second and third quarters respectively. The error term ε_t is an independent, identically-distributed process with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma^2$, constant.

a. If a dummy variable "q4" for the fourth quarter were also included, then what econometric problem would result?
b. Compute the forecast of y_{81}, the value of output in the first quarter of 2006.
c. Compute the forecast of y_{82}, the value of output in the second quarter of 2006.

(7) [Forecasting, forecast interval: 12 pts] We wish to use the equation

$$y_t = \beta_1 + \beta_2 x_{t-1} + \beta_3 y_{t-1} + \varepsilon_t$$

to forecast y_{201} . The last observations in our sample are $y_{200}=33$ and $x_{200}=48$. To compute the forecast, we first transform the data.

a. [2 pts] Should the dependent variable be transformed? If so, how?

b. [2 pts] Should the regressors be transformed? If so, how?

Suppose the equation has been estimated on the ***transformed data*** with the following results, standard errors in parentheses. The estimated variance of the error term is $\hat{\sigma}^2$ = 38.76.

y_t	=	65.0	+	0.61	x_{t-1}	+	0.26	y_{t-1}
		(3.2)		(0.21)			(0.07)	

c. [2 pts] Compute the forecast of y_{201} .

d. [2 pts] Compute the standard error of forecast error.

e. [4 pts] Compute a 95% forecast interval for y_{201} .

(8) [Forecasting, AR model: 6 pts] Suppose we have estimated the following model.

$$y_t = 2.4 + 0.6\, y_{t-1} + \varepsilon_t\ .$$

where ε_t denotes an independent, identically-distributed process with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma^2$, constant. In our data set, $y_{T-1} = 5$ and $y_T = 10$. Compute the following forecast values.

a. Compute the forecast of y_{T+1}.

b. Compute the forecast of y_{T+2}.

c. Compute the limit of the forecast y_{T+h} as h approaches infinity.
[Hint: this is the unconditional mean of the process.]

[end of exam]