

**MIDTERM EXAMINATION #4 VERSION A**  
**“Multiple Regression With Time-Series Data”**  
**April 24, 2008**

**INSTRUCTIONS:** This exam is closed-book, closed-notes. You may use a calculator for this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. Tables of the  $t$  distribution,  $F$  distribution, and chi-square distribution are attached.

**NOTATION:** In this exam,  $\hat{\beta}_j$  denotes the least-squares coefficient estimators of the linear model  $y_t = \beta_1 + \beta_2 x_{t2} + \dots + \beta_K x_{tK} + \varepsilon_t$ . The least-squares fitted value is denoted  $\hat{y}_t$ . The least-squares residual is denoted  $\hat{\varepsilon}_t$ . The array denoted  $X$  includes all regressors in all time periods in the sample. The sample size is denoted  $T$ . The true or population value of the variance of the unobserved error term  $\varepsilon_t$  is denoted  $\sigma^2$ . The (unbiased) least-squares estimator of  $\sigma^2$  is denoted  $\hat{\sigma}^2$ . The sample mean of  $y$  is denoted  $\bar{y}$ .

**I. MULTIPLE CHOICE:** Circle the one best answer to each question. Feel free to use margins for scratch work [2 pts each—18 pts total]

(1) Which is *not* a static model?

- a.  $q_t = 3.1 - 0.7 p_t - 0.3 i_t + \varepsilon_t$ .
- b.  $q_t = 6.5 - 2.7 p_t + \varepsilon_t$ .
- c.  $p_t = 0.7 - 0.8 q_t + \varepsilon_t$ .
- d.  $q_t = 1.8 - 1.1 p_{t-1} + \varepsilon_t$ .

(3) Which model below contains an exponential time trend?

- a.  $\ln(y_t) = 3.2 + 0.02 t + 4.1 x_t$ .
- b.  $\exp(y_t) = 12.3 + 0.83 t + 5.6 x_t$ .
- c.  $y_t = 5.9 + 0.04 t + 7.1 x_t$ .
- d.  $y_t = 14.2 + 0.19 t + 0.03 \exp(x_t)$ .
- e.  $y_t^2 = 4.3 + 0.27 t + 0.55 x_t + 0.07 x_t^2$ .

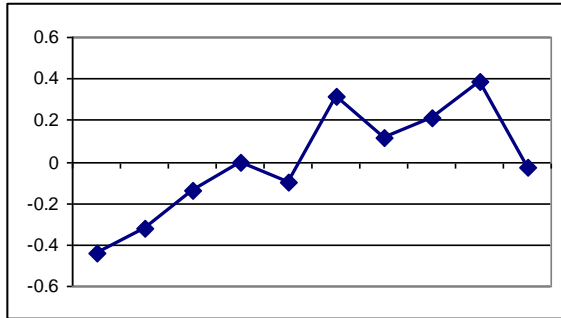
(2) In the time-series model

$$y_t = \beta_1 + \beta_2 x_{t,2} + \beta_3 x_{t,3} + \varepsilon_t,$$

the assumption  $E(\varepsilon_t | x_{t,2}, x_{t,3}) = 0$  means

- a. "strict exogeneity of regressors."
- b. "contemporaneous exogeneity of regressors."
- c. "homoskedasticity of the error term."
- d. "no serial correlation in the error term."
- e. "normality of the error term."

- (4) The time series  $\varepsilon_t$  graphed below appears to be
- positively serially-correlated.
  - negatively serially-correlated.
  - serially uncorrelated.
  - Cannot be determined from the information given.



- (5) Assume that  $\varepsilon_t$  is an IID (independent identically-distributed) random process. Which of the following is a stationary first-order autoregressive process [“AR(1)”]?
- $u_t = 0.4 u_{t-1} + 0.2 u_{t-2} + \varepsilon_t$ .
  - $u_t = \varepsilon_t + 0.5 \varepsilon_{t-1} + 0.2 \varepsilon_{t-2}$ .
  - $u_t = 0.2 + u_{t-1} + \varepsilon_t$ .
  - $u_t = \varepsilon_t + 0.5 \varepsilon_{t-1}$ .
  - $u_t = 0.3 u_{t-1} + \varepsilon_t$ .
  - $u_t = u_{t-1} + \varepsilon_t$ .

- (6) If there is no serial correlation, the Durbin-Watson test statistic is most likely
- less than zero.
  - approximately 0.
  - between 0 and 1.
  - approximately 1.
  - between 0 and 2.
  - approximately 2.
  - between 2 and 4.
  - approximately 4.

- (7) If the random process  $u_t$  has a unit root, then
- $u_t = 1$ .
  - $E(u_t) = 1$ .
  - $\text{Var}(u_t) = 1$ .
  - the square root of  $u_t = 1$ .
  - $u_t$  does not tend to revert back to its mean.

- (8) If  $y_t$  and  $x_t$  are two independent random walks, then a regression of  $y_t$  on  $x_t$  will typically produce
- an R-square value close to zero.
  - an excessively small (in absolute value) t-statistic for the coefficient of  $x_t$ .
  - an excessively large (in absolute value) t-statistic for the coefficient of  $x_t$ .
  - a valid t statistic for the coefficient of  $x_t$ .

- (9) Suppose we estimate the following vector autoregression model:

$$\text{wage}_t = \alpha_1 + \alpha_2 \text{wage}_{t-1} + \alpha_3 \text{cpi}_{t-1} + \varepsilon_t$$

$$\text{cpi}_t = \beta_1 + \beta_2 \text{cpi}_{t-1} + \beta_3 \text{wage}_{t-1} + \nu_t$$

If we reject the hypothesis that  $\alpha_3 = 0$ , then we conclude that

- wage "Granger-causes" cpi.
- cpi "Granger-causes" wage.
- both of the above.
- none of the above.

**II. MULTIPLE ANSWER:** The questions below may have more than one correct answer. Write “YES” next to all correct answers and “NO” next to all incorrect answers.

(1) [5 pts] For the model  $y_t = \beta_1 + \beta_2 x_t + \varepsilon_t$ , ordinary least squares yields consistent estimators of  $\beta_1$  and  $\beta_2$  if

- a.  $x_t$  and  $y_t$  are integrated processes but are not co-integrated.
- b.  $x_t$  and  $y_t$  are co-integrated processes.
- c.  $\varepsilon_t$  is a random walk.
- d.  $\varepsilon_t$  is serially-correlated, but stationary and weakly dependent, and  $x_t$  is weakly exogenous.
- e.  $\varepsilon_t$  is an independent, identically-distributed process, and  $x_t$  is strictly exogenous.


(2) [5 pts] Which is a test for serial correlation of the error term in a regression equation?

- a. Goldfeld-Quandt test.
- b. Durbin-Watson test.
- c. Dubin’s alternative test.
- d. Breusch-Pagan test.
- e. Breusch-Godfrey test.


**III. PROBLEMS:** Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Finite distributed lag: 6 pts] Suppose we have estimated the time-series model

$$y_t = 2.7 + 0.9 x_t + 0.2 x_{t-1} + \varepsilon_t .$$

- a. Is this a *static* model or a *dynamic* model?
- b. Compute the impact propensity (also called the "impact multiplier" or "short-run effect").
- c. Compute the long-run propensity (also called the "long-run multiplier" or "long-run effect").


(2) [Breusch-Godfrey test: 12 pts] Suppose we have estimated the regression model

$$y_t = \beta_1 + \beta_2 x_t + \beta_3 y_{t-1} + \varepsilon_t$$

using 91 observations, but we fear that  $\varepsilon_t$  might have first-order serial correlation. Accordingly, we must estimate an auxiliary regression.

- a. What should be the dependent variable of the auxiliary regression?
- b. What should be the regressors of the auxiliary regression?


(Note that this auxiliary regression must be estimated on observations 2 through 91 of the original data.) The  $R^2$  value from this auxiliary regression is 0.053 . Test the null hypothesis of no serial correlation at 5% significance. Give the value of the test statistic, its degrees of freedom, the critical point from the appropriate table, and your conclusion (whether you can reject the null hypothesis).

- c. Degrees of freedom =
- d. Value of test statistic =
- e. Critical point =
- f. Can you reject the null hypothesis?




(5) Dickey-Fuller test: 6 pts] We wish to test whether the GDP has a unit root using an augmented Dickey-Fuller test. We have estimated the following by ordinary least squares, using a large sample, standard errors in parentheses.

$$\Delta \text{GDP}_t = 10.5 - 0.47 \text{GDP}_{t-1} + 0.28 \Delta \text{GDP}_{t-1}$$

(2.1)                      (0.25)                      (0.08)

- a. Which is the null hypothesis of the Dickey-Fuller test: that GDP *does not have* a unit root, or that GDP *has* a unit root?
- b. Compute the Dickey-Fuller test statistic.
- c. The critical point at 5% significance for the augmented Dickey-Fuller test statistic is  $-2.86$ . Can you reject the null hypothesis at 5% significance?


(6) [Forecasting, trends and seasonality: 6 pts] Suppose we have estimated the following model for output, using 60 quarterly observations from the first quarter of 1991 to the fourth quarter of 2005.

$$\text{output}_t = 56.3 + 0.3 \text{trend} - 0.5 \text{q1}_t - 0.3 \text{q2}_t - 0.1 \text{q3}_t + \varepsilon_t .$$

The regressor “trend” equals 1 in the first quarter of 1991, equals 2 in the second quarter of 1991, and so forth, and equals 60 in the fourth quarter of 2005. The regressors “q1,” “q2,” and “q3,” are quarterly dummy variables for the first, second and third quarters respectively. The error term  $\varepsilon_t$  is an independent, identically-distributed process with  $E(\varepsilon_t) = 0$  and  $\text{Var}(\varepsilon_t) = \sigma^2$ , constant.

- a. If a dummy variable “q4” for the fourth quarter were also included, then what econometric problem would result?
- b. Compute the forecast of  $y_{61}$ , the value of output in the first quarter of 2006.
- c. Compute the forecast of  $y_{62}$ , the value of output in the second quarter of 2006.


(7) [Forecasting, forecast interval: 12 pts] We wish to use the equation

$$y_t = \beta_1 + \beta_2 x_{t-1} + \beta_3 y_{t-1} + \varepsilon_t$$

to forecast  $y_{201}$ . The last observations in our sample are  $y_{200}=53$  and  $x_{200}=18$ . To compute the forecast, we first transform the data.

a. [2 pts] Should the dependent variable be transformed? If so, how?

b. [2 pts] Should the regressors be transformed? If so, how?

Suppose the equation has been estimated on the *transformed data* with the following results, standard errors in parentheses. The estimated variance of the error term is  $\hat{\sigma}^2 = 29.75$ .

$y_t$	=	47.0	+	0.72	$x_{t-1}$	+	0.21	$y_{t-1}$
		(2.5)		(0.12)			(0.04)	

c. [2 pts] Compute the forecast of  $y_{201}$ .

d. [2 pts] Compute the standard error of forecast error.

e. [4 pts] Compute a 95% forecast interval for  $y_{201}$ .

(8) [Forecasting, AR model: 6 pts] Suppose we have estimated the following model.

$$y_t = 5.4 + 0.5 y_{t-1} - 0.1 y_{t-2} + \varepsilon_t$$

where  $\varepsilon_t$  denotes an independent, identically-distributed process with  $E(\varepsilon_t) = 0$  and  $\text{Var}(\varepsilon_t) = \sigma^2$ , constant. In our data set,  $y_{T-1} = 10$  and  $y_T = 12$ . Compute the following forecast values.

a. Compute the forecast of  $y_{T+1}$ .

b. Compute the forecast of  $y_{T+2}$ .

c. Compute the limit of the forecast  $y_{T+h}$  as  $h$  approaches infinity.

[Hint: this is the unconditional mean of the process.]


[end of exam]