Introduction to Econometrics (Econ 107)
Drake University, Spring 2008
William M. Boal

Signature: _____

Printed name: _____

MIDTERM EXAMINATION #2 VERSION B
"Two-Variable Regression"
March 4, 2008

INSTRUCTIONS: This exam is closed-book, closed-notes. You may use a calculator on this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. A table of the t-distribution is attached.

NOTATION: In this exam, $\hat{\beta}_1$ and $\hat{\beta}_2$ denote the least-squares estimators of the intercept and slope of the line $y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$, $\hat{y}_i$ denotes a least-squares fitted value, $\hat{\varepsilon}_i$ denotes a least-squares residual, and the sample size is denoted n. The true or population value of the variance of the unobserved error term ε_i is denoted σ^2. The (unbiased) least-squares estimate of σ^2 is denoted $\hat{\sigma}^2$. The sample means of x and y are denoted $\bar{x}$ and $\bar{y}$ respectively.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Feel free to use margins for scratch work [2 pts each—12 pts total]

(1) Suppose we wish to fit the equation $y = \beta_1 + \beta_2 x$ to data by the method of least squares. This method minimizes which function of the data?

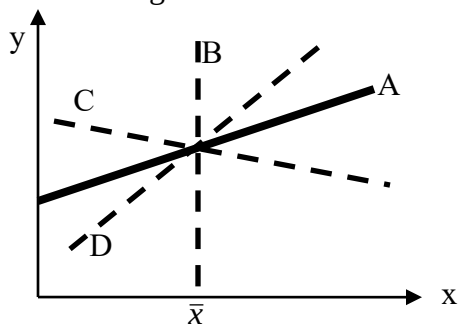
a. $f(\beta_1, \beta_2) = \sum (y_i - \beta_1 - \beta_2 x_i)^2$.
b. $f(\beta_1, \beta_2) = \sum |y_i - \beta_1 - \beta_2 x_i|$.
c. $f(\beta_1, \beta_2) = \sum (\beta_1 + \beta_2 x_i)^2$.
d. $f(\beta_1, \beta_2) = \sum (y_i - \beta_1 - \beta_2 x_i)$.
e. $f(\beta_1, \beta_2) = \sum \left(y_i^2 - (\beta_1 + \beta_2 x_i)^2\right)$.

(2) In the model $y_i = 2.7 + 0.4 x_i + \varepsilon_i$, assuming $E(\varepsilon_i | x_i)=0$, the conditional mean of y (that is, $E(y_i|x_i)$) is

a. $0.4 x_i$.
b. $2.7 + 0.4 x_i$.
c. 0.4 .
d. 2.7 .
e. zero.

(3) In the graph below, the solid line denoted "A" is the true population regression line. If the error term has mean zero but is *negatively correlated* with x, then the least-squares estimated line will tend to resemble

a. line A.
b. line B.
c. line C.
d. line D.
e. cannot be determined from the information given.



(4) If the equation $y = \beta_1 + \beta_2 y$ were estimated by mistake, then the least-squares estimate of β_1 would equal
a. -1.
b. 0.
c. 1.
d. n.
e. $\sum y_i$.

(5) The variance of the prediction error tends to decrease as the sample size n used for estimation increases, finally approaching
a. σ^2.
b. $\mathrm{Var}(\hat{\beta}_1)$.
c. $\mathrm{Var}(\hat{\beta}_2)$.
d. zero.
e. one.

(6) According to which model is the elasticity of y with respect to x constant and equal to 0.05?
a. $y = 4.5 + 0.05\ x$.
b. $y = 4.5 + 0.05\ (1/x)$.
c. $y = 4.5 + 0.05\ \ln(x)$.
d. $\ln(y) = 4.5 + 0.05\ x$.
e. $\ln(y) = 4.5 + 0.05\ \ln(x)$.

II. MULTIPLE ANSWER: The questions below may have more than one correct answer. Write "YES" next to all correct answers and "NO" next to all incorrect answers.

(1) [5 pts] Which equations hold necessarily, regardless of the data or the model?

a. $\sum x_i \hat{\varepsilon}_i = 0$	
b. $\sum (\hat{y}_i - \bar{y})^2 = \sum (y_i - \bar{y})^2 + \sum \hat{\varepsilon}_i^2$	
c. $\sum \hat{\varepsilon}_i = 0$	
d. $\sum \hat{y}_i = 0$	
e. $\sum \hat{\varepsilon}_i y_i = 0$	

(2) [5 pts] Which assumptions are required for the least-squares estimators to be consistent estimators?

a. Conditional mean of error term is zero: $E(\varepsilon_i \mid x_i) = 0$.	
b. No autocorrelation: $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$.	
c. Variance of error term is zero: $\mathrm{Var}(\varepsilon_i) = 0$.	
d. Error term is normally-distributed: $\varepsilon_i \sim N(0, \sigma^2)$	
e. Homoskedasticity: $\mathrm{Var}(\varepsilon_i) = \sigma^2$, a constant.	

(3) [5 pts] Which assumptions are required for the least-squares estimators to be maximum-likelihood estimators?

a. Conditional mean of error term is zero: $E(\varepsilon_i \mid x_i) = 0$.	
b. No autocorrelation: $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$.	
c. Variance of error term is zero: $\mathrm{Var}(\varepsilon_i) = 0$.	
d. Error term is normally-distributed: $\varepsilon_i \sim N(0, \sigma^2)$	
e. Homoskedasticity: $\mathrm{Var}(\varepsilon_i) = \sigma^2$, a constant.	

(4) [5 pts] Which assumptions are required for the least-squares estimators to have the lowest variance of all linear unbiased estimators ("BLUE")?

a. Conditional mean of error term is zero: $E(\varepsilon_i|x_i) = 0$. ☐
b. No autocorrelation: $Cov(\varepsilon_i,\varepsilon_j) = 0$ for $i \neq j$. ☐
c. Variance of error term is zero: $Var(\varepsilon_i) = 0$. ☐
d. Error term is normally-distributed: $\varepsilon_i \sim N(0, \sigma^2)$ ☐
e. Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant. ☐

(5) [4 pts] The variance of the least-squares slope estimator $\hat{\beta}_2$ is larger, and thus the true value of β_2 is estimated less precisely,

a. the smaller the variance of the error term σ^2. ☐
b. the smaller the sample variance of x: $\frac{1}{n}\sum(x_i - \bar{x})^2$. ☐
c. the smaller the sample size n. ☐
d. the smaller the sample mean of x, that is, $\bar{x}$. ☐

(6) [5 pts] Suppose we use the least-squares predictor ($\hat{y}_{n+1} = \hat{\beta}_1 + \hat{\beta}_2 x_{n+1}$) to predict y_{n+1}. The variance of the prediction error ($y_{n+1} - \hat{y}_{n+1}$) is smaller, and thus prediction is more precise,

a. the smaller the variance of the error term σ^2. ☐
b. the smaller the sample variance of x: $\frac{1}{n}\sum(x_i - \bar{x})^2$. ☐
c. the smaller the sample size n. ☐
d. the smaller the sample mean of x, that is, $\bar{x}$. ☐
e. the closer x_{n+1} is to $\bar{x}$. ☐

(7) [4 pts] Suppose a demand function for gasoline of the form $y_i = \beta_1 + \beta_2 x_i$, is estimated by least squares. Here, y_i denotes quantity demanded in gallons and x_i denotes the price of gasoline in dollars. Now suppose the price data are converted to cents (there are 100 cents in a dollar). How will the least-squares estimates change?

a. $\hat{\beta}_1$ will decrease by a factor of 100. ☐
b. $\hat{\beta}_2$ will decrease by a factor of 100. ☐
c. The r^2 value will decrease by a factor of 100. ☐
d. The t-statistic for $\hat{\beta}_2$ will decrease by a factor of 100. ☐

III. PROBLEMS: Write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [LS confidence intervals, tests, elasticity: 24 pts] The relationship between average test score and spending per pupil is estimated for a sample of n=300 school districts. For each district i, let y_i denote average test score and x_i denote spending per pupil (in thousands of dollars). The model $y_i = \beta_1 + \beta_2 x_i$ is estimated with the following results. The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors.

Average test score	=	45.6 (2.5)	+	3.2 (0.5)	Spending per pupil in thousands

a. [3 pts] Suppose a district spends \$10 thousand per pupil (that is, x = 10). According to these results, what would be this district's predicted average test score?

b. [3 pts] Suppose a district increased its spending by \$2 thousand. By how much would its average test score increase? That is, what is the predicted change Δy when $\Delta x = 2$?

c. [3 pts] Suppose the sample mean spending per pupil in this dataset is \$9.5 thousand and the sample mean test score is 76. (That is, $\bar{y} = 76.$ and $\bar{x} = 9.5.$) Compute the estimated elasticity of average test score with respect to spending per pupil at the sample means.

d. [6 pts] Compute a **95%** confidence interval for the **intercept, β_1** .

e. [9 pts] Test the hypothesis that spending has a positive effect on test scores, against the null hypothesis that spending has no effect (a **one-tailed test**) at **5%** significance. Give the value of the test statistic, the critical point(s) from a table, and your conclusion (whether you can reject null hypothesis).

Value of test statistic = _____. Critical point(s) = _____.

Reject null hypothesis? _____.

(2) [LS confidence intervals, prediction: 24 pts] The effect of the price of water on daily water consumption per capita is measured using a sample of n=16 cities. For each city i, let y_i denote its water consumption per capita and let x_i denote its price of water. The model $y_i = \beta_1 + \beta_2 x_i$ is estimated with the following results. The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors. Assume the error term is **normally distributed**.

y_i	=	92.5	–	250	x_i
		(6.5)		(40.0)	

a. [3 pts] What are the "degrees of freedom" for these estimates? Give an integer answer.

b. [6 pts] Compute a **95%** confidence interval for the **slope,** β_2 .

We wish to predict water consumption per capita (y_{n+1}) when the price (x_{n+1}) is 0.03. So we first transform the data.

c. [3 pts] Which variable (x_i or y_i) should be transformed? How?

Suppose the following equation has been estimated on the *transformed data* with the following results. (Numbers on top are the least-squares intercept and slope. Numbers at the bottom in parentheses are standard errors.) The estimated variance of the error term is $\hat{\sigma}^2 = 5.76$.

new y_i	=	85.0	–	250	new x_i
		(3.2)		(40.0)	

d. [3 pts] Predict water consumption per capita (y_{n+1}) when the price (x_{n+1}) is 0.03.

e. [3 pts] Compute the standard error of prediction error.

f. [6 pts] Compute a 95% prediction interval for water consumption when the price (x_{n+1}) is 0.03.

IV. CRITICAL THINKING: [7 pts] A researcher investigates the causes of house fires by running the following regression using data on U.S. states: $y_i = \beta_1 + \beta_2 x_i$, where y_i denotes the number of house fires reported in 2007 in state i, and x_i denotes the number of video games sold in state i in the same year. Using least-squares, the researcher finds a positive estimate for β_2, significantly different from zero at 1 percent. So the number of house fires is clearly *positively correlated* with the number of video games sold across states. Is this evidence that video games *cause* house fires? If yes, explain why. If no, explain why not and suggest a better way to estimate the regression equation using these state-level data.

[end of exam]