Introduction to Econometrics (Econ 107)
Drake University, Spring 2008
William M. Boal

Signature: ______________________

Printed name: ______________________

MIDTERM EXAMINATION #1 VERSION B
"Introduction and Statistics Review"
February 14, 2008

INSTRUCTIONS: This exam is closed-book, closed-notes. You may use a calculator on this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. A table of the t-distribution is attached.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Feel free to use margins for scratch work [3 pts each—42 pts total]

(1) Which of the following is *not* necessarily true?

a. $\sum(\alpha x_i) = \alpha \sum x_i$.

b. $\sum(x_i - \bar{x})^2 = \left(\sum x_i^2\right) - n\bar{x}^2$.

c. $\sum(x_i y_i) = \left(\sum x_i\right) \div \left(\sum y_i\right)$.

d. $\sum(x_i - \bar{x}) = 0$.

e. $\sum x_i = n\bar{x}$.

(2) $\dfrac{\partial}{\partial \alpha} \sum_{i=1}^{n} (3\alpha + x_i)^2 =$

a. $\sum_{i=1}^{n} (3\alpha + x_i)^2$.

b. $6\sum_{i=1}^{n} (3\alpha + x_i)$.

c. $2\sum_{i=1}^{n} (3\alpha + x_i)$.

d. $3\sum_{i=1}^{n} (3\alpha + 2x_i)$.

e. $6\sum_{i=1}^{n} x_i$.

(3) Suppose we wish to fit the equation $y = \beta_1 + \beta_2 x$ to data by the method of least squares. This method minimizes which function of the data?

a. $f(\beta_1, \beta_2) = \sum(y_i - \beta_1 - \beta_2 x_i)$.

b. $f(\beta_1, \beta_2) = \sum|y_i - \beta_1 - \beta_2 x_i|$.

c. $f(\beta_1, \beta_2) = \sum\left(y_i^2 - (\beta_1 + \beta_2 x_i)^2\right)$.

d. $f(\beta_1, \beta_2) = \sum(y_i - \beta_1 - \beta_2 x_i)^2$.

e. $f(\beta_1, \beta_2) = \sum(\beta_1 + \beta_2 x_i)^2$.

The next two questions assume the following. Suppose X is a Bernoulli random variable, with Prob{X=1} = 0.4 and Prob{X=0} = 0.6.

(4) The mean or expected value of X is
a. zero.
b. 0.24 .
c. 0.4 .
d. 0.6 .
e. one.

(5) The variance of X is
a. zero.
b. 0.24 .
c. 0.4 .
d. 0.6 .
e. one.

(6) Which of the following distributions has a symmetric bell-shaped density function?
a. chi-square distribution.
b. F distribution.
c. t distribution.
d. all of the above have symmetric bell-shaped density functions.
e. none of the above have symmetric bell-shaped density functions.

The next two questions assume the following. Suppose a random sample of size n is drawn from some population. The population has mean μ and variance σ^2. Consider the sample mean, defined as $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

(7) Var($\overline{X}$) =
a. zero .
b. σ^2 .
c. $\sigma^2 / (n-1)$.
d. σ^2 / n .
e. σ^2 / n^2 .
f. one.

(8) E($\overline{X}$) =
a. zero.
b. μ .
c. $\mu / (n-1)$.
d. μ / n .
e. μ / n^2 .
f. one.

(9) An estimator $\hat{\theta}$ of an unknown population parameter θ is said to be consistent if
a. $\lim_{n\to\infty} E(\hat{\theta}) = 0$.
b. $E(\hat{\theta}) = \theta$.
c. $E(\hat{\theta}) = 0$.
d. $\lim_{n\to\infty} E(\hat{\theta}) = \theta$.
e. $\lim_{n\to\infty} \text{Prob}(|\hat{\theta} - \theta| > \delta) = 0$, for all $\delta > 0$.

(10) An estimator $\hat{\theta}$ of an unknown population parameter θ is said to be unbiased if
a. $E(\hat{\theta}) = \theta$.
b. $E(\hat{\theta}) = 0$.
c. $\lim_{n\to\infty} E(\hat{\theta}) = 0$.
d. $\lim_{n\to\infty} E(\hat{\theta}) = \theta$.
e. $\lim_{n\to\infty} \text{Prob}(|\hat{\theta} - \theta| > \delta) = 0$, for all $\delta > 0$.

(11) The principle for finding an estimator that proceeds by setting the sample mean equal to the population mean, the sample variance equal to the population variance, etc., is called
a. the method of maximum likelihood.
b. the method of moments.
c. the scientific method.
d. Newton's method.

(12) A 95 percent confidence interval is necessarily
a. narrower than a 90 percent confidence interval.
b. wider than a 90 percent confidence interval.
c. identical to a 90 percent confidence interval.
d. answer cannot be determined from the information given.

(13) The probability that a test will correctly reject the null hypothesis when it is false is called the

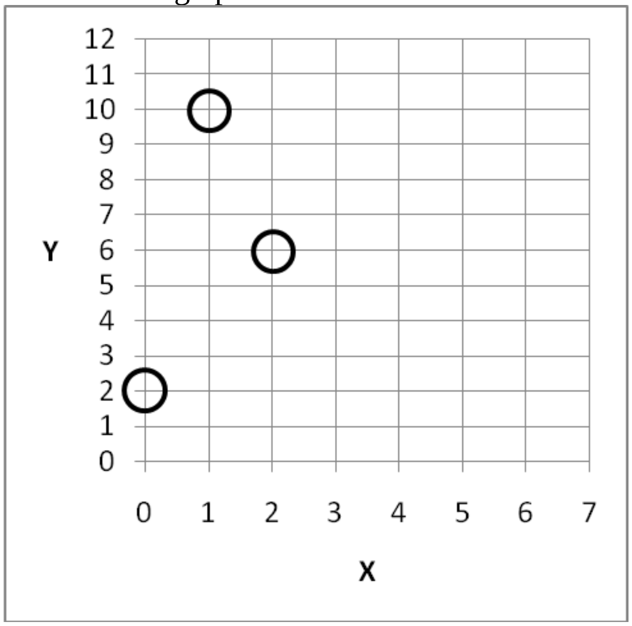
a. critical point of the test.
b. power of the test.
c. size or significance of the test.
d. test statistic.
e. standard error.

(14) Suppose the p-value for a test statistic is 0.037. If the size of the test is 1 percent, we

a. can reject the null hypothesis.
b. cannot reject the null hypothesis.
c. cannot compute the test statistic.
d. answer cannot be determined from the information given.

II. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Least-squares calculation: 15 pts] Suppose we have three observations on x_i and y_i as shown in the graph below.



a. Compute $\hat{\beta}_2$, the least-squares estimate of the slope of the line $y = \beta_1 + \beta_2 x$.

b. Compute $\hat{\beta}_1$, the least-squares estimate of the y-intercept of the same line.

c. Compute the three fitted values $\hat{y}_i$ of this least-squares estimated regression line.

d. Compute the three residuals $\hat{\varepsilon}_i$ of this estimated least-squares regression line.

e. Sketch the least-squares estimated line in the graph above.

(2) [Moments: 12 pts] Suppose X_1 and X_2 are random variables with the following moments.

$E(X_1) = 6$ $\quad$ $Var(X_1) = 4$ $\quad$ $Cov(X_1,X_2) = 0.5$
$E(X_2) = 2$ $\quad$ $Var(X_2) = 25$

Now let $Y = 3X_1 + X_2$. Compute the following and circle your final answers.

a. Compute E(Y).

b. Compute Var(Y).

c. Compute SD(Y).

d. Compute $Corr(X_1,X_2)$.

(3) [Estimation: 12 pts] Suppose we wish to estimate the mean of a population using the following (peculiar) estimator applied to a random sample of 10 observations.

$$\hat{\mu} = -5 + \frac{1}{8}\sum_{i=1}^{10} x_i$$

Compute the following properties of the estimator under the assumption that the true population mean is $E(X_i) = 12$ and the true population variance is $Var(X_i) = 32$. Circle your final answers.

a. Compute $E(\hat{\mu})$.

b. Compute $Bias(\hat{\mu})$.

c. Compute $Var(\hat{\mu})$.

d. Compute $MSE(\hat{\mu})$.

(4) [Inference for arbitrary distribution, large sample: 18 pts] Suppose we wish to analyze the distribution of the number of cars per household in a population. Let μ denote the unknown true population mean number of cars per household. Observations X_i have been collected on 400 households, with the following summary values. Here, $\overline{X}$ is the sample mean.

$$\sum_{i=1}^{400} X_i = 880 \qquad \sum_{i=1}^{400} \left(X_i - \overline{X}\right)^2 = 3136$$

a. [3 pts] Is the population distribution discrete or continuous? Justify your answer.

b. [3 pts] Compute an unbiased estimate of μ.

c. [3 pts] Compute the standard error of your estimate of μ.

d. [3 pts] Compute a 95% asymptotic confidence interval for μ.

e. [6 pts] Test the null hypothesis that $\mu = 2$ against the one-sided alternative hypothesis that $\mu > 2$, at 5% significance using an asymptotic test. Give the value of the test statistic, the critical point from a table, and your conclusion (whether you can reject null hypothesis).

Value of test statistic = ________________. Critical point(s) = __________________.

Reject null hypothesis? ______________________.

[end of exam]