Introduction to Econometrics (Econ 107)
Drake University, Fall 2007
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MIDTERM EXAMINATION #4 VERSION A
"Multiple Regression With Time-Series Data"
December 4, 2007

INSTRUCTIONS: This exam is closed-book, closed-notes. You may use a calculator for this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. Tables of the t distribution, F distribution, and chi-square distribution are attached.

NOTATION: In this exam, $\hat{\beta}_j$ denotes the least-squares coefficient estimators of the line $y_t = \beta_1 + \beta_2 x_{t2}, +\ldots+ \beta_K x_{tK} + \varepsilon_t$. The least-squares fitted value is denoted $\hat{y}_t$. The least-squares residual is denoted $\hat{\varepsilon}_t$. The sample size is denoted T. The true or population value of the variance of the unobserved error term ε_t is denoted σ^2. The (unbiased) least-squares estimator of σ^2 is denoted $\hat{\sigma}^2$. The sample mean of y is denoted $\bar{y}$.

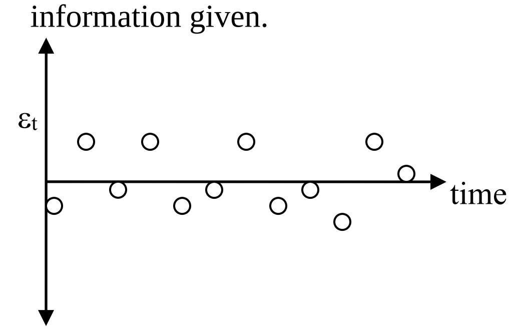
I. MULTIPLE CHOICE: Circle the one best answer to each question. Feel free to use margins for scratch work [2 pts each—24 pts total]

(1) Which is *not* a static model?
a. $y_t = 0.4 + 0.2\, x_t + \varepsilon_t$.
b. $y_t = 1.4 + 2.7\, x_{t-1} + \varepsilon_t$.
c. $y_t = 5.1 + 3.7\, x_{t,2} - 2.3\, x_{t,3} + \varepsilon_t$.
d. $y_t = 2.5 + 1.7\, x_t + \varepsilon_t$.

(2) In the time-series model
$$y_t = \beta_1 + \beta_2 x_{t,2} + \beta_3 x_{t,3} + \varepsilon_t ,$$
"strict exogeneity of regressors" means
a. $Var(\varepsilon_t) = \sigma^2$, a constant.
b. $E(\varepsilon_t) = 0$.
c. $E(X) = 0$.
d. $E(\varepsilon_t | x_{t,2}, x_{t,3}) = 0$.
e. $E(\varepsilon_t | X) = 0$.

(3) Which model below contains an exponential time trend?
a. $y_t = 5.2 + 0.02\, t + 2.1\, x_t$.
b. $\ln(y_t) = 0.2 + 0.03\, t + 1.1\, x_t$.
c. $\exp(y_t) = 0.3 + 0.43\, t + 1.6\, x_t$.
d. $y_t = 3.2 + 0.12\, t + 0.05 \exp(x_t)$.

(4) The time series ε_t graphed below appears to be
a. positively serially-correlated.
b. negatively serially-correlated.
c. serially uncorrelated.
d. Cannot be determined from the information given.



(5) Which of the following is a stationary first-order autoregressive process ["AR(1)"]?

a. $u_t = 0.3\, u_{t-1} - 0.1\, u_{t-2} + \varepsilon_t$.
b. $u_t = 0.07 + u_{t-1} + \varepsilon_t$.
c. $u_t = \varepsilon_t + 0.8\, \varepsilon_{t-1}$.
d. $u_t = u_{t-1} + \varepsilon_t$.
e. $u_t = 0.2\, u_{t-1} + \varepsilon_t$.
f. $u_t = \varepsilon_t + 0.3\, \varepsilon_{t-1} + 0.1\, \varepsilon_{t-2}$.

(6) Under the null hypothesis of no serial correlation, the Durbin-Watson test statistic is close to

a. minus one.
b. zero.
c. one.
d. two.
e. four.

(7) If the Durbin-Watson statistic equals 1.2, then an estimate of the serial correlation parameter $\rho = \text{Corr}(\varepsilon_t, \varepsilon_{t-1})$ is

a. 0.2
b. 0.3
c. 0.4.
d. 0.5.
e. 0.6.
f. 1.2.

(8) By definition, if the random process u_t has a unit root,

a. $u_t = 1$.
b. $E(u_t) = 1$.
c. $Var(u_t) = 1$.
d. the square root of $u_t = 1$.
e. u_t does not tend to revert back to its mean.

(9) If y_t and x_t are two independent random walks, then a regression of y_t on x_t will typically produce

a. an excessively small (in absolute value) t-statistic for the coefficient of x_t.
b. an excessively large (in absolute value) t-statistic for the coefficient of x_t.
c. a valid t statistic for the coefficient of x_t .
d. an R-square value close to zero.

(10) To test whether y_t and x_t are cointegrated, we apply a Dickey-Fuller test to

a. x_t .
b. y_t .
c. $y_t - \beta x_t$.
d. both x_t and y_t .

(11) Compared to the standard normal distribution, the Dickey-Fuller distribution is

a. more concentrated around zero.
b. virtually identical.
c. shifted to the right on the real number line.
d. shifted to the left on the real number line.

(12) Suppose we estimate the following pair of regression equations:

$$gdp_t = \alpha_1 + \alpha_2\, gdp_{t-1} + \alpha_3\, money_{t-1} + \varepsilon_{yt}$$
$$money_t = \beta_1 + \beta_2\, money_{t-1} + \beta_3\, gdp_{t-1} + \varepsilon_{zt}$$

If we reject the hypothesis that $\alpha_3 = 0$, then we conclude that

a. gdp "Granger-causes" money.
b. money "Granger-causes" gdp.
c. both of the above.
d. none of the above.

II. MULTIPLE ANSWER: The questions below may have more than one correct answer. Write "YES" next to all correct answers and "NO" next to all incorrect answers.

(1) [4 pts] Suppose ε_t is an independent, identically-distributed process. Which of the following u_t are stationary, weakly dependent processes?

a. $u_t = 0.6\, u_{t-1} + \varepsilon_t$.

b. $u_t = 3 + u_{t-1} + \varepsilon_t$.

c. $u_t = u_{t-1} + \varepsilon_t$.

d. $u_t = \varepsilon_t + 0.3\, \varepsilon_{t-1}$.

(2) [5 pts] For the model $y_t = \beta_1 + \beta_2\, x_t + \varepsilon_t$, ordinary least squares yields consistent estimators of β_1 and β_2 if

a. ε_t is serially-correlated, but stationary and weakly dependent.

b. ε_t is an independent, identically-distributed process.

c. x_t and y_t are integrated processes but are not co-integrated.

d. x_t and y_t are co-integrated processes.

e. ε_t is a random walk.

(3) [5 pts] Which is a test for serial correlation of the error term in a regression equation?

a. Durbin-Watson test.

b. Breusch-Pagan test.

c. Breusch-Godfrey test.

d. Dubin’s h test and alternative test.

e. Goldfeld-Quandt test.

III. PROBLEMS: Please write your answers in the boxes on this question sheet.

(1) [Finite distributed lag: 6 pts] Suppose we have estimated the time-series model

$$y_t = 3.5 + 1.6\, x_t + 2.1\, x_{t-1} - 0.4\, x_{t-2} + 0.3\, x_{t-3} + \varepsilon_t\ .$$

a. Is this a *static* model or a *dynamic* model?

b. Compute the impact propensity (also called the "impact multiplier" or "short-run effect").

c. Compute the long-run propensity (also called the "long-run multiplier" or "long-run effect").

(2) [Breusch-Godfrey test: 8 pts] Suppose we have estimated the regression model

$$y_t = \beta_1 + \beta_2\, x_t + \beta_3\, y_{t-1} + \varepsilon_t$$

using 84 quarterly observations, but we fear that ε_t might have fourth-order serial correlation. Accordingly, we have used the least-squares residuals to estimate the auxiliary regression

$$\hat{\varepsilon}_t = \alpha_1 + \alpha_2 x_t + \alpha_3 y_{t-1} + \alpha_4 \hat{\varepsilon}_{t-1} + \alpha_5 \hat{\varepsilon}_{t-2} + \alpha_6 \hat{\varepsilon}_{t-3} + \alpha_7 \hat{\varepsilon}_{t-4} + \nu_t\ ,$$

where ν_t denotes the error term in the auxiliary regression. (Note that this auxiliary regression must be estimated on observations 5 through 84 of the original data.) The R^2 value from this auxiliary regression is 0.12 . Test the null hypothesis of no serial correlation at 5% significance. Give the value of the test statistic, its degrees of freedom, the critical point from the appropriate table, and your conclusion (whether you can reject the null hypothesis).

Degrees of freedom = _____

Value of test statistic = ________________ Critical point = ______________________

Reject null hypothesis? ________________________.

(3) [Quasi-differencing: 12 pts] Suppose we wish to estimate the time-series regression model

$$y_t = \beta_1 + \beta_2\, x_t + u_t\ .$$

However, we believe the error term is serially correlated, following the process

$$u_t = 0.2\, u_{t-1} + \varepsilon_t\ ,$$

where ε_t is an independent, identically-distributed random error term. We need to transform the data to eliminate the serial correlation. The table below shows the first three observations on y_t and x_{tt}. Compute transformed values of the second and third observations using the Cochrane-Orcutt method.

Obs.	Raw data		Transformed data		
t	y_t	x_t	y_t	Replacement for intercept	x_t
1	15	5			
2	20	15			
3	22	18			

(4) [Random walk: 10 pts] Consider the following process:

$$u_t = 4.7 + u_{t-1} + \varepsilon_t \,.$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 4$. Assume $u_0 = 0$.

a. Find a formula in terms of *t* for the unconditional mean $E(u_t)$.
b. Find a formula in terms of *t* for the unconditional variance $Var(u_t)$.
c. Is u_t a *stationary* process or a *nonstationary* process?
d. Let $\Delta u_t = u_t - u_{t-1}$. Compute $E(\Delta u_t)$.
e. Compute $Var(\Delta u_t)$.

(5) Dickey-Fuller test: 6 pts] We wish to test whether the rate of inflation has a unit root using an augmented Dickey-Fuller test. We have estimated the following by ordinary least squares, using a large sample, standard errors in parentheses.

$$\Delta inflation_t = \underset{(0.52)}{1.30} - \underset{(0.10)}{0.31}\; inflation_{t-1} + \underset{(0.12)}{0.13}\; \Delta inflation_{t-1}$$

a. Which is the null hypothesis of the Dickey-Fuller test: that inflation *does not have* a unit root, or that inflation *has* a unit root?
b. Compute the Dickey-Fuller test statistic.
c. The critical point at 5% significance for the augmented Dickey-Fuller test statistic is –2.86. Can you reject the null hypothesis at 5% significance?

(6) [Forecasting: 6 pts] Suppose we have estimated the following model for sales, using 120 quarterly observations from the first quarter of 1978 to the fourth quarter of 2007.

$$y_t = 56.3 + 0.03 \text{ trend} - 0.5\ q1_t - 0.3\ q2_t - 0.1\ q3_t + \varepsilon_t \ .$$

The regressor "trend" equals 1 in the first quarter of 1978, equals 2 in the second quarter of 1978, and so forth, and equals 120 in the fourth quarter of 2007. The regressors "q1," "q2," and "q3," are quarterly dummy variables for the first, second and third quarters respectively. The error term ε_t is an independent, identically-distributed process with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma^2$, constant.

a. If a dummy variable "q4" for the fourth quarter were also included, then what econometric problem would result?

b. Compute the forecast of y_{121}, the value of sales in the first quarter of 2008.

c. Compute the forecast of y_{122}, the value of sales in the second quarter of 2008.

(7) [Forecasting: 6 pts] Suppose we have estimated the following model.

$$y_t = 23.4 + 0.6\ y_{t-1} - 0.2\ y_{t-2} + \varepsilon_t \ .$$

where ε_t denotes an independent, identically-distributed process with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma^2$, constant. In our data set, $y_{T-1} = 50$ and $y_T = 40$. Compute the following forecast values.

a. Compute the forecast of y_{T+1}.

b. Compute the forecast of y_{T+2}.

c. Compute the limit of the forecast y_{T+h} as h approaches infinity. [Hint: this is the unconditional mean of the process.]

IV. CRITICAL THINKING: Write a one-paragraph essay answering one question below (your choice). [8 pts]

(1) Suppose you believe the price p_t of a certain commodity follows a random walk:

$$p_t = 0.07 + p_{t-1} + \varepsilon_t ,$$

where $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 4$. You have 50 observations of the price, with $p_1 = 25.2$ and $p_{50} = 30.1$, and you must forecast p_{51}. Your research assistant suggests two methods of computing the forecast. The first method gives $\hat{p}_{51} = 30.1 + 0.07 = 30.8$. The second method gives $\hat{p}_{51} = 25.2 + 50(0.07) = 28.7$. Which is the better forecast? Why?

(2) Suggest a pair of prices in the real world that economic reasoning suggests are cointegrated. Explain why you think the pair are cointegrated.

Circle the question you are answering . Please write your answer below. Full credit requires correct reasoning, legible writing, good grammar including complete sentences, and accurate spelling.

[end of exam]