Introduction to Econometrics (Econ 107)
Drake University, Fall 2007
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FINAL EXAMINATION VERSION B
December 18, 2007

INSTRUCTIONS: This exam is closed-book, closed-notes. You may use a calculator for this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. Tables of the t distribution, F distribution, and chi-square distribution are attached.

NOTATION: In this exam, $\hat{\beta}_j$ denotes the least-squares coefficient estimators of the model $y = \beta_1 + \beta_2 x_2 + \ldots + \beta_K x_K + \varepsilon$. The least-squares fitted value is denoted $\hat{y}$. The least-squares residual is denoted $\hat{\varepsilon}$. In a cross-section sample, observations are indexed i=1,…,n. In a time-series sample, observations are indexed t=1,…,T. The true or population value of the variance of the unobserved error term ε is denoted σ^2. The (unbiased) least-squares estimator of σ^2 is denoted $\hat{\sigma}^2$. The sample mean of y is denoted $\bar{y}$.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Feel free to use margins for scratch work [1 pt each—16 pts total]

(1) Suppose we wish to fit the equation $y = \beta_1 + \beta_2 x$ to data by the method of least squares. This method minimizes which function of the data?

a. $f(\beta_1, \beta_2) = \sum \left(y_i^2 - (\beta_1 - \beta_2 x_i)^2\right)$.
b. $f(\beta_1, \beta_2) = \sum (y_i - \beta_1 - \beta_2 x_i)^2$.
c. $f(\beta_1, \beta_2) = \sum (y_i - \beta_1 - \beta_2 x_i)$.
d. $f(\beta_1, \beta_2) = \sum |y_i - \beta_1 - \beta_2 x_i|$.
e. $f(\beta_1, \beta_2) = \sum (\beta_1 + \beta_2 x_i)^2$.

(2) An estimator $\hat{\theta}$ of an unknown population parameter θ is said to be unbiased if

a. $E(\hat{\theta}) = \theta$.
b. $E(\hat{\theta}) = 0$.
c. $\lim_{n\to\infty} E(\hat{\theta}) = \theta$.
d. $\lim_{n\to\infty} \text{Prob}\left(|\hat{\theta} - \theta| > \delta\right) = 0$, for all $\delta > 0$.
e. $\lim_{n\to\infty} MSE(\hat{\theta}) = 0$.

(3) A wider confidence interval is obtained by

a. increasing the confidence level.
b. decreasing the confidence level.
c. increasing the sample size.
d. both (b) and (c).
e. none of the above.

(4) If the computed p-value for a test statistic is less than the size of the test, we

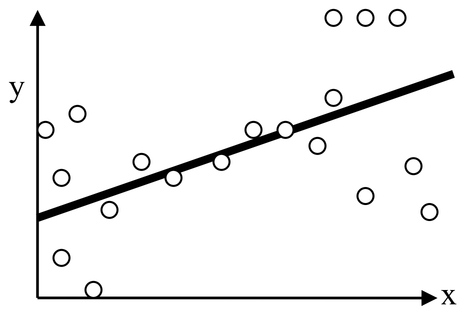
a. can reject the null hypothesis.
b. cannot reject the null hypothesis.
c. cannot compute the test statistic.
d. answer cannot be determined from the information given.

(5) In time-series data, any two variables are correlated in finite sample

a. if they both have trends.
b. only if one variable causes the other.
c. only if neither variable causes the other.
d. All of the above.
e. None of the above.

(6) According to which model is the elasticity of y with respect to x equal to 0.3?
a. $y = 2.5 + 0.3\,x$.
b. $y = 2.5 + 0.3\,(1/x)$.
c. $y = 2.5 + 0.3\ln(x)$.
d. $\ln(y) = 2.5 + 0.3\,x$.
e. $\ln(y) = 2.5 + 0.3\ln(x)$.

(7) In the graph below, the solid line is the true population regression line and the circles are observations in the sample. Which assumption appears to be violated in this sample?
a. $E(\varepsilon_i|x_i) = 0$.
b. Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant.
c. No autocorrelation: $Cov(\varepsilon_i,\varepsilon_j)=0$ for $i\neq j$.
d. All of the above.
e. None of the above.



(8) Adding another regressor to a regression equation will necessarily *increase*
a. the estimated coefficients.
b. the t statistics of the regressors.
c. the R^2 value.
d. Theil's adjusted R^2 value.
e. the sum of squared residuals.

(9) If two regressors x_{i2} and x_{i3} are *perfectly* correlated, then the least-squares estimators of their coefficients
a. will have large true standard errors.
b. will be zero.
c. cannot be computed.
d. will be biased.
e. will be inconsistent.

(10) The equation

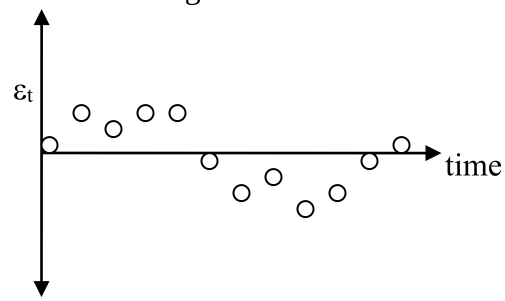
$$\ln(wage) = 2.8 + 0.09\,educ$$

implies that if *educ* increases by one unit, then *wage* will increase by about
a. \$9.00.
b. \$1.09.
c. \$0.09.
d. 9.0 percent.
e. 0.09 percent.

(11) Suppose Q = quantity demanded, P = price of the good, and I = consumer income. In which specification does 1.1 equal the income elasticity of demand?
a. $Q_i = 85.3 - 0.8\ln(P_i) + 1.1\ln(I_i)$.
b. $Q_i = 166.1 - 0.8\,P_i + 1.1\,I_i$.
c. $Q_i = 94.5 - 0.8\,(P_i/I_i)$.
d. $\ln(Q_i) = 4.5 - 0.8\,P_i + 1.1\,I_i$.
e. $\ln(Q_i) = 6.7 - 0.8\ln(P_i) + 1.1\ln(I_i)$.

(12) Which model below contains an exponential time trend?
a. $y_t = 5.2 + 0.02\,t + 2.1\,x_t$.
b. $\ln(y_t) = 0.2 + 0.03\,t + 1.1\,x_t$.
c. $\exp(y_t) = 0.3 + 0.43\,t + 1.6\,x_t$.
d. $y_t = 3.2 + 0.12\,t + 0.05\exp(x_t)$.

(13) The time series ε_t graphed below appears to be
a. positively serially-correlated.
b. negatively serially-correlated.
c. serially uncorrelated.
d. Cannot be determined from the information given.



(14) Which of the following is a stationary first-order autoregressive process ["AR(1)"]?
a. $u_t = 0.3\,u_{t-1} - 0.1\,u_{t-2} + \varepsilon_t$.

b. $u_t = 0.07 + u_{t-1} + \varepsilon_t$.
c. $u_t = \varepsilon_t + 0.8\,\varepsilon_{t-1}$.
d. $u_t = u_{t-1} + \varepsilon_t$.
e. $u_t = 0.2\,u_{t-1} + \varepsilon_t$.
f. $u_t = \varepsilon_t + 0.3\,\varepsilon_{t-1} + 0.1\,\varepsilon_{t-2}$.

(15) If y_t and x_t are two independent random walks, then a regression of y_t on x_t will typically produce

a. an excessively large (in absolute value) t-statistic for the coefficient of x_t.
b. a valid t statistic for the coefficient of x_t .
c. an excessively small (in absolute value) t-statistic for the coefficient of x_t.
d. an R-square value close to zero.

(16) To test whether y_t and x_t are cointegrated, we apply a Dickey-Fuller test to

a. x_t .
b. y_t .
c. $y_t - \beta x_t$.
d. both x_t and y_t .

II. MULTIPLE ANSWER: The questions below may have more than one correct answer. Write "YES" next to all correct answers and "NO" next to all incorrect answers.

(1) [5 pts] Which equations hold necessarily, regardless of the data or the model?

a. $\sum \hat{\varepsilon}_i y_i = 0$	
b. $\sum (y_i - \bar{y})^2 = \sum (\hat{y}_i - \bar{y})^2 + \sum \hat{\varepsilon}_i^2$	
c. $\sum \hat{\varepsilon}_i \hat{y}_i = 0$	
d. $\sum x_i \hat{\varepsilon}_i = 0$	
e. $\sum (x_i - \bar{x})(y_i - \bar{y}) = 0$	

(2) [5 pts] Which assumptions are required for the least-squares estimators to be "best linear unbiased estimators"?

a. Conditional mean of error term is zero: $E(\varepsilon_i \mid x_i) = 0$.	
b. Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant.	
c. No autocorrelation: $Cov(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$.	
d. Error term is normally-distributed: $\varepsilon_i \sim N(0, \sigma^2)$	
e. Sample mean of x variable is zero: $\bar{x} = 0$.	

(3) [4 pts] Suppose a demand function for milk of the form $y_i = \beta_1 + \beta_2 x_i$, is estimated by least squares. Here, y_i denotes quantity demanded in quarts and x_i denotes the price of milk in dollars. Now suppose the price data are converted to cents (there are 100 cents in a dollar).

a. $\hat{\beta}_1$ will decrease by a factor of 100.	
b. $\hat{\beta}_2$ will decrease by a factor of 100.	
c. The r^2 value will decrease by a factor of 100.	
d. The t-statistic for $\hat{\beta}_2$ will decrease by a factor of 100.	

(4) [4 pts] The variance of the least-squares slope estimator $\hat{\beta}_j$ is smaller, and thus the true value of β_j is estimated more precisely,

a. the larger the sample size.	
b. the larger the variance of the error term σ^2.	
c. the greater the variation of the x_{ij} values around the sample mean $\bar{x}_j$.	
d. the more closely correlated x_{ij} is with the other regressors.	

(5) [4 pts] Suppose ε_t is an independent, identically-distributed process. Which of the following u_t are stationary, weakly dependent processes?

a. $u_t = u_{t-1} + \varepsilon_t$.	
b. $u_t = \varepsilon_t + 0.7\,\varepsilon_{t-1}$.	
c. $u_t = 0.3\,u_{t-1} + \varepsilon_t$.	
d. $u_t = 2 + u_{t-1} + \varepsilon_t$.	

(6) [5 pts] For the model $y_t = \beta_1 + \beta_2 x_t + \varepsilon_t$, ordinary least squares yields consistent estimators of β_1 and β_2 if

a. ε_t is serially-correlated, but stationary and weakly dependent.	
b. ε_t is an independent, identically-distributed process.	
c. x_t and y_t are integrated processes but are not co-integrated.	
d. x_t and y_t are co-integrated processes.	
e. ε_t is a random walk.	

III. PROBLEMS: Please write your answers in the boxes on this question sheet.

(1) [Least-squares calculation: 12 pts] Suppose the following three observations on x_i and y_i are given.

Observation (i)	x_i	y_i
1	3	3
2	4	11
3	2	7

a. Compute $\hat{\beta}_2$, the least-squares estimate of the slope of the line $y = \beta_1 + \beta_2 x$.

b. Compute $\hat{\beta}_1$, the least-squares estimate of the y-intercept of the same line.

c. Compute the three fitted values $\hat{y}_i$ of this least-squares estimated regression line.

d. Compute the three residuals $\hat{\varepsilon}_i$ of this estimated least-squares regression line.

(2) [LS confidence intervals, tests, elasticity: 24 pts] The relationship between years of schooling and annual earnings is estimated for a sample of n=700 workers. For each worker i, let y_i denote weekly earnings and x_i denote the number of years of school completed. The model
$y_i = \beta_1 + \beta_2 x_i$ is estimated with the following results. The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors.

Weekly earnings	=	- 1045 (45.0)	+	165 (60.0)	Years of schooling completed

a. [3 pts] Suppose a worker had 12 years of schooling. According to these results, what would be this worker's predicted weekly earnings?

b. [3 pts] Suppose a worker acquired two additional years of schooling. By how much would earnings increase? That is, what is the predicted change Δy ?

c. [3 pts] Suppose the mean years of schooling in the sample is 13 and the mean weekly earnings is \$1100. (That is, $\bar{y} = 1100.$ and $\bar{x} = 13.$) Compute the estimated elasticity of earnings with respect to years of schooling at the sample means.

d. [6 pts] Compute a **95%** confidence interval for the **intercept, β_1** .

e. [9 pts] Test the hypothesis that schooling affects earnings, against the null hypothesis that schooling has no effect (a **two-tailed test**) at **5%** significance. Give the value of the test statistic, the critical points from a table, and your conclusion (whether you can reject null hypothesis).

Value of test statistic = ________________. Critical point(s) = __________________.

Reject null hypothesis? ______________________.

(3) [LS prediction: 12 pts] Using a sample of 400 houses, we have estimated an equation relating the selling price of a house (in thousands of dollars) to its size in square feet, its number of bathrooms, and whether the house has an attached two-car garage:

$$\underset{}{price} = \underset{(12.3)}{63.1} + \underset{(0.017)}{0.060}\ size + \underset{(1.2)}{5.2}\ baths + \underset{(3.3)}{19.5}\ garage$$

Here "garage" is a dummy variable equal to 1 if the house has an attached two-car garage, and equal to 0 otherwise.

a. Everything else equal, an increase in the size of the house by 500 square feet causes the price to increase by how much?

$ thousand

b. Everything else equal, an attached two-car garage causes the price to increase by how much?

$ thousand

Suppose we wish to predict the selling price of a house with size = 2000 square feet, three bathrooms, and an attached garage. So to simplify calculations, we first transform the data and estimate the same equation on the transformed data.

c. Which variables should be transformed? How?

Suppose the transformed data yield the following estimates:

$$\underset{}{price} = \underset{(5.0)}{218.2} + \underset{(0.017)}{0.060}\ size + \underset{(1.2)}{5.2}\ baths + \underset{(3.3)}{19.5}\ garage$$

The estimated variance of the error term is $\hat{\sigma}^2 = 11.0$.

d. Compute the predicted selling price of a house with size = 2000 square feet, three bathrooms, and an attached garage.

$ thousand

e. Compute the standard error of the prediction error.

f. Compute a 95% prediction interval for the selling price.

(4) [Finite distributed lag: 6 pts] Suppose we have estimated the time-series model

$$y_t = 3.5 + 1.6\ x_t + 2.1\ x_{t-1} - 0.4\ x_{t-2} + 0.3\ x_{t-3} + \varepsilon_t\ .$$

a. Is this a *static* model or a *dynamic* model?

b. Compute the impact propensity (also called the "impact multiplier" or "short-run effect").

c. Compute the long-run propensity (also called the "long-run multiplier" or "long-run effect").

IV. CRITICAL THINKING: Write a one-paragraph essay answering *one* question below (your choice). [3 pts]

(1) Suppose the following time-series model is estimated: $y_t = 5.3 + 1.7x_t + \varepsilon_t$. To check for serial correlation using Durbin's alternative test, an auxiliary regression equation must be estimated. By mistake the following auxiliary equation is estimated: $\hat{\varepsilon}_t = \alpha_1 + \alpha_2 x_t + \alpha_3 y_t + \nu_t$.

a. What is wrong with this auxiliary equation?
b. What values of α_1 , α_2 , α_3 , and ν_t will be obtained if this equation is estimated?

(2) Suppose you believe the price p_t of a certain commodity follows a random walk with drift:

$$p_t = 0.2 + p_{t-1} + \varepsilon_t\ ,$$

where $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 0.4$. You have 80 observations of the price, with $p_1 = 16.8$ and $p_{80} = 28.7$, and you must forecast p_{81}. Your research assistant suggests two methods of computing the forecast. The first method gives $\hat{p}_{81} = 16.8 + 80(0.2) = 32.8$. The second method gives $\hat{p}_{81} = 28.7 + 0.2 = 28.9$. Which method is the better forecast? Why?

[end of exam]