

FINAL EXAMINATION VERSION A
December 18, 2007

INSTRUCTIONS: This exam is closed-book, closed-notes. You may use a calculator for this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. Tables of the t distribution, F distribution, and chi-square distribution are attached.

NOTATION: In this exam, $\hat{\beta}_j$ denotes the least-squares coefficient estimators of the model $y = \beta_1 + \beta_2 x_2 + \dots + \beta_K x_K + \varepsilon$. The least-squares fitted value is denoted \hat{y} . The least-squares residual is denoted $\hat{\varepsilon}$. In a cross-section sample, observations are indexed $i=1, \dots, n$. In a time-series sample, observations are indexed $t=1, \dots, T$. The true or population value of the variance of the unobserved error term ε is denoted σ^2 . The (unbiased) least-squares estimator of σ^2 is denoted $\hat{\sigma}^2$. The sample mean of y is denoted \bar{y} .

I. MULTIPLE CHOICE: Circle the one best answer to each question. Feel free to use margins for scratch work [1 pt each—16 pts total]

(1) Suppose we wish to fit the equation $y = \beta_1 + \beta_2 x$ to data by the method of least squares. This method minimizes which function of the data?

- a. $f(\beta_1, \beta_2) = \sum (y_i - \beta_1 - \beta_2 x_i)$.
- b. $f(\beta_1, \beta_2) = \sum |y_i - \beta_1 - \beta_2 x_i|$.
- c. $f(\beta_1, \beta_2) = \sum (\beta_1 + \beta_2 x_i)^2$.
- d. $f(\beta_1, \beta_2) = \sum (y_i^2 - (\beta_1 - \beta_2 x_i)^2)$.
- e. $f(\beta_1, \beta_2) = \sum (y_i - \beta_1 - \beta_2 x_i)^2$.

(2) An estimator $\hat{\theta}$ of an unknown population parameter θ is said to be *consistent* if

- a. $E(\hat{\theta}) = \theta$.
- b. $E(\hat{\theta}) = 0$.
- c. $\lim_{n \rightarrow \infty} E(\hat{\theta}) = \theta$.
- d. $\lim_{n \rightarrow \infty} \text{Prob}(|\hat{\theta} - \theta| > \delta) = 0$, for all $\delta > 0$.
- e. $\lim_{n \rightarrow \infty} E(\hat{\theta}^2) = 0$.

(3) A narrower confidence interval is obtained by

- a. increasing the confidence level.
- b. decreasing the confidence level.
- c. increasing the sample size.
- d. both (b) and (c).
- e. none of the above.

(4) If the computed p-value for a test statistic is greater than the size of the test, we

- a. can reject the null hypothesis.
- b. cannot reject the null hypothesis.
- c. cannot compute the test statistic.
- d. answer cannot be determined from the information given.

(5) In time-series data, any two variables are correlated in finite sample

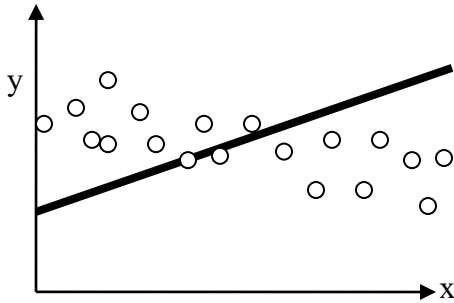
- a. only if one variable causes the other.
- b. only if neither variable causes the other.
- c. if they both have trends.
- d. All of the above.
- e. None of the above.

(6) According to which model does a one-unit change in x cause approximately a 0.03-unit increase in y ?

- a. $y = 2.5 + 0.03 x$.
- b. $y = 2.5 + 0.03 (1/x)$.
- c. $y = 2.5 + 0.03 \ln(x)$.
- d. $\ln(y) = 2.5 + 0.03 x$.
- e. $\ln(y) = 2.5 + 0.03 \ln(x)$.

(7) In the graph below, the solid line is the true population regression line and the circles are observations in the sample. Which assumption appears to be violated in this sample?

- a. $E(\varepsilon_i|x_i) = 0$.
- b. Homoskedasticity: $\text{Var}(\varepsilon_i) = \sigma^2$, a constant.
- c. No autocorrelation: $\text{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$.
- d. All of the above.
- e. None of the above.



(8) Adding another regressor to a regression equation will necessarily *decrease*

- a. the estimated coefficients.
- b. the t statistics of the regressors.
- c. the R^2 value.
- d. Theil's adjusted R^2 value.
- e. the sum of squared residuals.

(9) If two regressors x_{i2} and x_{i3} are closely *but not perfectly* correlated, then the least-squares estimators of their coefficients

- a. will have large standard errors.
- b. will be zero.
- c. cannot be computed.
- d. will be biased.
- e. will be inconsistent.

(10) The equation

$$\ln(\text{wage}) = 2.8 + 0.04 \text{educ}$$

implies that if *educ* increases by one unit, then *wage* will increase by about

- a. 4.0 percent.
- b. 0.04 percent.
- c. \$4.00.
- d. \$1.04.
- e. \$0.04.

(11) Suppose Q = quantity demanded, P = price of the good, and I = consumer income. In which specification does -0.8 equal the price elasticity of demand?

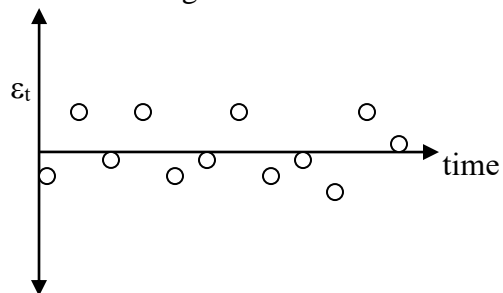
- a. $\ln(Q_i) = 4.5 - 0.8 P_i + 1.1 I_i$.
- b. $\ln(Q_i) = 6.7 - 0.8 \ln(P_i) + 1.1 \ln(I_i)$.
- c. $Q_i = 85.3 - 0.8 \ln(P_i) + 1.1 \ln(I_i)$.
- d. $Q_i = 166.1 - 0.8 P_i + 1.1 I_i$.
- e. $Q_i = 94.5 - 0.8 (P_i/I_i)$.

(12) Which model below contains an exponential time trend?

- a. $y_t = 5.2 + 0.02 t + 2.1 x_t$.
- b. $\exp(y_t) = 0.3 + 0.43 t + 1.6 x_t$.
- c. $y_t = 3.2 + 0.12 t + 0.05 \exp(x_t)$.
- d. $\ln(y_t) = 0.2 + 0.03 t + 1.1 x_t$.

(13) The time series ε_t graphed below appears to be

- a. positively serially-correlated.
- b. negatively serially-correlated.
- c. serially uncorrelated.
- d. Cannot be determined from the information given.



(14) Which of the following is a first-order moving-average process ["MA(1)"]?

- a. $u_t = u_{t-1} + \varepsilon_t$.

- b. $u_t = 0.2 u_{t-1} + \varepsilon_t$.
- c. $u_t = \varepsilon_t + 0.3 \varepsilon_{t-1} + 0.1 \varepsilon_{t-2}$.
- d. $u_t = 0.3 u_{t-1} - 0.1 u_{t-2} + \varepsilon_t$.
- e. $u_t = 0.07 + u_{t-1} + \varepsilon_t$.
- f. $u_t = \varepsilon_t + 0.8 \varepsilon_{t-1}$.

(15) If y_t and x_t are two independent random walks, then a regression of y_t on x_t will typically produce

- a. an excessively small (in absolute value) t-statistic for the coefficient of x_t .
- b. an excessively large (in absolute value) t-statistic for the coefficient of x_t .
- c. a valid t statistic for the coefficient of x_t .
- d. an R-square value close to zero.

(16) To test whether y_t and x_t are cointegrated, we apply a Dickey-Fuller test to

- a. both x_t and y_t .
- b. x_t alone.
- c. y_t alone.
- d. $y_t - \beta x_t$.

II. MULTIPLE ANSWER: The questions below may have more than one correct answer. Write “YES” next to all correct answers and “NO” next to all incorrect answers.

(1) [5 pts] Which equations hold necessarily, regardless of the data or the model?

- a. $\sum x_i \hat{\varepsilon}_i = 0$
- b. $\sum (\hat{y}_i - \bar{y})^2 = \sum (y_i - \bar{y})^2 + \sum \hat{\varepsilon}_i^2$
- c. $\sum x_i \hat{y}_i = 0$
- d. $\sum (x_i - \bar{x})(y_i - \bar{y}) = 0$
- e. $\sum \hat{\varepsilon}_i \hat{y}_i = 0$

(2) [5 pts] Which assumptions are required for the least-squares estimators to be unbiased estimators?

- a. Conditional mean of error term is zero: $E(\varepsilon_i | x_i) = 0$.
- b. Homoskedasticity: $\text{Var}(\varepsilon_i) = \sigma^2$, a constant.
- c. No autocorrelation: $\text{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$.
- d. Error term is normally-distributed: $\varepsilon_i \sim N(0, \sigma^2)$
- e. Sample mean of x variable is zero: $\bar{x} = 0$.

(3) [4 pts] Suppose a demand function for milk of the form $y_i = \beta_1 + \beta_2 x_i$, is estimated by least squares. Here, y_i denotes quantity demanded in quarts and x_i denotes the price of milk in dollars. Now suppose the quantity data are converted to gallons (there are 4 quarts in a gallon).

- a. $\hat{\beta}_1$ will decrease by a factor of 4.
- b. $\hat{\beta}_2$ will decrease by a factor of 4.
- c. The r^2 value will decrease by a factor of 4.
- d. The t-statistic for $\hat{\beta}_2$ will decrease by a factor of 4.

(4) [4 pts] The variance of the least-squares slope estimator $\hat{\beta}_j$ is larger, and thus the true value of β_j is estimated less precisely,

- a. the larger the sample size.
- b. the larger the variance of the error term σ^2 .
- c. the greater the variation of the x_{ij} values around the sample mean \bar{x}_j .
- d. the more closely correlated x_{ij} is with the other regressors.

(5) [4 pts] Suppose ε_t is an independent, identically-distributed process. Which of the following u_t are stationary, weakly dependent processes?

- a. $u_t = 0.6 u_{t-1} + \varepsilon_t$.
- b. $u_t = 3 + u_{t-1} + \varepsilon_t$.
- c. $u_t = u_{t-1} + \varepsilon_t$.
- d. $u_t = \varepsilon_t + 0.3 \varepsilon_{t-1}$.

(6) [5 pts] For the model $y_t = \beta_1 + \beta_2 x_t + \varepsilon_t$, ordinary least squares yields consistent estimators of β_1 and β_2 if

- a. x_t and y_t are integrated processes but are not co-integrated.
- b. x_t and y_t are co-integrated processes.
- c. ε_t is a random walk.
- d. ε_t is serially-correlated, but stationary and weakly dependent.
- e. ε_t is an independent, identically-distributed process.

III. PROBLEMS: Please write your answers in the boxes on this question sheet.

(1) [Least-squares calculation: 12 pts] Suppose the following three observations on x_i and y_i are given.

<i>Observation (i)</i>	x_i	y_i
1	2	6
2	1	7
3	3	8

a. Compute $\hat{\beta}_2$, the least-squares estimate of the slope of the line $y = \beta_1 + \beta_2 x$.

b. Compute $\hat{\beta}_1$, the least-squares estimate of the y-intercept of the same line.

c. Compute the three fitted values \hat{y}_i of this least-squares estimated regression line.

d. Compute the three residuals $\hat{\varepsilon}_i$ of this estimated least-squares regression line.

(2) [LS confidence intervals, tests, elasticity: 24 pts] The relationship between years of schooling and annual earnings is estimated for a sample of $n=600$ workers. For each worker i , let y_i denote weekly earnings and x_i denote the number of years of school completed. The model

$y_i = \beta_1 + \beta_2 x_i$ is estimated with the following results. The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors.

$$\begin{array}{rcccl} \text{Weekly} & = & -1920 & + & 240 & \text{Years of schooling} \\ \text{earnings} & & (55.0) & & (60.0) & \text{completed} \end{array}$$

- a. [3 pts] Suppose a worker had 12 years of schooling. According to these results, what would be this worker's predicted weekly earnings?

- b. [3 pts] Suppose a worker acquired two additional years of schooling. By how much would earnings increase? That is, what is the predicted change Δy ?

- c. [3 pts] Suppose the mean years of schooling in the sample is 13 and the mean weekly earnings is \$1200. (That is, $\bar{y} = 1200$. and $\bar{x} = 13$.) Compute the estimated elasticity of earnings with respect to years of schooling at the sample means.

- d. [6 pts] Compute a **95%** confidence interval for the **intercept, β_1** .

- e. [9 pts] Test the hypothesis that schooling affects earnings, against the null hypothesis that schooling has no effect (a **two-tailed test**) at **5%** significance. Give the value of the test statistic, the critical points from a table, and your conclusion (whether you can reject null hypothesis).

Value of test statistic = _____ . Critical point(s) = _____ .

Reject null hypothesis? _____ .

(3) [LS prediction: 12 pts] Using a sample of 400 houses, we have estimated an equation relating the selling price of a house (in thousands of dollars) to its size in square feet, its number of bathrooms, and whether the house has an attached garage:

$$price = 66.2 + 0.050 \text{ size} + 5.1 \text{ baths} + 22.4 \text{ garage}$$

(12.3)
(0.017)
(1.2)
(3.3)

Here, standard errors are in parentheses, and “garage” is a dummy variable equal to 1 if the house has an attached garage, and equal to 0 otherwise.

- a. Everything else equal, an increase in the size of the house by 500 square feet causes the price to increase by how much?
- b. Everything else equal, an attached two-car garage causes the price to increase by how much?

\$	thousand
\$	thousand

Suppose we wish to predict the selling price of a house with size = 2000 square feet, three bathrooms, and an attached garage. So to simplify calculations, we first transform the data and estimate the same equation on the transformed data.

- c. Which variables should be transformed? How?

Suppose the transformed data yield the following estimates:

$$price = 203.9 + 0.050 \text{ size} + 5.1 \text{ baths} + 22.4 \text{ garage}$$

(4.0)
(0.017)
(1.2)
(3.3)

The estimated variance of the error term is $\hat{\sigma}^2 = 9.0$.

- d. Compute the predicted selling price of a house with size = 2000 square feet, three bathrooms, and an attached garage.
- e. Compute the standard error of the prediction error.

\$	thousand

- f. Compute a 95% prediction interval for the selling price.

(4) [Finite distributed lag: 6 pts] Suppose we have estimated the time-series model

$$y_t = 2.7 + 1.4 x_t + 1.1 x_{t-1} - 0.3 x_{t-2} + 0.2 x_{t-3} + \varepsilon_t .$$

- a. Is this a *static* model or a *dynamic* model?
- b. Compute the impact propensity (also called the "impact multiplier" or "short-run effect").
- c. Compute the long-run propensity (also called the "long-run multiplier" or "long-run effect").

IV. CRITICAL THINKING: Write a one-paragraph essay answering *one* question below (your choice). [3 pts]

(1) Suppose the following time-series model is estimated: $y_t = 5.3 + 1.7x_t + \varepsilon_t$. To check for serial correlation using Durbin's alternative test, an auxiliary regression equation must be estimated. By mistake the following auxiliary equation is estimated: $\hat{\varepsilon}_t = \alpha_1 + \alpha_2 x_t + \alpha_3 y_t + v_t$.

- a. What is wrong with this auxiliary equation?
- b. What values of α_1 , α_2 , α_3 , and v_t will be obtained if this equation is estimated?

(2) Suppose you believe the price p_t of a certain commodity follows a random walk with drift:

$$p_t = 0.2 + p_{t-1} + \varepsilon_t ,$$

where $E(\varepsilon_t) = 0$ and $\text{Var}(\varepsilon_t) = 0.4$. You have 80 observations of the price, with $p_1 = 16.8$ and $p_{80} = 28.7$, and you must forecast p_{81} . Your research assistant suggests two methods of computing the forecast. The first method gives $\hat{p}_{81} = 16.8 + 80(0.2) = 32.8$. The second method gives $\hat{p}_{81} = 28.7 + 0.2 = 28.9$. Which method is the better forecast? Why?

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[end of exam]