

MIDTERM EXAMINATION #4 VERSION A
“Time Series”
April 25, 2006

INSTRUCTIONS: This exam is closed-book, closed-notes. You may use a calculator for this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. Tables of the t distribution chi-square distribution are attached.

NOTATION: In this exam, $\hat{\beta}_j$ denotes the least-squares coefficient estimators of the line $y_t = \beta_1 + \beta_2 x_{t,2} + \dots + \beta_K x_{t,K} + \varepsilon_t$. The least-squares fitted value is denoted \hat{y}_t . The least-squares residual is denoted $\hat{\varepsilon}_t$. The sample size is denoted T . The true or population value of the variance of the unobserved error term ε_t is denoted σ^2 . The (unbiased) least-squares estimator of σ^2 is denoted $\hat{\sigma}^2$. The sample mean of y is denoted \bar{y} .

I. MULTIPLE CHOICE: Circle the one best answer to each question. Feel free to use margins for scratch work [3 pts each—33 pts total]

(1) Which is not a static model?

- a. $y_t = 0.3 + 0.6 x_t + \varepsilon_t$.
- b. $y_t = 7.6 + 2.1 x_{t,2} + 4.3 x_{t,3} + \varepsilon_t$.
- c. $y_t = 4.6 - 2.1 x_t + \varepsilon_t$.
- d. $y_t = 6.2 + 4.1 x_{t-1} + \varepsilon_t$.

(2) In the time-series model

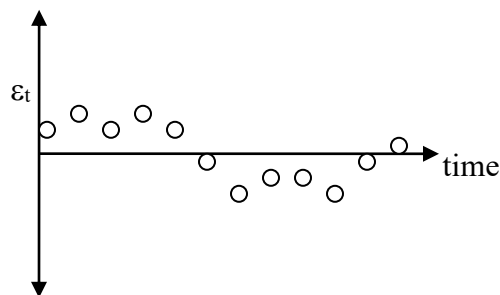
$$y_t = \beta_1 + \beta_2 x_{t,2} + \beta_3 x_{t,3} + \varepsilon_t,$$

"strict exogeneity of regressors" means

- a. $E(\varepsilon_t) = 0$.
- b. $E(\varepsilon_t | x_{t,2}, x_{t,3}) = 0$.
- c. $E(\varepsilon_t | X) = 0$.
- d. $E(X) = 0$.
- e. $\text{Var}(\varepsilon_t) = \sigma^2$, a constant.

(3) The time series ε_t graphed below is

- a. positively serially-correlated.
- b. negatively serially-correlated.
- c. serially uncorrelated.
- d. Cannot be determined from the information given.



(4) Consider the time-series regression model

$$y_t = \beta_1 + \beta_2 x_{t,2} + \beta_3 x_{t,3} + \varepsilon_t.$$

If the data are stationary and weakly dependent, and the regressors are contemporaneously exogenous:

$$E(\varepsilon_t | x_{t,2}, x_{t,3}) = 0,$$

then the least squares estimators for the β coefficients

- a. will be biased.
- b. will be inconsistent.
- c. will have large standard errors.
- d. cannot be computed.
- e. None of the above.

(5) Under the null hypothesis of no serial correlation, the Durbin-Watson test statistic is close to

- a. minus one.
- b. zero.
- c. one.
- d. two.
- e. four.

(6) If the Durbin-Watson statistic equals 1.4, then an estimate of the autocorrelation parameter $\rho = \text{Corr}(\varepsilon_t, \varepsilon_{t-1})$ is

- a. 0.2
- b. 0.3
- c. 0.4.
- d. 0.6.
- e. 1.4.

(7) By definition, if the random process u_t has a unit root,

- a. $E(u_t) = 1$.
- b. $\text{Var}(u_t) = 1$.
- c. it does not tend to revert back to its mean.
- d. $u_t = 1$.
- e. the square root of $u_t = 1$.

(8) If y_t and x_t are two independent random walks with drift, then a regression of y_t on x_t will typically produce

- a. a valid t statistic for the coefficient of x_t .
- b. an excessively small (in absolute value) t-statistic for the coefficient of x_t .
- c. an excessively large (in absolute value) t-statistic for the coefficient of x_t .
- d. an R-square value close to zero.

(9) To test whether y_t and x_t are cointegrated, we apply a Dickey-Fuller test to

- a. x_t .
- b. y_t .
- c. both x_t and y_t .
- d. $y_t - \beta x_t$.

(10) Compared to the standard normal distribution, the Dickey-Fuller distribution is

- a. virtually identical.
- b. shifted to the right on the real number line.
- c. shifted to the left on the real number line.
- d. more concentrated around zero.

(11) Suppose we estimate the following pair of regression equations:

$$\begin{aligned} \text{gdp}_t &= \alpha_1 + \alpha_2 \text{gdp}_{t-1} + \alpha_3 \text{msupply}_{t-1} + \varepsilon_{yt} \\ \text{msupply}_t &= \beta_1 + \beta_2 \text{msupply}_{t-1} + \beta_3 \text{gdp}_{t-1} + \varepsilon_{zt} \end{aligned}$$

If we reject the hypothesis that $\alpha_3 = 0$, then we conclude that

- a. gdp "Granger-causes" msupply.
- b. msupply "Granger-causes" gdp.
- c. both of the above.
- d. none of the above.

II. PROBLEMS: Please write your answers in the boxes on this question sheet.

(1) [Finite distributed lag: 9 pts] Suppose we have estimated the time-series model

$$y_t = 4.5 + 3.7 x_t + 2.5 x_{t-1} + 0.8 x_{t-2} + \varepsilon_t .$$

- a. Is this a *static* model or a *dynamic* model?
- b. Compute the impact propensity (also called the "impact multiplier" or "short-run effect").
- c. Compute the long-run propensity (also called the "long-run multiplier" or "long-run effect").

(2) [Breusch-Godfrey test: 12 pts] Suppose we have estimated the regression

$$y_t = \beta_1 + \beta_2 x_{t,2} + \beta_3 x_{t,3} + \beta_4 y_{t-1} + \varepsilon_t$$

using 61 annual observations, but we fear that ε_t might be serially correlated. Accordingly, we have used the residuals to estimate the auxiliary regression.

$$\hat{\varepsilon}_t = \alpha_1 + \alpha_2 x_{t,2} + \alpha_3 x_{t,3} + \alpha_4 y_{t-1} + \alpha_5 \hat{\varepsilon}_{t-1} + v_t$$

where v_t denotes the error term in the auxiliary regression. (Note that this auxiliary regression must be estimated on observations 2 through 61 of the original data.) The R^2 value from this auxiliary regression is 0.08 . Test the null hypothesis of no serial correlation at 5% significance. Give the value of the test statistic, its degrees of freedom, the critical point, and your conclusion (whether you can reject the null hypothesis).

Degrees of freedom = _____
Value of test statistic = _____ Critical point = _____
Reject null hypothesis? _____.

(3) [Quasi-differencing: 16 pts] Suppose we wish to estimate the time-series regression model

$$y_t = \beta_1 + \beta_2 x_{t,2} + \beta_3 x_{t,3} + u_t .$$

However, we believe the error term is serially correlated, following the AR(1) process

$$u_t = 0.4 u_{t-1} + \varepsilon_t ,$$

where ε_t is an independent, identically-distributed random error term. We need to transform the data to eliminate the serial correlation. The table below shows the first three observations on y_t , x_{2t} , and x_{3t} . Compute transformed values of the second and third observations using the Cochrane-Orcutt method.

Obs.	Raw data			Transformed data			
	y_t	x_{2t}	x_{3t}	y_t	Replacement for intercept	x_{2t}	x_{3t}
1	25	10	5				
2	30	15	10				
3	32	18	7				

(4) [Random walk: 15 pts] Consider the following random-walk process with drift:

$$u_t = 4.7 + u_{t-1} + \varepsilon_t .$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $\text{Var}(\varepsilon_t) = 4$. Assume $u_0 = 0$.

- a. Find a formula in terms of t for $E(u_t)$.
- b. Find a formula in terms of t for $\text{Var}(u_t)$.
- c. Is u_t a *stationary* process or a *nonstationary* process?
- d. Compute $E(\Delta u_t)$.
- e. Compute $\text{Var}(\Delta u_t)$.

(5) [Forecasting: 9 pts] Suppose we have estimated the following AR(2) model.

$$y_t = 23.4 + 0.6 y_{t-1} - 0.2 y_{t-2} + \varepsilon_t .$$

where ε_t denotes an independent, identically-distributed series with $E(\varepsilon_t) = 0$ and $\text{Var}(\varepsilon_t) = \sigma^2$, constant. In our data set, $y_{T-1} = 50$ and $y_T = 40$. Compute the following forecast values.

- a. Compute the forecast of y_{T+1} .
- b. Compute the forecast of y_{T+2} .
- c. Compute the limit of the forecast y_{T+h} as h approaches infinity.
[Hint: this is the unconditional mean of the process.]

III. CRITICAL THINKING [6 pts] Suggest a pair of prices in the real world that economic reasoning suggests might be cointegrated. Explain why you think the pair might be cointegrated.

[end of exam]