Introduction to Econometrics (Econ 107)
Drake University, Spring 2006
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MIDTERM EXAMINATION #2 VERSION B
"Two-Variable Regression"
February 23, 2006

INSTRUCTIONS: This exam is closed-book, closed-notes. You may use a calculator on this exam, but not a graphing calculator or a calculator with alphabetical keys. Point values for each question are noted in brackets. A table of the t-distribution is attached.

NOTATION: In this exam, $\hat{\beta}_1$ and $\hat{\beta}_2$ denote the least-squares estimators of the intercept and slope of the line $y_i = \beta_1 + \beta_2 x_i, + \varepsilon_i$, $\hat{y}_i$ denotes a least-squares fitted value, $\hat{\varepsilon}_i$ denotes a least-squares residual, and the sample size is denoted n. The true or population value of the variance of the unobserved error term ε_i is denoted σ^2. The (unbiased) least-squares estimate of σ^2 is denoted $\hat{\sigma}^2$. The sample means of x and y are denoted $\bar{x}$ and $\bar{y}$ respectively.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Feel free to use margins for scratch work [3 pts each—48 pts total]

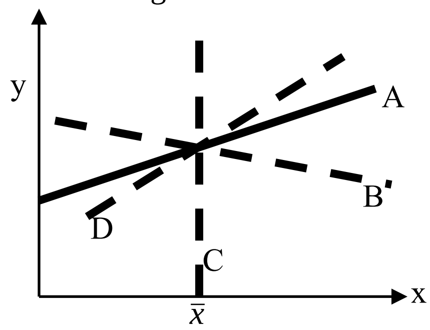
(1) Which equation holds necessarily, regardless of the data?
a. $\sum x_i y_i = 0$.
b. $\sum x_i \hat{y}_i = 0$.
c. $\sum y_i \hat{\varepsilon}_i = 0$.
d. $\sum (x_i - \bar{x})(y_i - \bar{y}) = 0$.
e. $\sum (y_i - \bar{y})^2 = \sum (\hat{y}_i - \bar{y})^2 + \sum \hat{\varepsilon}_i^2$.

(2) In the model $y_i = \beta_1 + \beta_2 x_i, + \varepsilon_i$, assuming $E(\varepsilon_i|x_i)=0$, the conditional mean of y (that is, $E(y_i|x_i)$) is
a. β_1 .
b. β_2 .
c. $\beta_2 x_i$.
d. $\beta_1 + \beta_2 x_i$.
e. one.

(3) Which assumption is required for the least-squares estimators to be unbiased?
a. $E(\varepsilon_i|x_i) = 0$.
b. Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant.
c. No autocorrelation: $Cov(\varepsilon_i,\varepsilon_j) = 0$ for $i \neq j$.
d. All of the above.
e. None of the above.

(4) Which assumption is required for the sum of the least-squares residuals to equal zero?
a. $E(\varepsilon_i|x_i) = 0$.
b. Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant.
c. No autocorrelation: $Cov(\varepsilon_i,\varepsilon_j) = 0$ for $i \neq j$.
d. All of the above.
e. None of the above.

(5) In the graph below, the solid line denoted "A" is the true population regression line. If the error term has mean zero but is *negatively correlated* with x, then the least-squares estimated line will tend to resemble
a. line A.
b. line B.
c. line C.
d. line D.
e. cannot be determined from the information given.



(6) Which assumption is required for the least-squares estimators to be method-of-moments estimators?
a. $E(\varepsilon_i|x_i) = 0$.
b. Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant.
c. No autocorrelation: $Cov(\varepsilon_i,\varepsilon_j) = 0$ for $i \neq j$.
d. All of the above.
e. None of the above.

(7) Which assumption is required for the least-squares estimators to be Best Linear Unbiased Estimators?
a. $E(\varepsilon_i|x_i) = 0$.
b. Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant.
c. No autocorrelation: $Cov(\varepsilon_i,\varepsilon_j) = 0$ for $i \neq j$.
d. All of the above.
e. None of the above.

(8) If the data observations fit exactly on a straight line, then the least-squares estimate of the variance of the error term would be
a. $\hat{\sigma}^2 = -1$.
b. $\hat{\sigma}^2 = 0$.
c. $\hat{\sigma}^2 = 1$.
d. $\hat{\sigma}^2 = n$.
e. None of the above.

(9) The variance of the least-squares slope estimator $\hat{\beta}_2$ is larger, and thus the true value of β_2 is estimated less precisely,
a. the larger the sample size.
b. the larger the variance of the error term σ^2.
c. the larger the variation of the x values around the sample mean $\bar{x}$.
d. All of the above.
e. None of the above.

(10) The assumption that the error term is normally distributed:
$$\varepsilon_i \sim N(0, \sigma^2)$$
is required in order for the least-squares estimators to be
a. consistent.
b. best linear unbiased estimators.
c. best unbiased estimators.
d. asymptotically normally-distributed (that is, approximately normally-distributed in large samples).
e. All of the above.

(11) Suppose we use the least-squares predictor $(\hat{y}_{n+1} = \hat{\beta}_1 + \hat{\beta}_2 x_{n+1})$ to predict y_{n+1}. The variance of the prediction error $(y_{n+1} - \hat{y}_{n+1})$ is smaller, and thus prediction is more precise,
a. the smaller the variance of the error term σ^2.
b. the farther x_{n+1} is from $\bar{x}$.
c. the smaller the variation of the x-values in our sample around $\bar{x}$.
d. All of the above.
e. None of the above.

(12) The variance of the prediction error tends to decrease as the sample size used for estimation increases, finally approaching

a. σ^2.
b. zero.
c. one.
d. Var($\hat{\beta}_1$).
e. Var($\hat{\beta}_2$).

(13) In time-series data, any two variables that each have trends must be

a. orthogonal.
b. causally related.
c. correlated in any finite sample.
d. All of the above.
e. None of the above.

(14) According to which model does a one-unit change in x cause approximately a two percent increase in y?

a. $y = 5.7 + 0.02\ (1/x)$.
b. $y = 5.7 + 0.02 \ln(x)$.
c. $\ln(y) = 5.7 + 0.02\ x$.
d. $\ln(y) = 5.7 + 0.02 \ln(x)$.
e. $y = 5.7 + 0.02\ x$.

(15) According to which model is the elasticity of y with respect to x equal to 1.2?

a. $\ln(y) = 3.7 + 1.2\ x$.
b. $\ln(y) = 3.7 + 1.2 \ln(x)$.
c. $y = 3.7 + 1.2\ x$.
d. $y = 3.7 + 1.2\ (1/x)$.
e. $y = 3.7 + 1.2 \ln(x)$.

(16) Suppose a production function is estimated of the form $y_i = \beta_1 + \beta_2 x_i$, where y_i denotes weekly output and x_i denotes labor input, measured as labor-hours. Now suppose the input data are converted to workers (each worker works 40 hours per week) and the equation is re-estimated. Which of the following are true?

a. $\hat{\beta}_1$ will increase by a factor of 40 .
b. $\hat{\beta}_2$ will increase by a factor of 40 .
c. The sum of squared residuals will increase by a factor of $(40)^2$.
d. The r^2 value will increase by a factor of $(40)^2$.
e. All of the above.

II. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [LS confidence intervals, tests, prediction: 24 pts] The relationship between per-capita income and population growth is estimated using a sample of n=15 countries. For each country i, let y_i denote its fertility rate and let x_i denote its per capita income in thousands of U.S. dollars. For example, if country #1 has a fertility rate of 2.5 and per-capita income of $2,700 then $y_1 = 2.5$ and $x_1 = 2.7$.

The model $y_i = \beta_1 + \beta_2 x_i$ is estimated with the following results. (The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors.)

Fertility rate	=	3.25	-	0.025	Per-capita income
		(0.08)		(0.010)	

a. [3 pts] Suppose a country's per-capita income rose from $2 thousand dollars to $5 thousand dollars. By how much would its fertility rate decrease? That is, predict Δy .

b. [3 pts] Suppose a country had a per-capita income of $1 thousand dollars. According to these results, what would be its predicted fertility rate?

For the rest of this problem, assume the error term is normally-distributed.

c. [3 pts] What are the "degrees of freedom" for these estimates? Give an integer answer.

d. [6 pts] Compute a **95%** confidence interval for the intercept.

e. [9 pts] Test the null hypothesis that per-capita income has no effect on fertility, against the alternative hypothesis that per-capita income has a negative effect (a **one-tailed test**) at **5%** significance. Give the value of the test statistic, the critical point from a table, and your conclusion (whether you can reject null hypothesis).

Value of test statistic = ________________. Critical point(s) = ___________________.

Reject null hypothesis? _______________________.

(2) [LS confidence intervals, tests, elasticity: 24 pts] The relationship between electricity rates and electricity usage is estimated using a sample of n=1000 households in a variety of electric utility districts. For each household i, let y_i denote the average monthly consumption of electricity (in kilowatt-hours) and x_i denote the rate or price per kilowatt-hour. For example, if household #1 faces a rate of $0.07 per kilowatt-hour and consumes two thousand kilowatt-hours, then $x_1 = 0.07$ and $y_1 = 2000$.

The model $y_i = \beta_1 + \beta_2 x_i$ is estimated with the following results. The numbers on top are the least-squares estimates of the intercept and slope, and the numbers at the bottom in parentheses are standard errors.

Electricity usage per month in kilowatt-hours	=	3400 (585)	-	17500 (6250)	Rate per kilowatt-hour

a. [3 pts] Suppose electricity were free. According to these results, how much electricity is a typical household predicted to consume per month?

kilowatt-hours

b. [3 pts] Suppose the electricity rate increased from $0.06 per kilowatt-hour to $0.10 per kilowatt-hour. By how many kilowatt-hours would electricity consumption decrease? That is, predict Δy .

kilowatt-hours

c. [3 pts] Suppose the mean electricity usage in the sample is 2000 kilowatt-hours and the mean electricity rate is $0.08 per kilowatt-hour. (That is, $\bar{y} = 2000.$ and $\bar{x} = 0.08$.) Compute the estimated elasticity of demand for electricity at the sample means.

d. [6 pts] Compute a **95%** confidence interval for the intercept.

e. [9 pts] Test the null hypothesis that electricity rates have no effect on electricity usage, against the alternative hypothesis that rates have a negative effect (a **one-tailed test**) at **5%** significance. Give the value of the test statistic, the critical point from a table, and your conclusion (whether you can reject null hypothesis).

Value of test statistic = ________________. Critical point(s) = __________________.

Reject null hypothesis? ______________________.

III. CRITICAL THINKING [4 pts] A researcher wants to estimate the effect of alcohol consumption on health status. Using a sample of 100 Drake employees, the researcher estimates the equation $y_i = \beta_1 + \beta_2 x_i$, where y_i denotes an index of the health status of employee i, and x_i denotes the number of alcoholic drinks consumed per month by the same employee. The researcher is shocked to find that the least squares estimator of β_2 is estimated as positive! Assuming that alcohol consumption, *ceteris paribus*, should actually *decrease* health status, why might least-squares give the "wrong" answer here? Which of the following assumptions would you suspect is violated in these data? Why? [Hint: The researcher has just discovered that a number of Drake employees suffer from serious medical conditions such as diabetes, cirrhosis, etc. and consequently have been forbidden by their doctors from consuming alcohol.]

- $E(\varepsilon_i|x_i) = 0$.
- Homoskedasticity: $Var(\varepsilon_i) = \sigma^2$, a constant.
- No autocorrelation: $Cov(\varepsilon_i,\varepsilon_j) = 0$ for $i \neq j$.

[end of exam]