Introduction to Econometrics (Econ 107)
Drake University, Spring 2006
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MIDTERM EXAMINATION #1 ANSWER KEY
"Introduction and Statistics Review"
February 7, 2006

VERSION A

I. MULTIPLE CHOICE: [3 pts each—42 pts total]
(1)c. (2)c. (3)c. (4)b. (5)d. (6)c. (7)b. (8)d. (9)a. (10)b. (11)a. (12)d. (13)c. (14)a.

II. PROBLEMS

(1) [Least-squares calculation: 12 pts] a. -3. b. 14. c. 5, 11, 8. d. -3, -3, 6.

(2) [Moments: 12 pts] a. 33. b. 49. c. 7. d. 0.375.

(3) [Estimation: 12 pts] a. mean=8. b. bias=2. c. variance=1.25. d. MSE=5.25 .

(4) [Inference for arbitrary distribution, large sample: 18 pts]
  a. Discrete distribution because can only take values that are nonnegative integers.
  b. estimate=2.05 .
  c. SE=0.03, so CI = 2.05±1.96(0.03) = 2.05±0.0588 = (1.9912, 2.1088).
  d. Test statistic = (2.05-2)/0.03 = 1.667; critical point = 1.645; conclusion = reject null hypothesis

III. CRITICAL THINKING [4 pts]

What is wrong with this estimator is that it is *inconsistent.*

This estimator is unbiased because $E(\hat{\mu}) = \frac{1}{5}\sum_{i=1}^{5} E(x_i) = \frac{1}{5}\sum_{i=1}^{5}\mu = \mu$. Put differently, the bias is $Bias(\hat{\mu}) = 0$

This estimator is also asymptotically unbiased because, trivially, $\lim_{n\to\infty} Bias(\hat{\mu}) = 0$.

But this estimator is inconsistent. The distribution of the estimator does not change after the first five observations are sampled, because only the first five observations are used. So the distribution of the estimator does not collapse around the true value. Also note that the mean square error is given by $MSE(\hat{\mu}) = Bias(\hat{\mu})^2 + Var(\hat{\mu}) = 0 + \frac{Var(x_i)}{5}$, which does not converge to zero.

VERSION B

I. MULTIPLE CHOICE: [3 pts each—42 pts total]
(1)e. (2)d. (3)b. (4)d. (5)b. (6)a. (7)c. (8)b. (9)c. (10)b. (11)b. (12)a. (13)b. (14)b.

II. PROBLEMS

(1) [Least-squares calculation: 12 pts] a. 0.5. b. 2. c. 4, 6, 5. d. -1, -1, 2.

(2) [Moments: 12 pts] a. 27. b. 81. c. 9. d. 0.125.

(3) [Estimation: 12 pts] a. mean=8. b. bias=2. c. variance=5. d. MSE=9.

(4) [Inference for arbitrary distribution, large sample: 18 pts]

a. Discrete distribution because can only take values that are nonnegative integers.
b. estimate=2.1 .
c. SE=0.02, so CI = 2.1±1.96(0.02) = 2.1±0.0392 = (2.0608, 2.1392).
d. Test statistic = (2.1-2)/0.02 = 5; critical point = 1.645; conclusion = reject null hypothesis

III. CRITICAL THINKING [4 pts]

What is wrong with this estimator is that it is *biased in small sample*.
This estimator is biased because $E(\hat{\mu}) = \frac{1}{n-5}\sum_{i=1}^{n} E(x_i) = \frac{1}{n-5}\sum_{i=1}^{n}\mu = \frac{n\mu}{n-5} \neq \mu$. The bias is given by $Bias(\hat{\mu}) = E(\hat{\mu}) - \mu = \frac{-5\mu}{n-5}$.
However, this estimator is asymptotically unbiased because $\lim_{n\to\infty} Bias(\hat{\mu}) = 0$.
Moreover, this estimator is consistent. Its mean square error is given by
$MSE(\hat{\mu}) = Bias(\hat{\mu})^2 + Var(\hat{\mu}) = \left(\frac{-5\mu}{n-5}\right)^2 + \frac{n\,Var(x_i)}{(n-5)^2}$. Note that both terms converge to zero as n approaches infinity. Since the MSE converges to zero, the estimator is consistent.

VERSION C

I. MULTIPLE CHOICE: [3 pts each—42 pts total]
(1)a. (2)a. (3)c. (4)b. (5)a. (6)c. (7)a. (8)c. (9)d. (10)e. (11)b. (12)b. (13)a. (14)a.

II. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Least-squares calculation: 12 pts] a. -1. b. 9. c. 7, 3, 5. d. -2, -2, 4.

(2) [Moments: 12 pts] a. 31. b. 100. c. 10. d. 5/12 or 0.4167.

(3) [Estimation: 12 pts] a. mean=5. b. bias=-1. c. variance=0.8. d. MSE=1.8.

(4) [Inference for arbitrary distribution, large sample: 18 pts]

a. Discrete distribution because can only take values that are nonnegative integers.
b. estimate=2.12 .
c. SE=0.04, so CI = 2.12±1.96(0.04) = 2.12±0.0784 = (2.0416, 2.1984).
d. Test statistic = (2.12-2)/0.04 = 3; critical point = 1.645; conclusion = reject null hypothesis

III. CRITICAL THINKING [4 pts]

What is wrong with this estimator is that it is *biased in small sample*.

This estimator is biased because $E(\hat{\mu}) = \frac{1}{n+1}\sum_{i=1}^{n} E(x_i) = \frac{1}{n+1}\sum_{i=1}^{n} \mu = \frac{n\mu}{n+1} \neq \mu$. The bias is given by $Bias(\hat{\mu}) = E(\hat{\mu}) - \mu = \frac{-\mu}{n+1}$.

However, this estimator is asymptotically unbiased because $\lim_{n\to\infty} Bias(\hat{\mu}) = 0$.

Moreover, this estimator is consistent. Its mean square error is given by $MSE(\hat{\mu}) = Bias(\hat{\mu})^2 + Var(\hat{\mu}) = \left(\frac{-\mu}{n+1}\right)^2 + \frac{n\,Var(x_i)}{(n+1)^2}$. Note that both terms converge to zero as n approaches infinity. Since the MSE converges to zero, the estimator is consistent.

[end of answer key]