LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

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Part 5: Further topics

Chapter 19: Input markets

Problems

(19.1) [Input demand] Suppose a firm has the hourly production function $q = 3\, x_1^{1/3}\, x_2^{2/3}$, where x_1 denotes the number of machines and x_2 denotes the number of workers. The price of the firm's product is given at \$20.

a. Find an expression for the marginal product of workers (x_2) in terms of x_1 and x_2.

In the short run, the firm's quantity of machines is fixed at $x_1 = 8$.

b. Find an expression for the firm's marginal revenue product (MRP) of workers in terms of x_2 alone.
c. Find the firm's short-run labor demand equation, showing how many workers will be hired (x_2) as a function of the workers' wage (w_2): $x_2 = f(w_2)$.
d. What is the short-run elasticity of demand for workers (the elasticity of x_2 with respect to w_2) according to this equation?
e. Compute the number of workers hired if the wage is \$10. Compute the number of workers hired if the wage is \$20.

(19.2) [Input demand] Suppose a firm has the hourly production function $q = 2\, x_1^{1/2}\, x_2^{1/2}$, where x_1 denotes the number of machines and x_2 denotes the number of workers. The price of the firm's product is given at \$4.

a. Find an expression for the marginal product of workers (x_2) in terms of x_1 and x_2.

In the short run, the firm's quantity of machines is fixed at $x_1 = 25$.

b. Find an expression for the firm's marginal revenue product (MRP) of workers in terms of x_2 alone.
c. Find the firm's short-run labor demand equation, showing how many workers will be hired (x_2) as a function of the workers' wage (w_2): $x_2 = f(w_2)$.
d. What is the short-run elasticity of demand for workers (the elasticity of x_2 with respect to w_2) according to this equation?
e. Compute the number of workers hired if the wage is \$5. Compute the number of workers hired if the wage is \$10.

(19.3) [Elasticity of substitution] Suppose the elasticity of substitution between capital and labor in a certain industry is 0.8. Suppose the price of labor (w_2) increases by 10 percent but the price of capital inputs (w_1) remains unchanged.

a. Will the input price ratio (w_2/w_1) increase or decrease? By approximately how much?
b. Will the industry capital/labor ratio (x_1/x_2) increase or decrease? By approximately how much?

(19.4) [Elasticity of substitution] Suppose the elasticity of substitution between capital and labor in a certain industry is 1.2. Suppose the price of labor (w_2) increases by 10 percent but the price of capital inputs (w_1) remains unchanged.

a. Will the input price ratio (w_2/w_1) increase or decrease? By approximately how much?
b. Will the industry capital/labor ratio (x_1/x_2) increase or decrease? By approximately how much?

The next two questions use Hicks' formula for the industry elasticity of input demand:

$$|\varepsilon_I^D| = \sigma + S_I (|\varepsilon_Q^D| - \sigma).$$

(John R. Hicks, *The Theory of Wages*, London: Macmillan and Co, 1935, pp. 242-244.)

(19.5) [Input demand, Hicks's formula] See information in the box above. For the United States as a whole, the following are approximate values:

- the elasticity of substitution in production between capital and labor = 1.
- labor's share of total costs = 0.70.

Further assume the elasticity of output demand (in absolute value) = 0.1. Compute the elasticity of labor demand for the U.S. as a whole.

(19.6) [Hicks-Marshall rules] See information in the box above. Assume that S_I, the input's share of total cost, lies between zero and one; that $|\varepsilon_Q^D|$, the absolute value of output demand, is positive; and that σ, the elasticity of substitution in production, is positive.

a. Find the partial derivative of $|\varepsilon_I^D|$ with respect to $|\varepsilon_Q^D|$. Explain why your formula proves that elasticity of demand for an input is larger, the larger the elasticity of demand for the output.
b. Find the partial derivative of $|\varepsilon_I^D|$ with respect to σ. Explain why your formula proves that elasticity of demand for an input is larger, the more easily inputs can be substituted for each other.
c. Find the partial derivative of $|\varepsilon_I^D|$ with respect to S_I. Explain why your formula proves that elasticity of demand for an input is larger, the larger the share of the input in total costs, provided $|\varepsilon_Q^D| > \sigma$.

(19.7) [Value of the firm] Suppose a firm is expected to generate $10 million in (economic) profits every year perpetually.

a. Compute the value of the firm if the interest rate is 4%.
b. Compute the value of the firm if the interest rate is 5%.
c. Explain intuitively why your answer to part (b) is less than your answer to part (a).

(19.8) [Demand for capital] Suppose the purchase price of a piece of capital equipment is $150 (and has no other costs). Over its three-year life, the equipment is expected to generate the following additional revenue for the firm: $50 one year from today, $60 two years from today, and $70 three years from today.

a. Suppose the interest rate is 5%. Should the firm buy this piece of equipment? Justify your answer.
b. Suppose the interest rate is 10%. Should the firm buy this piece of equipment? Justify your answer.
c. Explain intuitively why your answer to part (b) is different from your answer to part (a).

(19.9) [Demand for capital] Suppose a new computer costs $1000, depreciation is 20% per year, and the interest rate is 7%. Assume for simplicity that there are no other costs of ownership (hah!). Compute the equilibrium annual rental rate for the computer during the first year.

[end of problem set]