

LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

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Part 5: Further topics

Chapter 19: Input markets

Problems

(19.1) [Input demand] Suppose a firm has the hourly production function $q = 3 x_1^{1/3} x_2^{2/3}$, where x_1 denotes the number of machines and x_2 denotes the number of workers. The price of the firm's product is given at \$20.

- a. Find an expression for the marginal product of workers (x_2) in terms of x_1 and x_2 .

In the short run, the firm's quantity of machines is fixed at $x_1 = 8$.

- b. Find an expression for the firm's marginal revenue product (MRP) of workers in terms of x_2 alone.
- c. Find the firm's short-run labor demand equation, showing how many workers will be hired (x_2) as a function of the workers' wage (w_2): $x_2 = f(w_2)$.
- d. What is the short-run elasticity of demand for workers (the elasticity of x_2 with respect to w_2) according to this equation?
- e. Compute the number of workers hired if the wage is \$10. Compute the number of workers hired if the wage is \$20.

(19.2) [Input demand] Suppose a firm has the hourly production function $q = 2 x_1^{1/2} x_2^{1/2}$, where x_1 denotes the number of machines and x_2 denotes the number of workers. The price of the firm's product is given at \$4.

- a. Find an expression for the marginal product of workers (x_2) in terms of x_1 and x_2 .

In the short run, the firm's quantity of machines is fixed at $x_1 = 25$.

- b. Find an expression for the firm's marginal revenue product (MRP) of workers in terms of x_2 alone.
- c. Find the firm's short-run labor demand equation, showing how many workers will be hired (x_2) as a function of the workers' wage (w_2): $x_2 = f(w_2)$.
- d. What is the short-run elasticity of demand for workers (the elasticity of x_2 with respect to w_2) according to this equation?
- e. Compute the number of workers hired if the wage is \$5. Compute the number of workers hired if the wage is \$10.

(19.3) [Elasticity of substitution] Suppose the elasticity of substitution between capital and labor in a certain industry is 0.8. Suppose the price of labor (w_2) increases by 10 percent but the price of capital inputs (w_1) remains unchanged.

- Will the input price ratio (w_2/w_1) increase or decrease? By approximately how much?
- Will the industry capital/labor ratio (x_1/x_2) increase or decrease? By approximately how much?

(19.4) [Elasticity of substitution] Suppose the elasticity of substitution between capital and labor in a certain industry is 1.2. Suppose the price of labor (w_2) increases by 10 percent but the price of capital inputs (w_1) remains unchanged.

- Will the input price ratio (w_2/w_1) increase or decrease? By approximately how much?
- Will the industry capital/labor ratio (x_1/x_2) increase or decrease? By approximately how much?

The next two questions use Hicks' formula for the industry elasticity of input demand:

$$|\varepsilon_I^D| = \sigma + S_I (|\varepsilon_Q^D| - \sigma).$$

(John R. Hicks, *The Theory of Wages*, London: Macmillan and Co, 1935, pp. 242-244.)

(19.5) [Input demand, Hicks's formula] See information in the box above. For the United States as a whole, the following are approximate values:

- the elasticity of substitution in production between capital and labor = 1.
- labor's share of total costs = 0.70.

Further assume the elasticity of output demand (in absolute value) = 0.1. Compute the elasticity of labor demand for the U.S. as a whole.

(19.6) [Hicks-Marshall rules] See information in the box above. Assume that S_I , the input's share of total cost, lies between zero and one; that $|\varepsilon_Q^D|$, the absolute value of output demand, is positive; and that σ , the elasticity of substitution in production, is positive.

- Find the partial derivative of $|\varepsilon_I^D|$ with respect to $|\varepsilon_Q^D|$. Explain why your formula proves that elasticity of demand for an input is larger, the larger the elasticity of demand for the output.
- Find the partial derivative of $|\varepsilon_I^D|$ with respect to σ . Explain why your formula proves that elasticity of demand for an input is larger, the more easily inputs can be substituted for each other.
- Find the partial derivative of $|\varepsilon_I^D|$ with respect to S_I . Explain why your formula proves that elasticity of demand for an input is larger, the larger the share of the input in total costs, provided $|\varepsilon_Q^D| > \sigma$.

(19.7) [Value of the firm] Suppose a firm is expected to generate \$10 million in (economic) profits every year perpetually.

- a. Compute the value of the firm if the interest rate is 4%.
- b. Compute the value of the firm if the interest rate is 5%.
- c. Explain intuitively why your answer to part (b) is less than your answer to part (a).

(19.8) [Demand for capital] Suppose the purchase price of a piece of capital equipment is \$150 (and has no other costs). Over its three-year life, the equipment is expected to generate the following additional revenue for the firm: \$50 one year from today, \$60 two years from today, and \$70 three years from today.

- a. Suppose the interest rate is 5%. Should the firm buy this piece of equipment? Justify your answer.
- b. Suppose the interest rate is 10%. Should the firm buy this piece of equipment? Justify your answer.
- c. Explain intuitively why your answer to part (b) is different from your answer to part (a).

(19.9) [Demand for capital] Suppose a new computer costs \$1000, depreciation is 20% per year, and the interest rate is 7%. Assume for simplicity that there are no other costs of ownership (hah!). Compute the equilibrium annual rental rate for the computer during the first year.

[end of problem set]