LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

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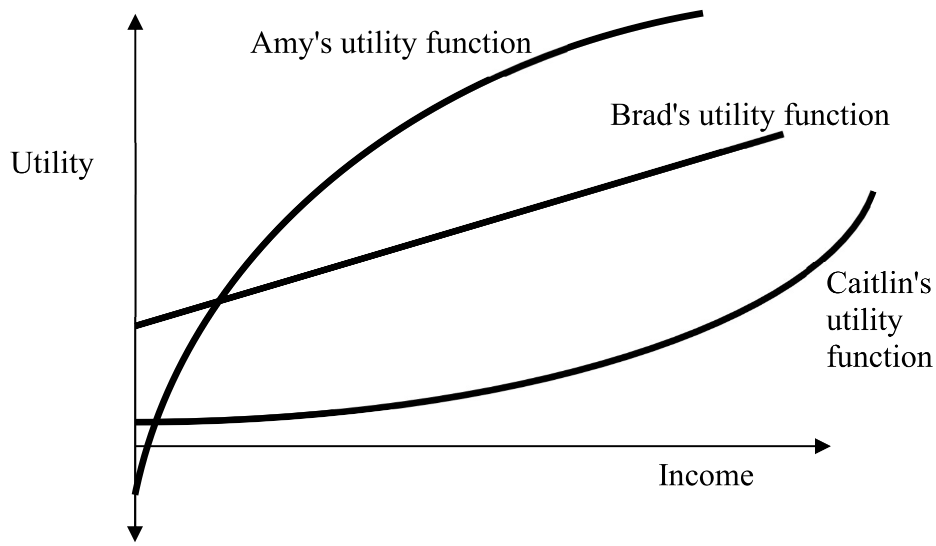
Part 5: Further topics

Chapter 18: Uncertainty and information

Problems

(18.1) [Risk aversion] Utility functions for Amy, Brad, and Caitlin are shown in the graph below.



a. Is Amy risk-averse, risk-loving, or risk neutral? Why?
b. Is Brad risk-averse, risk-loving, or risk neutral? Why?
c. Is Caitlin risk-averse, risk-loving, or risk neutral? Why?

(18.2) [Risk aversion] For each utility function below, verify that utility increases with income by finding the formula for the first derivative (dU/dW) and verifying that it is positive, assuming that W itself is positive. Then determine whether this utility function shows *risk aversion* by finding the formula for the second derivative (d^2U/dW^2) and determining whether it is negative.

a. $U(W) = -1/W$.
b. $U(W) = 5W$.
c. $U(W) = 3W + W^2$.
d. $U(W) = (1/a)W^a$, where $0 < a < 1$.
e. $U(W) = W^3 - 30\ W^2 + 500W$.

(18.3) [Expected income and expected utility] Suppose a person's income next year is uncertain. The person has a 50% chance of receiving \$10,000 and a 50% chance of receiving \$20,000 income. The person's utility function is $U(W) = \ln(W)$, where W denotes income.

a. Determine whether this person is risk-averse by determining whether the second derivative of the utility function is negative.
b. Compute the person's expected income (in dollars). Call it EW.
c. Compute the person's expected utility (in "utils"). Call it EU.
d. Compute the person's utility of their expected income—that is, U(EW). Is this less than or greater than their expected utility—that is, EU ?

(18.4) [Expected income and expected utility] Suppose a person's income next year is uncertain. The person has a 20% chance of receiving \$40,000 and a 80% chance of receiving \$160,000 income. The person's utility function is $U(W) = W^{1/2}$, where W denotes income.

a. Determine whether this person is risk-averse by determining whether the second derivative of the utility function is negative.
b. Compute the person's expected income (in dollars). Call it EW.
c. Compute the person's expected utility (in "utils"). Call it EU.
d. Compute the person's utility of their expected income—that is, U(EW). Is this less than or greater than their expected utility—that is, EU ?

(18.5) [Expected income and expected utility] Suppose a person's income next year is uncertain. The person has a 40% chance of receiving \$10,000 and a 60% chance of receiving \$20,000 income. The person's utility function is $U(W) = 2 - (10{,}000/W)$, where W denotes income.

a. Determine whether this person is risk-averse by determining whether the second derivative of the utility function is negative.
b. Compute the person's expected income (in dollars). Call it EW.
c. Compute the person's expected utility (in "utils"). Call it EU.
d. Compute the person's utility of their expected income—that is, U(EW). Is this less than or greater than their expected utility—that is, EU ?

(18.6) [Expected income and expected utility] Consider the following game. A coin is flipped until a heads appears. When it does, the person is paid an amount equal to 2 raised to the power of the number of flips. For example, if heads appears on the first flip of the coin, $2 is paid. If the first flip is a tails and the second flip is heads, $4 is paid. If the first two flips are tails and the third flip is a heads, $8 is paid, and so forth.

a. How much would you, personally, be willing to pay to play this game? Give an honest answer.

Now the probability of any particular sequence of length n is equal to $(1/2)^n$. A table of probabilities and payoffs would begin as follows.

Coin sequence	Length of sequence (n)	Probability	Payoff (W)
H	1	$(1/2) = 0.5$	\$2
TH	2	$(1/2)^2 = 0.25$	\$4
TTH	3	$(1/2)^3 = 0.125$	\$8
TTTH	4	$(1/2)^4 = 0.0625$	\$16
TTTTH	5	$(1/2)^5 = 0.03125$	\$32
Etc.			

b. Write the expected value of the payoff (EW) as an infinite series using Σ notation.
c. Show that the series does not converge, that the expected value of the payoff (EW) is infinite.

The so-called "St. Petersburg paradox," several centuries old, is that most people are willing to pay only a finite amount to play this game. In the eighteenth century, mathematician Daniel Bernoulli[1] proposed a resolution of this paradox. He suggested people make decisions under uncertainty based on expected *utility*, not expected *value*. He proposed the utility function U(W) = ln(W).

d. Suppose a person has utility function U(W) = ln(W). Write the expected utility of the game (EU) as an infinite series.
e. It can be shown that $\sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^i i = 2$. Use that fact to prove that EU = 2 ln(2) utils.
f. How much would this person be willing to pay to play the game? [Hint: Set U(W) = EU and solve for W.]

[1] Daniel Bernoulli, "Exposition of a New Theory on the Measurement of Risk," *Econometrica, Vol. 22*, No. 1 (Jan., 1954), pp. 23-36. Translated from Latin into English by Louise Sommer. Originally published as "Specimen Theoriae Novae de Mensura Sortis," *Commentarii Academiae Scientiarum Imperialis Petropolianae, Tomus V* [*Papers of the Imperial Academy of Sciences in Petersburg, Vol. V*], 1738, pp. 175-192. The paradox is described on page 31.

(18.7) [Demand for insurance] Suppose someone has utility function $U(W) = -1000/W$, where W denotes wealth. Suppose this person now has wealth of $60,000 but faces a 20% chance of losing $30,000 of that wealth.

a. Compute this person's expected wealth without insurance (in dollars). Call this value EW.
b. Compute this person's expected utility without insurance (in "utils"). Call this value EU.
c. Compute the amount of wealth that, if had for certain, would bring the same utility as the person's current situation. [Hint: Solve $U(W^*) = EU$ for W^*.]
d. Compute the maximum premium amount this person would be willing to pay for full insurance against losing $30,000.
e. Compute the actuarially fair premium amount for full insurance against losing $30,000.

(18.8) [Demand for insurance] Suppose someone has utility function $U(W) = -(1000/W)^2$, where W denotes wealth. Suppose this person now has wealth of $10,000 but faces a 10% chance of losing $6,000 of that wealth.

a. Compute this person's expected wealth without insurance (in dollars). Call this value EW.
b. Compute this person's expected utility without insurance (in "utils"). Call this value EU.
c. Compute the amount of wealth that, if had for certain, would bring the same utility as the person's current situation. [Hint: Solve $U(W^*) = EU$ for W^*.]
d. Compute the maximum premium amount this person would be willing to pay for full insurance against losing $6,000.
e. Compute the actuarially fair premium amount for full insurance against losing $6,000.

(18.9) [Hidden action and moral hazard] Suppose a person has utility function $U = W^{1/2}$, where W denotes wealth. This person has initial wealth of $160,000 but faces a 25% chance of losing $70,000 of that wealth. Assume that only full insurance is available.

a. Compute the actuarially fair premium amount for full insurance.
b. Compute the maximum premium amount this person would be willing to pay for full insurance against losing $70,000.
c. Suppose now that whenever people buy insurance, they fail to take proper care and the risk of loss rises to 30%. Compute the actuarially fair premium amount in this case. If offered full insurance at the actuarially fair rate, would this person buy insurance or not? [Hint: Compare your answer here to your answer for part (b).]

(18.10) [Hidden characteristics and adverse selection] Suppose everyone in the country has an annual income of W = $40,000 but faces some risk of losing $30,000. Everyone has the same utility function, namely $U(W) = W^{1/2}$, where W denotes income. But half the people face a 10% chance of loss while half the people face a 50% chance of loss.

a. Compute expected income for each group of people (in dollars).
b. Compute expected utility for each group of people (in "utils").
c. Compute the maximum insurance premium amount (in dollars) that each group would be willing to pay to insure against this loss.
d. Suppose insurance companies cannot distinguish between low-risk and high-risk people, and so initially set an insurance premium based on average risk of their customers, which is obviously 30%. Compute the actuarially fair premium amount (in dollars) based on the average risk.
e. Assume insurance companies have no costs other than benefits, and so, under competition, offer insurance at the actuarially fair amount. Describe the equilibrium outcome in this market. What premium is charged? Who buys insurance?

(18.11) [Hidden characteristics and adverse selection] Suppose everyone in the country has an annual income of W = $40,000 but faces some risk of losing $30,000. Everyone has the same utility function, namely $U(W) = W^{1/2}$, where W denotes income. But half the people face a 10% chance of loss while half the people face a 50% chance of loss.

a. Compute expected income for each group of people (in dollars).
b. Compute expected utility for each group of people (in "utils").
c. Compute the maximum insurance premium amount (in dollars) that each group would be willing to pay to insure against this loss.
d. Suppose insurance companies cannot distinguish between low-risk and high-risk people, and so initially set an insurance premium based on average risk of their customers, which is obviously 30%. Compute the actuarially fair premium amount (in dollars) based on the average risk.
e. Assume insurance companies have no costs other than benefits, and so, under competition, offer insurance at the actuarially fair amount. Describe the equilibrium outcome in this market. What premium is charged? Who buys insurance?

(18.12) [Hidden characteristics and adverse selection] Suppose the market for auto insurance consists of 10,000 people. Order these people from high-risk to low-risk, and index them by $Q = 0$ to 10,000. The expected loss of the Qth person is given by $EL = 300 - 0.01\ Q$. (Thus the last person's expected loss is about \$200.) Everyone is risk-averse, and willing to pay \$20 more than their expected loss (EL) for insurance.

a. Give an equation for the demand for insurance P_D or willingness-to-pay, as a function of Q.
b. Give an equation for the marginal cost of insurance MC, as a function of Q. Assume there are no administrative costs.
c. If the market were efficient, how many people would get insurance?
d. Give an equation for the average cost of insurance AC as a function of Q.
e. Assume the market is competitive, but that insurance companies cannot observe individual persons' expected loss. Find the equilibrium price P and quantity Q of insurance.
f. Compute the deadweight loss from adverse selection.

(18.13) [Hidden characteristics and adverse selection with administrative costs] Suppose the market for homeowners insurance consists of 50,000 people. Order these people from high-risk to low-risk, and index them by $Q = 0$ to 50,000. The expected loss of the Qth person is given by $EL = 500 - 0.01\ Q$. (Thus the last person's expected loss is about zero.) Everyone is risk-averse, and willing to pay 50% more than their expected loss (EL) for insurance. In other words, person Q is willing to pay $1.5(500-0.01Q) = 750 - 0.015\ Q$. The administrative cost for each customer is \$50.

a. Give an equation for the marginal cost of insurance MC as a function of Q.
b. If the market were efficient, how many people would get insurance?
c. Give an equation for the average cost of insurance AC as a function of Q.
d. Assume the market is competitive, but that insurance companies cannot observe individual persons' expected loss. Find the equilibrium price P and quantity Q of insurance.
e. Compute the deadweight loss from adverse selection.

(18.14) [Hidden action and moral hazard] Suppose the market for renter's insurance consists of 1000 people. Assume each person faces the same risk, and the expected cost of insuring each person, including administrative cost, is MC**=$20. However, some people are more risk-averse than others, so they are willing to pay more for renter's insurance. If people are ordered from high to low willingness-to-pay, the demand for renter's insurance is given by $P = 30 - (Q/50)$.

a. What will be the market price of renter's insurance?
b. How many people (Q**) will buy renter's insurance?

Now suppose there is some simple action people could take—such as replacing batteries in smoke detectors—that reduces the cost of insuring people from MC**=$20 to MC*=$10. Suppose replacing batteries costs just $1, so the demand for renter's insurance by people who take this action becomes $P = 30 - (Q/50) - 1$. Initially assume insurance companies can verify that people have replaced their batteries.

c. Now what will be the market price of renter's insurance?
d. Now how many people (Q*) will buy renter's insurance?

Finally suppose the action of replacing batteries is *hidden* from insurance companies. If companies cannot verify that people have replaced their batteries, they must price insurance as if people did not do so, as in parts (a) and (b). In fact, no one who buys insurance will actually replace their batteries, because they have no incentive to do so, a *moral hazard* problem, which creates two kinds of costs.

e. Compute the increase in cost (net of the cost of batteries) of insuring people who do not replace their batteries. This equals $(MC^{**}-MC^{*}-1) \times Q^{**}$.
f. Compute the loss in welfare from too few people buying insurance. This equals the area between the demand curve $P = 30 - (Q/50) - 1$ and the supply curve P=$10, from Q** to Q*.

[end of problem set]