

# LECTURE NOTES ON MICROECONOMICS

## ANALYZING MARKETS WITH BASIC CALCULUS

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### Part 5: Further topics

#### Chapter 17: Externalities and public goods

#### Problems

(17.1) [Marginal benefit and marginal cost] Let  $Q$  denote the total number of miles of a highway system. Suppose the total benefit from the system, in millions of dollars, is given by the following function of  $Q$ :

$$TB(Q) = 50Q - (Q^2/10)$$

while total cost of the system, also in millions of dollars, is given by the following function of  $Q$ :

$$TC(Q) = 5Q + (Q^2/20)$$

- Find an expression for the marginal benefit of the highway system  $MB(Q)$ .
- Find an expression for the marginal cost of the highway system  $MC(Q)$ .
- Compute the optimal length of the highway system  $Q^*$ .
- Compute the total cost and total benefit of the optimal highway system. Compute the net benefit.

Note: The next two problems both refer to the following information. A factory and a fishing club share the same lake. The factory's marginal benefit from pollution (reflecting increased profits) is given by:  $MB = 10 - (Q/100)$ , where  $Q$  = tons of pollution per year. The fishing club's marginal cost of pollution (reflecting degradation of the fish habitat) is given by:  $MC = 1 + (Q/50)$ . It is recommended that you sketch these curves on a graph and find their intersection before working the problems.

(17.2) [Bargaining and social cost] See the information above. Suppose the factory owns the lake.

- a. If for some reason the factory were unable to bargain with the fishing club (and the government did not intervene) what quantity of pollution would the factory choose? [Hint: It would keep polluting until the marginal benefit from polluting fell to zero.]
- b. Starting from this level of pollution, if the factory were able to bargain with the fishing club, what is the minimum payment per ton that it would accept to reduce pollution by a small amount?
- c. What is the maximum payment per ton the fishing club would offer to reduce pollution by a small amount?

Suppose (for parts d, e, and f) that bargaining were efficient. That is, the factory would continue to reduce pollution as long as it received sufficient payment from the fishing club for each ton of reduction to compensate it for lost marginal benefit.

- d. What quantity of pollution would result?
- e. What is the minimum total payment the factory would accept for this reduction in pollution? [Hint: This is the area of a triangle under the MB curve.]
- f. What is the maximum total payment the fishing club would offer for this reduction in pollution? [Hint: This is the area of a trapezoid under the MC curve.]
- g. What are the total gains from trade to both parties combined?

(17.3) [Bargaining and social cost] See the information above. Suppose the fishing club owned the lake.

- a. If for some reason the fishing club were unable to bargain with the factory (and the government did not intervene) what quantity of pollution would the fishing club permit?
- b. Starting from this level of pollution, if the fishing club were able to bargain with the factory, what is the minimum payment per ton of pollution it would accept to permit a small amount of pollution?
- c. What is the maximum payment per ton of pollution the factory would offer to pay for permission to release a small amount of pollution?

Suppose (for parts d, e, and f) that bargaining were efficient. That is, the fishing club would continue to permit additional pollution as long as it received sufficient payment from the factory for each additional ton to cover its marginal cost.

- d. What quantity of pollution would result?
- e. What is the minimum total payment the fishing club would accept for permitting this pollution? [Hint: This is the area of a trapezoid under the MC curve.]
- f. What is the maximum total payment the factory would offer to for permission to release this pollution? [Hint: This is the area of a trapezoid under the MB curve.]
- g. What are the total gains from trade to both parties combined?

Note: The next two problems both refer to the following information. A factory is located near a housing development. The factory's marginal benefit from noise (reflecting increased profits) is given by:  $MB = 8 - (Q/500)$ , where  $Q$  is a measure of the amount of noise created by the factory. The housing development's marginal cost of noise (reflecting decreased quality of life) is given by:  $MC = 2 + (Q/1000)$ . It is recommended that you sketch these curves on a graph and find their intersection before working the problems.

(17.4) [Bargaining and social cost] See the information above. Suppose there is no law restricting noise.

- a. Compute the economically efficient amount of noise  $Q^*$ .
- b. How much noise will the factory make if bargaining is impossible?
- c. Starting from this level of noise, compute the maximum amount the housing development would be willing to pay the factory to reduce noise to the economically efficient amount [Hint: This is the area under the MC curve, from the efficient amount of noise up to the actual amount.]
- d. Compute the minimum amount the factory would be willing to accept to reduce noise to the economically efficient amount. [Hint: This is the area under the MB curve, from the efficient amount of noise up to the actual amount.]
- e. Is there room for agreement between the factory and the housing development? Why or why not?

(17.5) [Bargaining and social cost] See the information above. Suppose the law does not permit any noise without the consent of those affected by the noise.

- a. Compute the economically efficient amount of noise  $Q^*$ .
- b. Compute the maximum amount the factory would be willing to pay the housing development to permit noise at the economically efficient amount? [Hint: This is the area under the MB curve, from zero up to the efficient amount]
- c. Compute the minimum amount the housing development would be willing to accept to permit noise to the economically efficient amount? [Hint: This is the area under the MC curve, from the zero up to the efficient amount.]
- d. Is there room for agreement between the factory and the housing development? Why or why not?

(17.6) [External cost and Pigou tax] Suppose demand and supply for a particular chemical is given by the following equations:  $P_D = 18 - (Q/1000)$  and  $P_S = 4 + (Q/1000)$ . Unfortunately, use of this chemical damages the earth's ozone layer, creating an external cost. Marginal external cost per unit is estimated to be  $MEC = 2 + (Q/500)$ .

- a. Compute the (unregulated) competitive equilibrium price ( $P$ ) and quantity ( $Q$ ) in this market.
- b. Find a formula for the marginal social cost of the chemical.
- c. Compute the socially-optimal quantity of output ( $Q$ ).
- d. Compute the deadweight social loss from this external cost.
- e. Compute the Pigou tax rate on this chemical, in dollars per unit, that would result in the socially-optimal quantity of the chemical. [Hint: The Pigou tax equals the marginal external cost at the socially-optimal quantity.]

(17.7) [External benefit and Pigou subsidy] Suppose demand and supply for a particular vaccine is given by the following equations:  $P_D = 20 - (Q/1000)$  and  $P_S = 4 + (Q/1000)$ . This vaccine creates an external benefit because vaccinated people are less likely to carry the disease and spread it to other people. Marginal external benefit per unit is estimated to be  $MEB = 17 - (Q/1000)$ .

- a. Compute the (unregulated) competitive equilibrium price (P) and quantity (Q) in this market.
- b. Find a formula for the marginal social benefit of the vaccine.
- c. Compute the socially-optimal quantity of output (Q).
- d. Compute the deadweight social loss from this external benefit.
- e. Compute the Pigou subsidy rate for this vaccine, in dollars per unit, that would result in the socially-optimal quantity of the vaccine. [Hint: The Pigou subsidy equals the marginal external benefit at the socially-optimal quantity.]

(17.8) [Auctioning pollution permits] Suppose twenty factories are producing pollution (Q). Ten old factories each have marginal benefits from pollution (reflecting increased profit) given by the equation  $Q_{old} = 100 - 10 MB$ . Ten new factories each have marginal benefits from pollution given by the equation  $Q_{new} = 50 - 5 MB$ .

- a. Find an equation for the total demand for pollution by all factories. [Hint: Set  $Q_{tot} = 10 Q_{old} + 10 Q_{new}$ . Substitute and simplify.]
- b. If there is no penalty for pollution, how much will all factories produce, in total ( $Q_{tot}$ )?

Now suppose the government has determined that pollution must be reduced to  $Q_{tot}=600$  units. It has created 600 permits to emit one unit of pollution and will sell them at auction.

- c. Compute the equilibrium auction price of a pollution permit.
- d. How many permits will a typical old factory buy? How much money will it spend on pollution permits?
- e. How many permits will a typical new factory buy? How much money will it spend on pollution permits?

(17.9) [Tradeable pollution permits] Suppose twenty factories are producing pollution ( $Q$ ). Ten old factories each have marginal benefits from pollution (reflecting increased profit) given by the equation  $Q_{old} = 100 - 10 MB$ . Ten new factories each have marginal benefits from pollution given by the equation  $Q_{new} = 50 - 5 MB$ .

- Find an equation for the total demand for pollution by all factories. [Hint: Set  $Q_{tot} = 10 Q_{old} + 10 Q_{new}$ . Substitute and simplify.]
- If there is no penalty for pollution, how much will all factories produce, in total ( $Q_{tot}$ )?

Now suppose the government has determined that pollution must be reduced to  $Q_{tot}=600$  units. It has created 600 permits to emit one unit of pollution and has distributed 30 permits to each of the twenty factories. However, all factories are permitted to trade permits in a well-organized market.

- Compute the equilibrium price of a tradable permit. [Hint: Set total demand equal to total supply (600).]
- Will a typical old factory want to buy or sell permits, or will it be content with the 30 permits it was given? If it buys or sells permits, how many?
- Will a typical new factory want to buy or sell permits, or will it be content with the 30 permits it was given? If it buys or sells permits, how many?

(17.10) [Nonrival good] Suppose 1000 people live near a proposed bike trail. The trail will cost \$20,000 per mile to build. Let  $Q$  denote the length of the bike trail in miles. A typical individual person's marginal benefit from this bike trail is given by the following expression:

$$MB = 60 - 5 Q .$$

- Find an expression for the marginal social benefit from the bike trail  $MSB(Q)$ .
- Compute  $Q^*$  the socially-optimal length of the bike trail.

(17.11) [Pure public good] Five neighbors live on a certain street. Unfortunately, the street accumulates trash, which nobody likes. But picking up trash takes time--about one hour for each large bag--and the opportunity cost of each person's time is \$10 per hour. Suppose each neighbor's demand (or marginal benefit) for trash pick-up is  $MB = 12 - (Q/2)$ , where  $MB$  denotes marginal benefit and  $Q$  denotes the number of bags picked up. Note that trash picked up by one neighbor benefits all neighbors simultaneously, so trash pickup is a nonrival good.

- Suppose one neighbor, taking initiative, decides to pick up trash. How many hours will that neighbor spend picking up trash, if that neighbor considers only her or his own benefit?
- After that first neighbor picks up trash, will other neighbors have an incentive to pick up more trash? Why or why not?
- Find an expression for the entire street's social marginal benefit from trash pick-up. [Hint: Because trash pick-up is a public good, social marginal benefit is the *vertical* sum of individual marginal benefits.]
- Compute the economically efficient (or socially optimal) amount of trash to pick up ( $Q$ ).
- Note the difference between your answers to parts (b) and (d). How are problems like this solved in the real world? Describe at least two ways.

(17.12) [Congestion pricing] Freeways typically save time for drivers versus taking an alternate route. But with too many cars, freeways become congested. Therefore, the average amount of time saved depends negatively on the number of cars on a freeway. Suppose that for a particular freeway, the average time saved in minutes (ATS) depends on the number of cars (Q) according to the following function:

$$ATS(Q) = 30 - (Q/100) .$$

- a. Assume that cars enter the freeway until the average time saved drops to zero. How many cars (Q) will use the freeway?

Suppose our objective is to maximize total time saved (TTS), defined as average time saved times the number of cars on the freeway. (Note that TTS is zero when cars are permitted to enter the freeway without restriction as in part (a) above.)

- b. Find an expression for TTS(Q).
- c. Compute the number of cars  $Q^*$  that maximizes total time saved.
- d. Compute TTS( $Q^*$ ) and ATS( $Q^*$ ).
- e. Suppose we wish to control the number of cars so as to maximize the total time saved by imposing a fee or toll on every car entering the freeway. If each driver is willing to pay \$0.50 for every minute of time saved, what should that fee be? In other words, what fee would make the average driver indifferent between taking the freeway and paying the fee, versus taking an alternate route and not paying the fee?

(17.13) [Common property resource] Suppose a particular species of bird inhabits a certain park and is attractive to hunters. But if too many hunters pursue the bird, the sustainable number that are shot begins to fall. The function relating the sustainable number of birds shot (Q) and the number of hunters admitted to the park (n) is given by  $Q = (-n^2/40) + 5n$ . Assume each hunter takes home the average number of birds shot per hunter ( $Q/n$ ).

- a. How many hunters would completely wipe out the bird population (drive Q to zero)?
- b. How many hunters would maximize the sustainable number of birds shot? What is that maximum?
- c. Find a formula for the number of birds each hunter takes home ( $Q/n$ ), as a function of the number of hunters (n).

Suppose each hunter incurs \$50 in his or her own costs to hunt and that each bird shot is worth \$25 to the hunter.

- d. How many hunters will want to enter the park and hunt this particular species of bird? [Hint: Set value of average catch ( $\$25 \times Q/n$ ) equal to \$50 and solve for n.]
- e. The government has decided to charge a fee to each hunter (in addition to the \$50) so as to maximize the sustainable total number of birds shot. What should that fee be? [Hint: Calculate the value of the average catch when n equals the number you found in part (d). Set this equal to \$50 plus the fee. Now solve for the fee.]

[end of problem set]