LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

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Part 4: General equilibrium and market power

Chapter 16: Game theory

Problems

In this problem set, the phrase "Nash equilibrium" refers to a Nash equilibrium in *pure strategies*, not mixed or randomized strategies. In other words, players must choose a particular strategy—they are not permitted to randomly switch between strategies.[1]

(16.1) [Game theory] Two neighboring countries must decide whether to have armies. Each feels safer if it has an army *provided the other country has none*. However, armies are expensive to maintain. The following table represents their situation as a game in normal form.

		Country B: Have no army	Country B: Have army
Country A	Have no army	A gets zero, B gets zero.	A gets -100, B gets 100.
	Have army	A gets 100, B gets -100.	A gets -50, B gets -50.

Note that if both countries have armies, then both countries must spend a lot of money maintaining their armies and neither country feels safer.

a. Is this a zero-sum game? Why or why not?
b. List any Pareto-efficient outcomes for this game. (An outcome is defined by specifying what strategy each player chooses.)
c. What is Country A's best reply if Country B has no army?
d. What is Country A's best reply if Country B has an army?
e. What is Country B's best reply if Country A has no army?
f. What is Country B's best reply if Country A has an army?
g. Is there a dominant-strategy equilibrium for this game? If so what is it? (An equilibrium outcome is defined by specifying what strategy each player chooses.)
h. Is there a Nash equilibrium for this game? If so, what is it? (An equilibrium outcome is defined by specifying what strategy each player chooses.)
i. How do you think this sort of game would be played in real life?

[1] Nash himself showed that every game has an equilibrium in mixed strategies (John Nash, "Non-Cooperative Games," *Annals of Mathematics*, Second Series, Vol. 54, No. 2 (September 1951), pp. 286-295.) But some games have no Nash equilibrium in pure strategies—for example, "Rock, Paper, Scissors."

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(16.2) [Game theory] Here is a simple model of bank runs before the availability of deposit insurance. Suppose the National Rock-Solid Bank has just two depositors, each with $50,000 on deposit. If the bank calls in all its loans immediately, many of its debtors will default and the bank will have liquid assets of only $60,000. But if the bank waits to call in loans till next month, more of its debtors will be able to pay and it will have liquid assets of $90,000. The two depositors are faced with the following situation, represented as a game in normal form.

		Depositor B	
		Withdraw money today	Withdraw money next month
Depositor A	Withdraw money today	A gets $30,000, B gets $30,000.	A gets $50,000, B gets $10,000.
	Withdraw money next month	A gets $10,000, B gets $50,000.	A gets $45,000, B gets $45,000.

a. Is this a zero-sum game? Why or why not?
b. List any Pareto-efficient outcomes for this game. (An outcome is defined by specifying what strategy each player chooses.)
c. What is Depositor A's best reply if Depositor B withdraws money next month?
d. What is Depositor A's best reply if Depositor B withdraws money today?
e. What is Depositor B's best reply if Depositor A withdraws money next month?
f. What is Depositor B's best reply if Depositor A withdraws money today?
g. Is there a dominant-strategy equilibrium for this game? If so what is it? (An equilibrium outcome is defined by specifying what strategy each player chooses.)
h. Is there a Nash equilibrium for this game? If so, what is it? (An equilibrium outcome is defined by specifying what strategy each player chooses.)
i. How do you think this sort of game would be played in real life?

(16.3) [Game theory] Two restaurant chains—O'Donalds and Burger Queen—are planning to build restaurants in Centerville. There are two prime locations in this town: intersection #1 and intersection #2. Now intersection #1 has more traffic and is therefore likely to be more profitable. A chain that locates at intersection #1 all by itself will enjoy $50,000 in profit. By contrast, a chain that locates at intersection #2 all by itself will enjoy only $40,000 in profit. If both chains locate at the same intersection, the profit will be divided evenly between them. Write out a table of this game in normal form and use it to answer the following questions.

a. Is this a zero-sum game? Why or why not?
b. List any Pareto-efficient outcomes for this game. (An outcome is defined by specifying what strategy each player chooses.)
c. What is O'Donalds' best reply if Burger Queen chooses intersection #1?
d. What is O'Donalds' best reply if Burger Queen chooses intersection #2?
e. What is Burger Queen's best reply if O'Donalds chooses intersection #1?
f. What is Burger Queen's best reply if O'Donalds chooses intersection #2?
g. Is there a dominant-strategy equilibrium for this game? If so what is it? (An equilibrium outcome is defined by specifying what strategy each player chooses.)
h. Is there a Nash equilibrium for this game? If so, what is it? (An equilibrium outcome is defined by specifying what strategy each player chooses.)
i. How do you think this sort of game would be played in real life?

(16.4) [Game theory] Two companies "Brand Name" and "Copycat" are each designing a product. Brand Name has a solid consumer reputation, so any product it designs is sure to sell. Copycat cannot sell a product on its own, but if its product looks just like Brand Name's it will take away one-fourth of Brand Name's profit. The following table represents their situation as a game in normal form. The choice of product appearance is reduced here to two color choices.

		Copycat: Red	Copycat: Blue
Brand Name	Red	Brand Name gets $750,000, Copycat gets $250,000.	Brand Name gets $1 million, Copycat gets zero.
	Blue	Brand Name gets $1 million, Copycat gets zero.	Brand Name gets $750,000, Copycat gets $250,000.

a. Is this a zero-sum game? Why or why not?
b. List any Pareto-efficient outcomes for this game. (An outcome is defined by specifying what strategy each player chooses.)
c. What is Brand Name's best reply if Copycat chooses red?
d. What is Brand Name's best reply if Copycat chooses blue?
e. What is Copycat's best reply if Brand Name chooses red?
f. What is Copycat's best reply if Brand Name chooses blue?
g. Is there a dominant-strategy equilibrium for this game? If so what is it? (An equilibrium outcome is defined by specifying what strategy each player chooses.)
h. Is there a Nash equilibrium for this game? If so, what is it? (An equilibrium outcome is defined by specifying what strategy each player chooses.)
i. How do you think this sort of game would be played in real life?

(16.5) [Game theory] Suppose the games given in problems (3) and (4) above were played *sequentially* rather than simultaneously. That is, suppose one player chose a strategy first and could not change it later in response to the second player's reply.

a. In the restaurant-location game of problem (3), what would the outcome be if Chain A moved first? What would the outcome be if Chain B moved first? Is it more advantageous to move first or to move second in this game? Why?
b. In the product-design game of problem (4), what would the outcome be if Brand Name moved first? What would the outcome be if Copycat moved first? Is it more advantageous to move first or to move second in this game? Why?

(16.6) [Game theory] Biologist John Maynard Smith reports an experiment in which two pigs—one weak and the other strong—were kept in a pen with a lever at one end and a food dispenser at the other.[2] When the lever was pushed (requiring a slight effort) a small amount of food appeared at the dispenser. Who do you think got most of the food—the strong pig or the weak pig? The surprising answer was in fact the Nash equilibrium to a simple game. Each pig had two strategies. The pig could push the lever and then run to the food dispenser at the other end of the pen. (This strategy uses up some calories from the effort.) Or the pig could wait by the food dispenser. The payoffs were as follows.

- If both pigs wait, neither gets any food, so each pig receives zero calories.
- If both pigs push the lever, Strong Pig pushes Weak Pig out of the way and gets all the food, so Strong Pig receives 90 calories and Weak Pig *loses* 10 calories from the effort.
- If Weak Pig pushes the lever while Strong Pig waits, then Strong Pig keeps Weak Pig away and again gets all the food, so Strong Pig receives 100 calories and Weak Pig again loses 10 calories.
- If Strong Pig pushes the lever while Weak Pig waits, Weak Pig eats most of the food before Strong Pig arrives and pushes it out of the way, so Weak Pig receives 75 calories and Strong Pig receives 15 calories.

Write a table of this game in normal form and use it to answer the following questions.

a. What is Strong Pig's best reply if Weak Pig chooses "push lever"?
b. What is Strong Pig's best reply if Weak Pig chooses "wait by dispenser"?
c. What is Weak Pig's best reply if Strong Pig chooses "push lever"?
d. What is Weak Pig's best reply if Strong Pig chooses "wait by dispenser"?
e. Is there a dominant-strategy equilibrium for this game? If so what is it? (An equilibrium outcome is defined by specifying what strategy each player chooses.)
f. Is there a Nash equilibrium for this game? If so, what is it? (An equilibrium outcome is defined by specifying what strategy each player chooses.)

[end of problem set]

[2] Steven E. Landsburg, *Price Theory and Applications*, fourth edition, 1999, page 433