LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

William M. Boal

Part 4: General equilibrium and market power

Chapter 15: Imperfect competition

Problems

(15.1) [Joint profit maximization] Two firms in a cartel each produce memory chips. Firm A has daily total cost function $TC_A = 9q_A + 0.02\ q_A^2$ Firm B has daily total cost function $TC_B = q_B + 0.01\ q_B^2$

a. Find the marginal cost function for Firm A: $MC_A(q_A)$.
b. Find the marginal cost function for Firm B: $MC_B(q_B)$.

Suppose the cartel intends to produce 1000 memory chips. Assume the cartel sets output levels for each firm so as to maximize jointly the total profits of the cartel.

c. How many memory chips will Firm A produce?
d. How many memory chips will Firm B produce?
e. Compute the cartel's joint marginal cost when producing 1000 memory chips.

The next three problems all refer the following information. Suppose the bottle-opener industry is served by two firms (#1 and #2) with identical, constant marginal cost given by: MC = AC = \$2. Market demand is given by $P = 14 - (Q/10)$. It is recommended that you sketch these curves on a graph and find their intersection before working the problems.

(15.2) [Competition] See information on the bottle-opener industry in the box above. Suppose the two firms engage in *price competition* (also called the "Bertrand model").

a. Compute equilibrium price P* and equilibrium total market quantity Q*, and plot them on your graph. [Hint: In competitive equilibrium, P* = MC.]
b. Compute consumer surplus and producer surplus.
c. Is there any deadweight loss? Does every demander who is willing to pay the marginal cost for the good get served?

(15.3) [Joint profit maximization] See information on the bottle-opener industry in the box above. Suppose the two firms engage in *collusion* to maximize the sum of their profits.

a. Find an expression for the two firm's joint marginal cost. [Hint: The horizontal sum of identical horizontal lines is another horizontal line just like them.]
b. Find an expression for the market marginal revenue function MR(Q).
c. Compute the firms' total quantity Q* and price P*. [Hint: Set MC = MR(Q), just as a monopolist would.]
d. Compute the combined total profit of the two firms.
e. Compute the social deadweight loss. [Hint: Sketch a graph first.]

(15.4) [Cournot duopoly] See information on the bottle-opener industry in the box above. Suppose the two firms engage in a symmetric *Cournot duopoly*, each firm setting its own quantity while taking the other firm's quantity as given. Let q_1 = firm #1's quantity and q_2 = firm #2's quantity, so that total market quantity $Q = q_1 + q_2$.

a. Find an expression for firm #1's total revenue, as a function of its own quantity and the quantity produced by the other firm: $TR_1(q_1,q_2)$. [Hint: By definition, $TR_1 = P\, q_1$. Here, replace P by the equation for the demand curve, and then replace Q by $(q_1 + q_2)$.]
b. Find an expression for firm #1's marginal revenue, as a function of its own quantity and the quantities produced by the other firm: $MR_1(q_1,q_2)$. [Hint: $MR_1(q_1,q_2) = \partial TR_1(q_1,q_2) / \partial q_1$.]
c. Find an expression for firm #1's reaction function, showing how much firm #1 will produce for any given level of quantity set by the other firm: $q_1^* = f(q_2)$. [Hint: Set MR_1 = MC and solve for q_1 as a function of q_2 .]
d. Assume the equilibrium is symmetric (that is, assume $q_1^* = q_2^*$) and compute firm #1's equilibrium quantity q_1^*.
e. Compute total market quantity Q* and the equilibrium price P*.
f. Compute the combined total profit of both firms.
g. Compute the social deadweight loss. [Hint: Sketch a graph first.]

The next three problems all refer the following information. Suppose the shoe-horn industry is served by two firms (Firm A and Firm B) with identical, constant marginal cost given by: MC = AC = \$3. Market demand is given by $P = 21 - (Q/10)$. It is recommended that you sketch these curves on a graph and find their intersection before working the problems.

(15.5) [Competition] See information on the shoe-horn industry in the box above. Suppose the two firms engage in *price competition* (also called the "Bertrand model").

a. Compute equilibrium price P* and equilibrium total market quantity Q*, and plot them on your graph. [Hint: In competitive equilibrium, P* = MC.]
b. Compute consumer surplus and producer surplus.
c. Is there any deadweight loss? Does every demander who is willing to pay the marginal cost for the good get served?

(15.6) [Joint profit maximization] See information on the shoe-horn industry in the box above. Suppose the two firms engage in *collusion* to maximize the sum of their profits.

a. Find an expression for the two firm's joint marginal cost. [Hint: The horizontal sum of identical horizontal lines is another horizontal line just like them.]
b. Find an expression for the market marginal revenue function MR(Q).
c. Compute the firms' total quantity Q* and price P*. [Hint: Set MC = MR(Q), just as a monopolist would.]
d. Compute the combined total profit of the two firms.
e. Compute the social deadweight loss. [Hint: Sketch a graph first.]

(15.7) [Cournot duopoly] See information on the shoe-horn industry in the box above. Suppose the two firms engage in a symmetric *Cournot duopoly*, each firm setting its own quantity while taking the other firm's quantity as given. Let q_A = firm A's quantity and q_B = firm B's quantity, so that total market quantity $Q = q_A + q_B$.

a. Find an expression for firm A's total revenue, as a function of its own quantity and the quantity produced by the other firm: $TR_A(q_A,q_B)$. [Hint: By definition, $TR_A = P\ q_A$. Here, replace P by the equation for the demand curve, and then replace Q by $(q_A + q_B)$.]
b. Find an expression for firm A's marginal revenue, as a function of its own quantity and the quantities produced by the other firm: $MR_A(q_A,q_B)$. [Hint: $MR_A(q_A,q_B) = \partial TR_A(q_A,q_B) / \partial q_A$.]
c. Find an expression for firm A's reaction function, showing how much firm A will produce for any given level of quantity set by the other firm: $q_A^* = f(q_B)$. [Hint: Set MR_A = MC and solve for q_A as a function of q_B .]
d. Assume the equilibrium is symmetric (that is, assume $q_A^* = q_B^*$) and compute firm A's equilibrium quantity q_A^*.
e. Compute total market quantity Q* and the equilibrium price P*.
f. Compute the combined total profit of both firms.
g. Compute the social deadweight loss. [Hint: Sketch a graph first.]

The next four problems refer to the Lerner index of monopoly power, which measures the percent of price represented by markup over marginal cost (P−MC). Recall that the Lerner index of market power takes the following form under Cournot oligopoly:

$$\text{Lerner index} = \frac{P - MC_i}{P} = \frac{S_i}{|\varepsilon|}$$

where P denotes the equilibrium price, MC_i denotes the marginal cost of firm #i, S_i denotes the market share of firm #i, and ε denotes the market elasticity of demand.

(15.8) [Symmetric Cournot oligopoly] See information in box above. Suppose in a certain market the elasticity of demand is $\varepsilon = -2$ and all n firms have the same marginal cost MC=$15. Assume the equilibrium is *symmetric* so that all n firms have the same market share $S_i = 1/n$. Use the equation above to calculate the market equilibrium price P when the number of firms is …

a. $n = 1$ (monopoly).
b. $n = 2$ (duopoly).
c. $n = 3$ (sometimes called "triopoly").
d. $n = 8$.
e. $n \to \infty$ (a very large number of firms, so that $S_i \to 0$).

(15.9) [Asymmetric Cournot oligopoly] See information in box above. Suppose in a certain market there are three firms with *different* marginal costs, and the elasticity of demand is –2. Because the firms have unequal marginal costs, this is *not* a symmetric oligopoly, but the equation for the Lerner index still applies to each firm. We observe that the market price is $10. We also observe that the market share of firm #1 is S_1=0.6 (or 60%), the market share of firm #2 is S_2=0.3 (or 30%), and the market share of firm #3 is S_3=0.1 (or 10%).

a. Use the equation for the Lerner index to compute the marginal cost of each firm.
b. Does the biggest firm have the highest marginal cost, or vice versa? Or is size unrelated to marginal cost?

(15.10) [Asymmetric Cournot oligopoly] See information in box above. Suppose in a certain market there are four firms with possibly *different* marginal costs, and the elasticity of demand is –1.5. Because the firms have unequal marginal costs, this is *not* a symmetric oligopoly, but the equation for the Lerner index still applies to each firm. We observe that the market price is $30. We also observe that the market share of Firm A is S_A=0.3 (or 30%), the market share of Firm B is S_B=0.3 (or 30%), the market share of Firm C is S_C=0.3 (or 30%), and the market share of Firm D is S_D=0.1 (or 10%).

a. Use the equation for the Lerner index to compute the marginal cost of each firm.
b. Do the big firms have the highest marginal cost, or does the small firm? Or is size unrelated to marginal cost?
c. Assume average cost equals marginal cost for each firm. If this were a competitive market rather than a Cournot oligopoly, would all four firms survive?

(15.11) [Asymmetric Cournot oligopoly] There are essentially two producers of CPU chips for personal computers. A recent report indicates the Intel has about 80% market share in the CPU market and AMD has about 20% market share.[1] Assume the firms are a Cournot duopoly.

a. Which firm evidently has higher marginal cost? Explain your reasoning.
b. Use the information given in the box above to prove that, in a Cournot oligopoly, the ratio of any firm's marginal cost to the market price is given by
$$\frac{MC_i}{P} = \frac{|\varepsilon| - S_i}{|\varepsilon|}$$
c. Assume that the elasticity of demand for CPUs is -1, and compute the ratio of marginal cost to price each firm.

(15.12) [Monopoly and Cournot oligopoly] The market for health care is known to have inelastic demand (that is, a demand elasticity less than 1 in absolute value) at the current price.[2]

a. Economist A claims that the firms in the health care industry are colluding to set prices at the joint profit-maximizing level. Is this claim possible or impossible? Explain your reasoning. [Hint: The pricing equation for monopoly or collusion is P = MC / (1– (1/|ε|)). What would the price be if demand were inelastic? What would be the value of the Lerner index if demand were inelastic?]
b. Economist B claims that the firms are interacting as Cournot oligopolists. Is this claim possible or impossible? Explain your reasoning. [Hint: Use the Lerner index given in the box above to derive a pricing equation for Cournot oligopolists. Show that if demand is inelastic, the Cournot oligopoly price can still be finite if there is more than one firm.]

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[1] https://www.eteknix.com/amd-gains-2-2-cpu-market-share-intel-q1-2017/, accessed 7/11/2017.

[2] This question is inspired by Kenneth J. Arrow, "Uncertainty and the Welfare Economics of Medical Care," American Economic Review, Vol. 53, No. 5 (December 1963), p. 957.

(15.13) [Cournot duopoly pricing] In this problem we prove that, if demand is linear and marginal cost is constant, then a symmetric Cournot duopoly results in a price that is one-third of the way from competitive outcome to the price-intercept of the demand curve. Let the demand curve be given by $P = a - bQ$, where $Q = q_1 + q_2$, and let c denote marginal cost.

a. Find expressions for the competitive equilibrium price and quantity in terms of a, b, and c.
b. Find an expression for Cournot duopolist #1's reaction function in terms of a, b, c, and q_2.
c. Prove that the Cournot duopoly equilibrium quantity is $Q^* = (2/3)\,(a-c)/b$.
d. Prove that the Cournot duopoly equilibrium price is $P^* = (1/3)a + (2/3)c$.

(15.14) [Cournot triopoly pricing] In this problem we prove that, if demand is linear and marginal cost is constant, then a symmetric Cournot triopoly results in a price that is one-fourth of the way from marginal cost to the price-intercept of the demand curve. Let the demand curve be given by $P = a - bQ$, where $Q = q_1 + q_2 + q_3$, and let c denote marginal cost.

a. Find expressions for the competitive equilibrium price and quantity in terms of a, b, and c.
b. Find an expression for Cournot triopolist #1's reaction function in terms of a, b, c, q_2 , and q_3 .
c. Prove that the Cournot triopoly equilibrium quantity is $Q^* = (3/4)\,(a-c)/b$.
d. Prove that the Cournot triopoly equilibrium price is $P^* = (1/4)a + (3/4)c$.

(15.15) [Antitrust policy] Suppose a market has three firms (Firm A, Firm B, and Firm C) each with constant marginal cost equal to $30 (equal to average cost). The market elasticity of demand is -2. Assume the Cournot model of oligopoly applies to this market.

a. What will be the equilibrium market shares of Firm A, Firm B, and Firm C?
b. What is the equilibrium price?

Suppose Firms A and B propose to merge into a single firm—call it "AB." There would then be just two firms in the market (Firm AB and Firm C). Again assume the Cournot model of oligopoly would apply to the outcome.

c. What would be the new equilibrium market shares of Firm AB and Firm C?
d. What would be the new equilibrium price?
e. It can be shown that Firm C's quantity would increase as a result of this merger. Do you think Firm C would object to this merger? Why or why not?

(15.16) [Cournot duopoly with differentiated products] Firm A and Firm B produce similar items, but their designs are distinctive, so their products are not perfect substitutes. Firm A's demand is given by $p_A = 19 - (q_A/10) - (q_B/20)$. Firm B's demand is given by $p_B = 19 - (q_B/10) - (q_A/20)$. As in an ordinary Cournot duopoly, each firm sets its own quantity, taking as given the quantity of the other firm. Note however that here the firms can have different prices. Both firms have constant marginal cost equal to \$4 .

a. Find an expression for Firm A's total revenue in terms of q_A and q_B . [Hint: $TR_A(q_A,q_B) = p_A\, q_A$.]
b. Find an expression for firm A's marginal revenue in terms of q_A and q_B: $MR_A(q_A,q_B)$. [Hint: $MR_A(q_A,q_B) = \partial TR_A(q_A,q_B) / \partial q_A$.]
c. Find an expression for firm A's reaction function, showing how much firm A will produce for any given level of quantity set by the other firm: $q_A^* = f(q_B)$. [Hint: Set $MR_A = MC$ and solve for q_A as a function of q_B .]
d. Assume the equilibrium is symmetric (that is, assume $q_A^* = q_B^*$) and compute firm A's equilibrium quantity q_A^*.
e. Compute firm A's equilibrium price p_A^* and the Lerner index.

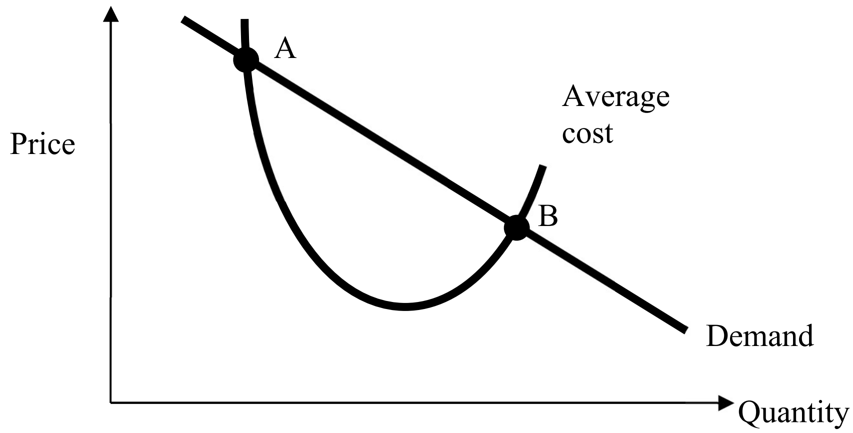
(15.17) [Price-setting (Bertrand) duopoly with differentiated products] Firm A and Firm B produce similar items, but their designs are distinctive, so their products are not perfect substitutes. Firm A's demand is given by $Q_A = 600 - 20\, P_A + 10\, P_B$. Firm B's demand is given by $Q_B = 600 - 20\, P_B + 10\, P_A$. Each firm sets its own *price*, taking as given the *price* of the other firm. Assume the firms have no costs, so each firm simply seeks to maximize its own total revenue.

a. Give an expression for Firm A's total revenue TR_A, in terms of P_A and P_B .
b. What price should Firm A set, given Firm B's price P_B ? Give an expression in terms of P_B. In other words, give Firm A's *best reply function* .
c. Compute the (Nash) equilibrium prices P_A^* and P_B^*. [Hint: you may assume this equilibrium is symmetric: $P_A^* = P_B^*$.]
d. Compute Q_A^* and Q_B^*. [Again, assume symmetry.]
e. Compute the total revenue of each firm.

(15.18) [Collusive price-setting with differentiated products] Use the same assumptions as in the previous problem, but now assume that Firm A and Firm B *coordinate* their prices to maximize the combined total revenue ($TR=TR_A+TR_B$) of both companies.

a. Give an expression for the companies' combined total revenue, in terms of P_A and P_B .
b. Compute the prices P_A^* and P_B^* that maximize TR. [Hint: Set $\partial TR/\partial P_A = 0$, and set $\partial TR/\partial P_B = 0$. Solve these equations jointly for P_A and P_B .]
c. Compute Q_A^* and Q_B^*, the corresponding quantities.
d. Compute TR*, the maximum combined total revenue.

(15.19) [Monopoly, monopolistic competition] The graph below shows the demand curve faced by a firm and its average cost curve.



a. Explain why the firm would never choose to operate at point A.
b. Explain why the firm would never choose to operate at point B.

(15.20) [Monopolistic competition] Long-run equilibrium in the model of monopolistic competition is similar to long-run equilibrium in the model of perfect competition in that two conditions hold:

(i) Marginal revenue = marginal cost for each firm.
(ii) Price (sometimes called "average revenue") = average cost for each firm.

a. Which condition is *desired* by each firm? Why?
b. Which condition is *forced* on each firm by market forces outside its control? Why?

(15.21) [Monopolistic competition] In this problem, we prove that the tangency condition for monopolistic competition incidentally implies that marginal cost = marginal revenue. At the tangency, the slope of the AC curve equals the slope of the demand curve, and also AC=P. But what is the slope of the AC curve? Since AC=TC/Q by definition, the quotient rule from calculus implies that the slope of the AC curve is given by the following:

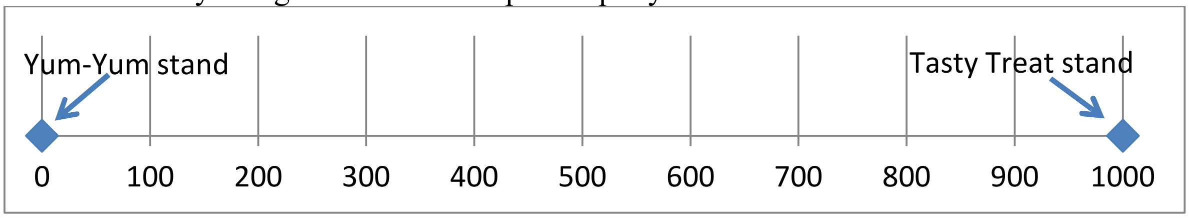
(i) $\frac{dAC}{dQ} = \frac{(dTC/dQ)\,Q - TC}{Q^2} = \frac{(dTC/dQ)\,Q - (AC \times Q)}{Q^2}$.

Also, recall from our study of monopoly that marginal revenue can be written as

(ii) $MR = P + Q\frac{dP}{dQ}$

a. Solve (ii) to give an expression for dP/dQ, the slope of the demand curve.
b. Set dP/dQ = dAC/dQ, and substitute AC=P. Use algebra to show that MR=MC. Show your work.

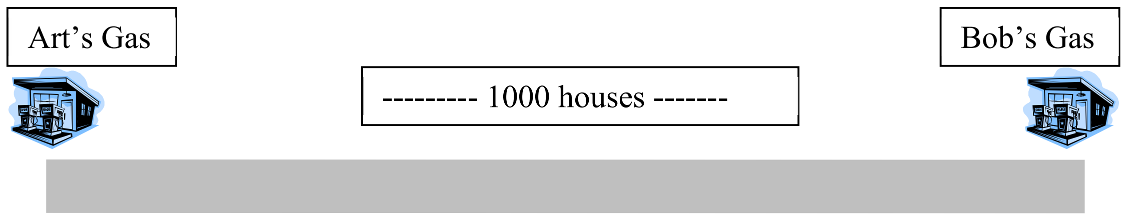
(15.22) [Monopolistic competition] Two ice-cream stands are positioned at opposite ends of a beach. Yum-Yum is at the extreme left and Tasty Treat is at the extreme right, as shown in the diagram below. The beach is 1000 yards long and there are people scattered evenly along the beach—one person per yard.



Everyone wants to buy one ice-cream cone. The only question is where they will buy it. No one wants to walk far. In fact, each person is willing to pay one cent to avoid walking (back and forth) one yard. In other words, each person's total price of an ice cream cone is $p + 0.01\ D$, where p is the money price they pay and D is the distance to the stand. Naturally, each person chooses the stand with the lowest total price. Suppose that Tasty Treat sets a money price of P_T=\$6.

a. Find an equation relating quantity of customers that choose Yum-Yum q_Y to Yum-Yum's price p_Y. [Hint: set the total price of an ice-cream cone from Yum-Yum equal to the total price of a cone from Tasty-Treat, and solve for D.]
b. Find an equation relating Yum-Yum's total revenue TR_Y to Yum-Yum's price p_Y.
c. Compute the price p_Y that maximizes Yum-Yum's total revenue TR_Y.
d. Now suppose alternatively that no one minds walking, so they just choose the stand with the lowest money price P. Now what price p_Y would maximize Yum-Yum's total revenue?

(15.23) [Monopolistic competition] Two gas stations are situated at opposite ends of a road. Art's Gas is at one end and Bob's Gas is at the other. Along the road live 1000 customers in evenly-spaced houses. Art and Bob are each setting prices for a tankful of gas (say, 10 gallons) to maximize their *individual* profits. Assume the marginal and average cost of a tankful of gas (for both Art and Bob) is \$20. Of course, Art and Bob will likely set their prices higher than \$20 because they have market power. In this problem, you must find out how much higher.



Assume everyone wants to buy one tankful of gas. The only question is where they will buy it. No one wants to drive far. In fact, each person is willing to pay one cent to avoid driving (back and forth) past each house. In other words, each person's total price of a tankful of gas is $p + 0.01\ D$, where p is the money price they pay for a tankful of gas and D is the distance (in houses) to the gas station. Naturally, each person goes to the gas station which offers, for that person, the lowest total price.

a. Find an equation relating quantity of customers that go to Art's gas station as a function of p_A and p_B. [Hint: Set the total price of Art's Gas equal to the total price of Bob's Gas, and solve for D*.]
b. Find an equation relating Art's total revenue TR_A to p_A and p_B. [Hint: $TR_A = D^* \times p_A$.]
c. Find an equation relating Art's profit to p_A and p_B. [Hint: Art's profit = $TR_A - D^* \times \$20$.]
d. Find an equation for Art's best reply price p_A to Bob's price p_B. [Hint: Maximize Art's profit with respect to p_A.]
e. Assume symmetry ($p_A = p_B$) and find equilibrium prices p_A and p_B.

[end of problem set]