LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

William M. Boal

Part 3: Firms and competition

Chapter 10: Cost

Problems

(10.1) [Minimizing cost] Suppose a firm wishes to produce 30 units of output per hour at minimum cost. Machines cost $18 per hour and workers cost $8 per hour. The firm's production function is $q = x_1^{1/2} x_2^{1/2}$ where q denotes the quantity of output per hour, x_1 denotes the number of machines, and x_2 denotes the number of workers.

a. Find an algebraic expression for the firm's marginal rate of substitution in production (MRSP) of workers for machines. [Hint: This is the slope of the firm's isoquants with machines on the vertical axis and workers on the horizontal axis.]
b. Compute the slope of the firm's isocost lines, with machines on the vertical axis and workers on the horizontal axis.
c Give an equation for the firm's target isoquant.
d. Find the number of machines x_1* and the number of workers x_2* required to produce 30 units per hour at minimum cost.
e. Compute the total cost of producing 30 units of output per hour—that is, TC(30).
f. Compute the average cost of producing 30 units of output per hour—that is, AC(30).

(10.2) [Minimizing cost] Suppose a firm wishes to produce 500 units of output per hour at minimum cost. Machines cost $40 per hour and workers cost $10 per hour. The firm's production function is $q = x_1^{1/3} x_2^{2/3}$ where q denotes the quantity of output per hour, x_1 denotes the number of machines, and x_2 denotes the number of workers.

a. Find an algebraic expression for the firm's marginal rate of substitution in production (MRSP) of workers for machines. [Hint: This is the slope of the firm's isoquants with machines on the vertical axis and workers on the horizontal axis.]
b. Compute the slope of the firm's isocost lines, with machines on the vertical axis and workers on the horizontal axis.
c Give an equation for the firm's target isoquant.
d. Find the number of machines x_1* and the number of workers x_2* required to produce 500 units per hour at minimum cost.
e. Compute the total cost of producing 500 units of output per hour—that is, TC(500).
f. Compute the average cost of producing 500 units of output per hour—that is, AC(500).

(10.3) [Minimizing cost] Suppose a firm wishes to produce 300 units of output per hour at minimum cost. Machines cost $10 per hour and workers cost $15 per hour. The firm's production function is $q = 3\,(x_1^{1/2} + x_2^{1/2})^2$ where q denotes the quantity of output per hour, x_1 denotes the number of machines, and x_2 denotes the number of workers.

a. Find an algebraic expression for the firm's marginal rate of substitution in production (MRSP) of workers for machines. [Hint: This is the slope of the firm's isoquants with machines on the vertical axis and workers on the horizontal axis.]
b. Compute the slope of the firm's isocost lines, with machines on the vertical axis and workers on the horizontal axis.
c Give an equation for the firm's target isoquant.
d. Find the number of machines x_1^* and the number of workers x_2^* required to produce 300 units per hour at minimum cost.
e. Compute the total cost of producing 300 units of output per hour—that is, TC(300).
f. Compute the average cost of producing 300 units of output per hour—that is, AC(300).

(10.4) [Minimizing cost with fixed-proportions technology] Suppose a particular machine can make 50 parts per hour if it is operated by four workers. The machine cannot be operated by fewer than four workers, and extra workers on the same machine add nothing to output. A firm can use as many machines as desired, with no loss in output per machine, provided each machine is operated by at least four workers. Machines cost $15 per hour. Competent workers may be hired for $10 per hour.

a. Compute the slope of the firm's isocost lines, with machines on the vertical axis and workers on the horizontal axis.
b. Find the number of machines x_1^* and the number of workers x_2^* required to produce 200 parts per hour at minimum cost. [Hint: Calculus is no help here, because the isoquants are not smooth curves. The solution is at the kink in the isoquant.]
c. Compute the total cost of producing 200 parts per hour—that is, TC(200). Also compute the average cost per unit AC(200).
d. Compute the total cost of producing 1000 parts per hour—that is, TC(1000). Also compute the average cost per unit AC(1000).
e. Find an algebraic expression for the total cost function TC(q). Also find formulas for the firm's average cost function AC(q) and marginal cost function MC(q). [Hint: In this problem, AC(q) and MC(q) are *very* simple functions—constants!]

(10.5) [Deriving input demand functions] Suppose a firm's daily production function is $q = x_1^{1/2} x_2^{1/2}$ where q denotes the quantity of output, x_1 denotes the number of machines, and x_2 denotes the number of workers. Machines cost \$15 per day and workers can be hired for \$60 per day.

a. Compute the slope of the firm's isocost lines, with machines on the vertical axis and workers on the horizontal axis.
b. Find an algebraic expression for the firm's marginal rate of substitution in production (MRSP) of workers for machines. [Hint: This is the slope of the firm's isoquants with machines on the vertical axis and workers on the horizontal axis.]
c. Find an equation for the firm's expansion path given these input prices—that is, an equation for the number of machines it will use (x_1) as a function of the number of workers (x_2). [Hint: Set the slope of the isocost line equal to the MRSP and solve for x_1.]
d. Find an equation for the number of workers the firm will use (x_2) as a function of its output target (q). [Hint: Substitute your answer to part (c) into the production function and solve for x_2.]
e. Find an equation for the number of machines the firm will use (x_1) as a function of its output target (q).

(10.6) [Deriving long-run and short-run cost curves] Suppose a firm's weekly production function is $q = 10\, x_1^{1/2} x_2^{1/2}$ where q denotes the quantity of output per week, x_1 denotes the number of machines, and x_2 denotes the number of workers. Machines cost \$100 per week and workers can be hired for \$400 per week. Assume initially that the firm can set any level of machines and workers it chooses—this is the long-run situation. It can be shown (using the methods described in problem (3) above) that the cost-minimizing input demand functions for this problem are: $x_1^* = q/5$ and $x_2^* = q/20$.

a. Use these input demand functions to find an algebraic expression for the long-run total cost function $TC(q)$. Also find formulas for the firm's long-run average cost function $AC(q)$ and long-run marginal cost function $MC(q)$. [Hint: In this case, these are *very* simple functions.]

Now assume a short-run situation where the number of machines is fixed at $x_1=25$ but the number of workers is variable.

b. Compute the firm's weekly short-run fixed cost SFC.
c. Find an algebraic expression for the number of workers required to produce any given level of output. That is, find x_2 as a function of q. [Hint: Substitute $x_1=25$ into the production function and solve for x_2.]
d. Find an algebraic expression for the cost of these workers at any given level of output. That is, find $SVC(q)$.
e. Find an algebraic expression for the firm's short-run total cost $STC(q)$.
f. Find an algebraic expression for the firm's short-run average variable cost $SAVC(q)$.
g. Find an algebraic expression for the firm's short-run average fixed cost $SAFC(q)$.
h. Find an algebraic expression for the firm's short-run marginal cost $SMC(q)$.

(10.7) [Long-run cost curves] Suppose a firm has a total cost function given by:

$$TC(q) = 0.01\, q^3 - 2\, q^2 + 150\, q\ .$$

a. What are the firm's (long run) costs when $q = 0$?
b. Find an algebraic expression for the firm's average cost function $AC(q)$.
c. Find the firm's so-called "efficient scale of operation" q_{ES}. [Hint: The efficient scale is the value of q that minimizes the function $AC(q)$. Find the minimum by setting the slope (derivative) of $AC(q)$ equal to zero.]
d. Compute the firm's minimum average cost—that is, its average cost of production when it operates at the efficient scale. That is, compute $AC(q_{ES})$.
e. For what values of q does the firm enjoy economies of scale (falling $AC(q)$)? For what values of q does the firm suffer diseconomies of scale (rising $AC(q)$)?
f. Find an algebraic expression for the firm's marginal cost function $MC(q)$.

(10.8) [Long-run cost curves] Suppose a firm has a total cost function given by:

$$TC(q) = 0.05\, q^3 - 2\, q^2 + 50\, q\ .$$

a. What are the firm's (long run) costs when $q = 0$?
b. Find an algebraic expression for the firm's average cost function $AC(q)$.
c. Find the firm's so-called "efficient scale of operation" q_{ES}. [Hint: The efficient scale is the value of q that minimizes the function $AC(q)$. Find the minimum by setting the slope (derivative) of $AC(q)$ equal to zero.]
d. Compute the firm's minimum average cost—that is, its average cost of production when it operates at the efficient scale. That is, compute $AC(q_{ES})$.
e. For what values of q does the firm enjoy economies of scale (falling $AC(q)$)? For what values of q does the firm suffer diseconomies of scale (rising $AC(q)$)?
f. Find an algebraic expression for the firm's marginal cost function $MC(q)$.

(10.9) [Long-run cost with minimum capacity] Production of a certain item requires a machine, labor, electricity, and raw materials. The daily cost of labor, electricity, and raw materials, denoted C_1 is strictly proportional to daily output at \$5 per unit: $C_1(q) = 5\, q$. Machines come in different sizes, and the daily cost of these machines, denoted C_2, is proportional to daily capacity at \$10 per unit for machines with at least 100 units of capacity: $C_2(q) = 10\, q$, if $q \geq 100$. Unfortunately, machines with less than 100 units of capacity are not available.

a. Give equations defining total cost $TC = C_1 + C_2$.
b. Give equations defining average cost $AC = TC/q$.
c. Compute average cost when q equals 10 units, 20 units, 50 units, 100 units, and 200 units.
d. The efficient scale q_{ES} is a set or range of values here, not a single number. What is that range?
e. Give equations defining marginal cost $MC = dTC/dq$.

(10.10) [Short-run cost curves] Suppose a firm has daily fixed costs of $100 per day. The same firm has daily short-run variable costs (that is, costs that are affected by the level of output it chooses) given by:

$$SVC(q) = 0.01\, q^3 - q^2 + 40\, q$$

a. Find an expression for the firm's short-run total cost STC(q).
b. Compute the firm's short run total cost when $q = 0$.
c. Find an expression for the firm's short-run average variable cost SAVC(q).
d. For what level of output q is SAVC(q) at its minimum? [Hint: Find the minimum by setting the slope (derivative) of SAVC(q) equal to zero.]
e. Compute the firm's shut-down price.
f. Find an expression for the firm's short-run average fixed cost SAFC(q).
g. Find an expression for the firm's short-run marginal cost SMC(q).
h. Find an expression for the firm's short-run average total cost SATC(q).

(10.11) [Short-run cost curves] Suppose a firm has daily fixed costs of $200 per day. The same firm has daily short-run variable costs (that is, costs that are affected by the level of output it chooses) given by:

$$SVC(q) = 0.05\, q^3 - 3q^2 + 60\, q$$

a. Find an expression for the firm's short-run total cost STC(q).
b. Compute the firm's short run total cost when $q = 0$.
c. Find an expression for the firm's short-run average variable cost SAVC(q).
d. For what level of output q is SAVC(q) at its minimum? [Hint: Find the minimum by setting the slope (derivative) of SAVC(q) equal to zero.]
e. Compute the firm's shut-down price.
f. Find an expression for the firm's short-run average fixed cost SAFC(q).
g. Find an expression for the firm's short-run marginal cost SMC(q).
h. Find an expression for the firm's short-run average total cost SATC(q).

[end of problem set]