LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

William M. Boal

Part 2: Consumers and demand

Chapter 8: Measuring consumer welfare

Section 8.1: The impact on consumers of price changes

Many actions of government increase or decrease the prices paid by consumers. For example, the government might increase the tax on gasoline, causing a rise the total price paid by consumers. Or the government might eliminate quotas on imports of clothing, causing a fall in the price of clothing paid by consumers. Or the government might seek to break up a cartel of computer-chip producers, causing a fall in the price of computers paid by consumers.

To evaluate the impact of such government actions, it would be useful to measure their effect on the well-being or welfare of consumers. The most obvious indicator of welfare is utility. However, utility is not directly measurable, as noted earlier (see chapter 3 section 3.2). In any case, a dollar-based measure of consumer benefit is more useful than a util-based measure because it can be compared to the dollar cost of government actions.

Compensating variation. How can we *quantify in dollars* the change in consumer welfare from a change in prices? Put differently, *how much better or worse off* is the consumer after the price change? A natural measure of the change in consumer welfare is the amount of additional income we must give (or take away from) the consumer, in compensation for the price change, to allow the consumer to maintain the same level of utility as before the change. Formally, this income adjustment is called the *compensating variation in income*.[1]

Section 8.2: Changes in many prices

In this section, we apply the idea of compensating variation to a situation where many prices change simultaneously. In this situation, the consumer's budget line will shift and

---

[1] Compensating variation was first defined by Hicks (John R. Hicks, *Value and Capital: An Inquiry Into Some Fundamental Principles of Economic Theory*, 2nd edition, Oxford, United Kingdom: Oxford University Press, 1946, pp. 40, 330-332). It should be emphasized that *compensating variation* is a theoretical concept. Actual income adjustments by government programs or labor contracts are influenced by political or bargaining power—they rarely equal Hicks's compensating variation. Throughout this chapter, we use as a benchmark the level of utility attained before the price change. If instead the benchmark is the new level of utility *after* the price change, a similar concept called the *equivalent variation in income* results (Hicks, 1946, p. 331).

possibly rotate if prices do not all change at the same rate. This situation is often described as a *change in the cost of living*.

Consider first the case of an increase in prices. If prices generally rise, then the budget line will shift toward the origin, and the consumer will not be able to attain that consumer's old indifference curve, as in figure 8.1. The compensating variation in this case is the increase in income required to push the consumer's budget line back up until it just barely touches the old indifference curve. The compensating variation is thus the same change in income used to define the Hicks substitution effect in section 6.5 and illustrated in figure 6.10 of the previous chapter.

The combined result of the increase in prices and the (positive) compensating variation in income is shown as the "hypothetical budget line" in figure 8.2. Note that the compensating variation in income is sufficient to allow the consumer to regain the old indifference curve, but is not sufficient to afford the old bundle A at the new prices. This is seen clearly in figure 8.2, where bundle A lies above the "hypothetical budget line" passing through bundle B. If given this compensating variation in income, the consumer would choose a new bundle B on the same indifference curve as the old bundle A, but costing less than bundle A at the new prices.[2]

Exact cost-of-living index. The fall in consumer welfare resulting from an increase in many prices can be expressed as the required change in income, but is more commonly expressed as the ratio of required total income to old income. Let the *required income at new prices* be the total income required to reach the old indifference curve at the new prices—that is, the sum of old income and the compensating variation.[3] Thus for example in figure 8.2, the required income at new prices is the income corresponding to the "hypothetical budget line." The *exact cost-of-living (COL) index* is the ratio of required income at new prices, to old income before the price change, usually multiplied by 100 for convenience:

$$
\begin{aligned}
\text{Exact COL index} &= \frac{\text{old income} + \text{compensating variation}}{\text{old income}} \times 100 \\
&= \frac{\text{required income at new prices}}{\text{old income}} \times 100
\end{aligned} \qquad (8.1)
$$

The exact COL index will be greater than 100 if prices are generally increasing.

As a numerical example, suppose in figure 8.2 that the solid budget lines reflect an income of $2000 and the hypothetical budget line requires an income of $2500. Then the compensating variation for this consumer is $500. The exact COL index is ($2500/$2000) x 100 = 125.

[2] Note that the change from bundle A to bundle B is just the Hicks substitution effect of the price change, as defined in section 6.5 and illustrated in figure 6.10 of the previous chapter.

[3] The required level of income can be viewed as a function that depends on the target level of utility and the current prices. This function is known as the *expenditure function*.

Figure 8.1. An increase in prices shifts budget line inward and reduces consumer welfare

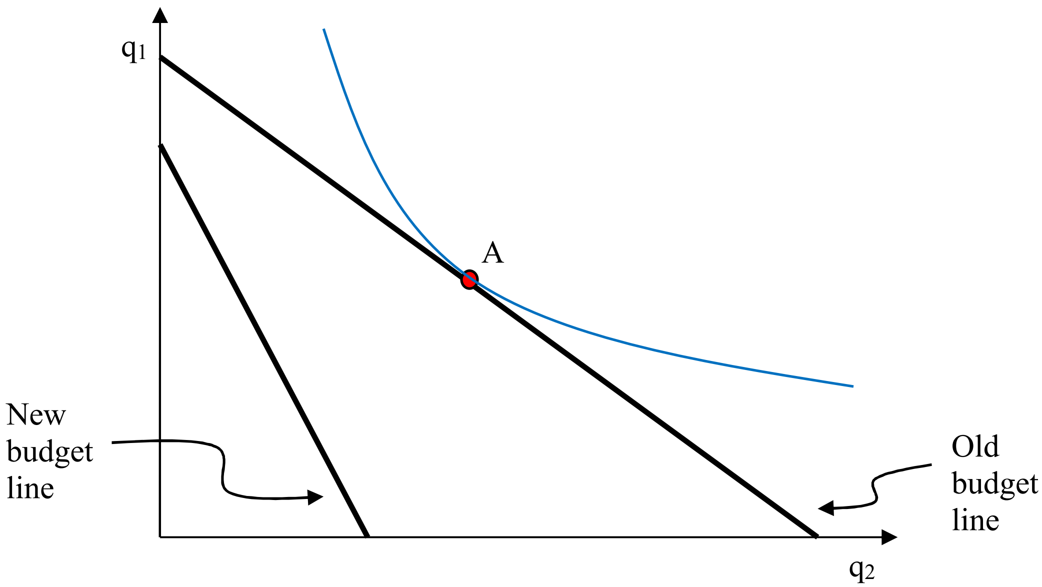
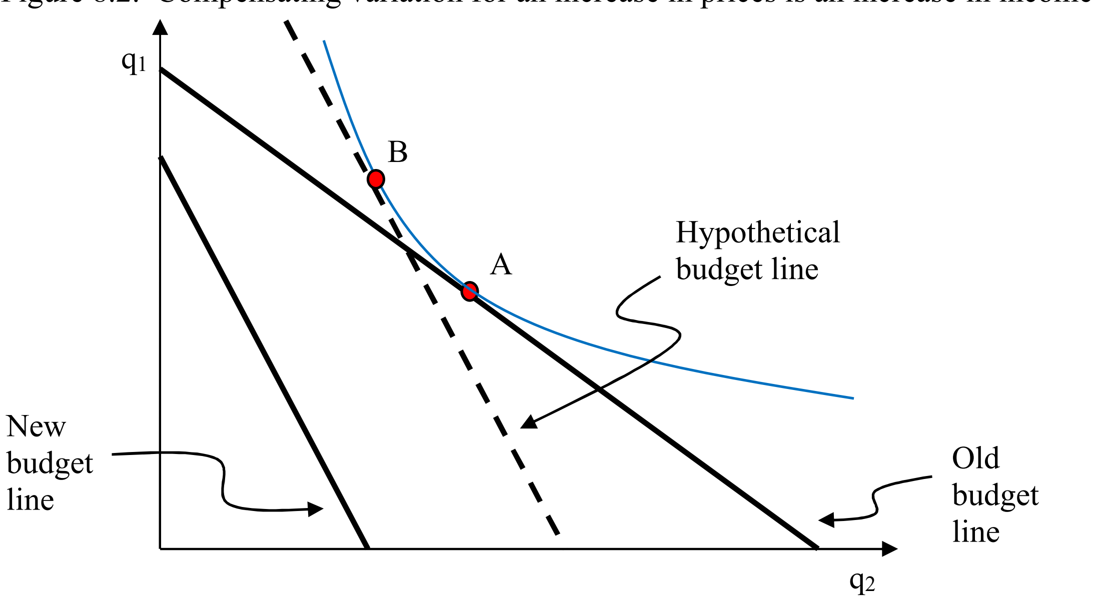


Figure 8.2. Compensating variation for an increase in prices is an increase in income



Consider next the case of a decrease in prices. If prices generally fall, then the budget line will shift out, away from the origin, and the consumer will be able to attain a much higher indifference curve than before, as in figure 8.3. The compensating variation in this case is the *decrease* in income required to push the consumer's budget line back down until it just barely touches the old indifference curve. The combined result of the decrease in prices and the (negative) compensating variation in income is shown as the "hypothetical budget line" in figure 8.4. Again, the compensating variation in income is sufficient to allow the consumer to regain the old indifference curve, but is not sufficient to afford the old bundle A. If required to give up this compensating variation in income, the consumer would choose a new bundle B on the same indifference curve as the old bundle A, but costing less than bundle A at the new prices. We know that bundle B costs less than bundle A at the new prices, because bundle A lies above the "hypothetical budget line" passing through bundle B.

The same definition (8.1) for the exact COL index still applies to a decrease in prices. However, since the compensating variation income is negative, the exact COL index will be less than 100.

Section 8.2: Practical COL indexes

Computing the compensating variation in income or the exact COL index in the real world is tricky. We must either know the shapes of the indifference curves (if we use graphical methods similar to figures 8.2 and 8.4) or the form of the utility function (if we use calculus as in problem (8.1) at the end of this chapter). In many situations, indifference curves and utility functions are not known. Instead, we typically know only the quantities of goods purchased before or after the price change.[4] Unfortunately, the old and new quantities actually observed are not usually on the same indifference curve, unlike bundles A and B in figures 8.2 and 8.4. Consumer buying patterns certainly respond to changes in prices (and perhaps simultaneous changes in income) but there is no reason to believe that consumers are just as well off as before the price changes. Given such limited information, the exact COL index cannot be computed.

Laspeyres[5] cost-of-living index. An approximate COL index can be computed using only the original bundle. The Laspeyres COL index is defined as the ratio of the cost of the *old* bundle at new prices to its cost at old prices, times 100. For two goods, the formula is

(8.2) $$\text{Laspeyres COL index} = \frac{p_1^{new} q_1^{old} + p_2^{new} q_2^{old}}{p_1^{old} q_1^{old} + p_2^{old} q_2^{old}} \times 100 \ .$$

[4] These are obtained from surveys of consumers like the Consumer Expenditure Survey in the United States <http://www.bls.gov/cex/home.htm>.

[5] Ernst Louis Étienne Laspeyres, *Geshichete der Volkswirtshaftlichen Anschauungen der Niederlander und ihrer Literatur zur Zeit der Republik*, Leipzig, 1863. Also, "Die Berechnung einer mittleren Waarenpreissteigerung," *Jahrbucher fur Nationalokonomie und Statistik 16*, (1871), pp. 296-315.

Figure 8.3. An decrease in prices shifts budget line outward and increases consumer welfare

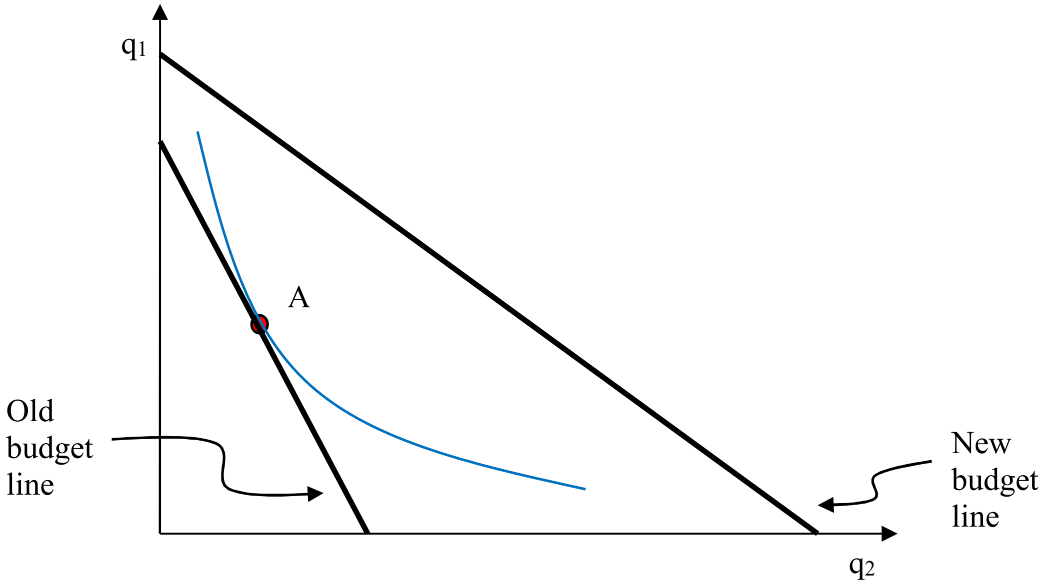
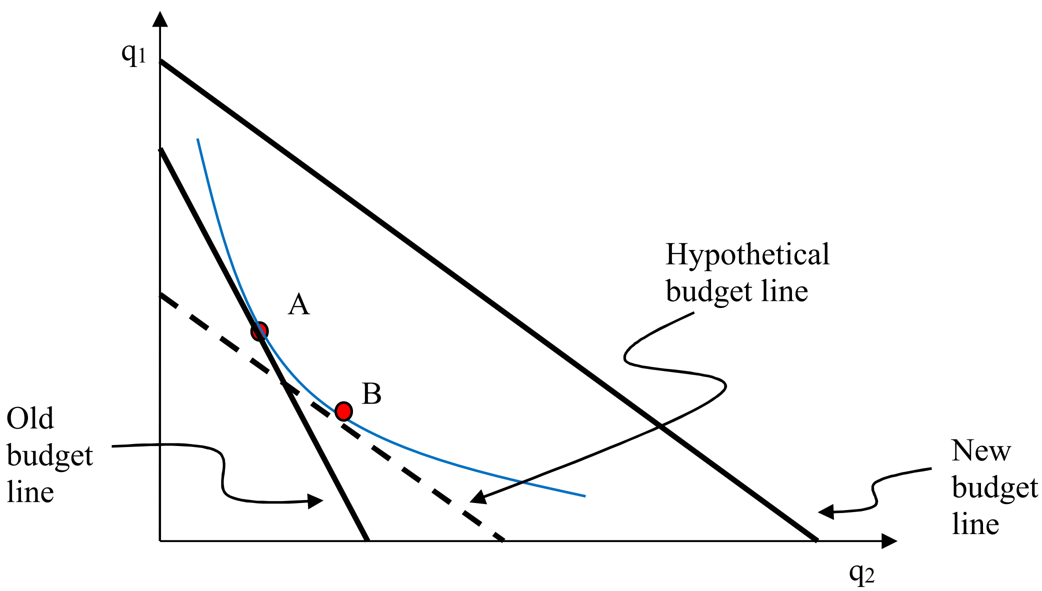


Figure 8.4. Compensating variation for decrease in prices is a decrease in income



With more than two goods, the Laspeyres formula becomes

(8.3) $$\text{Laspeyres COL index} = \frac{p_1^{new} q_1^{old} + p_2^{new} q_2^{old} + \cdots + p_n^{new} q_n^{old}}{p_1^{old} q_1^{old} + p_2^{old} q_2^{old} + \cdots + p_n^{old} q_n^{old}} \times 100 .$$

Note that the numerator of the Laspeyres index is the cost of the old bundle at new prices while the denominator is the cost of the old bundle at old prices. Thus the difference between the numerator and the denominator is the change in income required to keep the old bundle affordable at new prices—that is, the change in income corresponding to the Slutsky substitution effect, described in sections 7.2 and 7.3 of these lecture notes.

Example: Table 8.1 shows hypothetical data on new and old prices and quantities of food and clothing. Note that the price of food rose modestly, but the price of clothing rose sharply. As a consequence, consumers substituted from clothing into food. Application of the formula for the Laspeyres COL index gives the following:

(8.4) $$\text{Laspeyres COL index} = \frac{\$7 \times 200 + \$10 \times 200}{\$6 \times 200 + \$4 \times 200} \times 100 = 170 .$$

Substitution bias. When prices are increasing, the Laspeyres index is generally greater than the exact COL index because its numerator overestimates the expenditure required to reach the old indifference curve at new prices. Put differently, the income adjustment for the Slutsky substitution effect is *greater* than the income adjustment for the Hicks substitution effect, as noted in section 7.5 and illustrated in figure 7.11.

For example, in figure 8.2, the cost of old bundle A is clearly greater than the cost of new bundle B at new prices because bundle A lies above the hypothetical budget line passing through bundle B. This tendency of the Laspeyres index to overstate increases in the cost of living is known as *substitution bias* because it ignores consumers' ability to substitute between goods so as to attain a target indifference curve at lowest cost. Thus, if prices rise, any upward COL adjustments based on the Laspeyres index in the real world are too large. [6]

The Laspeyres COL index formulas (8.2) and (8.3) can also be applied when prices are decreasing. Figure (8.4) shows that here again, the Laspeyres index is generally greater than the exact COL index because its numerator overestimates the cost of attaining the old indifference curve at new prices. Thus, if prices fell, any downward COL adjustments based on the Laspeyres index in the real world are too small.

[6] The U.S. Consumer Price Index (CPI) is basically a Laspeyres index. The so-called Boskin Commission estimated that the CPI overstated the annual increase in the cost of living by about 0.5 percentage points because of substitution bias. U.S. Senate, Committee on Finance, *Final Report of the Advisory Commission to Study the Consumer Price Index.* Print 104-72, 104 Cong., 2 sess., (Washington, D.C., Government Printing Office, 1996).

Table 8.1. New and old prices and quantities—example

	Food		Clothing	
	p_1	q_1	p_2	q_2
Old	$6	200	$4	200
New	$7	600	$10	111

Paasche[7] cost-of-living index . An alternative approximate COL index can be computed using only the new bundle. The Paasche COL index is the ratio of the cost of the *new* bundle at new prices to its cost at old prices, times 100. For two goods, the formula is

$$(8.5) \qquad \text{Paasche COL index} = \frac{p_1^{new} q_1^{new} + p_2^{new} q_2^{new}}{p_1^{old} q_1^{new} + p_2^{old} q_2^{new}} \times 100 \ .$$

With more than two goods, the Paasche formula becomes

$$(8.6) \qquad \text{Paasche COL index} = \frac{p_1^{new} q_1^{new} + p_2^{new} q_2^{new} + \cdots + p_n^{new} q_n^{new}}{p_1^{old} q_1^{new} + p_2^{old} q_2^{new} + \cdots + p_n^{old} q_n^{new}} \times 100 \ .$$

The formulas for the Paasche index are identical to those for the Laspeyres index except that the Paasche index uses new quantities everywhere while the Laspeyres index uses old quantities.

Example: Inserting the data from table 8.1 into the formula for the Paasche COL index gives the following:

$$(8.7) \qquad \text{Paasche COL index} = \frac{\$7 \times 600 + \$10 \times 100}{\$6 \times 600 + \$4 \times 100} \times 100 = 130 \, .$$

Note that the Paasche COL index is smaller than the Laspeyres COL index.

Substitution bias again. The Paasche index is also vulnerable to substitution bias, but in the opposite direction. When prices are increasing, the Paasche index is generally less than the exact COL index because its denominator overestimates the cost of attaining the old indifference curve at old prices. For example, in figure 8.2, the cost of new bundle B is clearly greater than the cost of old bundle A at old prices because bundle B lies above the old budget line passing through bundle A. Thus, if prices rise, any upward COL adjustments based on the Paasche index in the real world are too small.

The Paasche COL index formulas (8.5) and (8.6) can also be applied when prices are decreasing. Figure (8.4) shows that here again, the Paasche index is generally less than the exact COL index because its denominator overestimates the cost of attaining the old indifference curve at old prices. Thus, if prices fell, any downward COL adjustments based on a Paasche index in the real world are too large.

[7] Hermann Paasch, "Uber die Preisentwicklung der letzten Jahre nach den Hamburger Borennotirungen," *Jahrbucher fur Nationalokonomie und Statistik 23*, (1874), pp. 168-178.

Fisher cost-of-living index . Since the Laspeyres COL index is biased up and the Paasche COL index is biased down, one might be tempted to construct some sort of average of the two. The Fisher index is just that, and is given by[8]

(8.8) $$\text{Fisher COL index} = \sqrt{\begin{pmatrix}\text{Laspeyres}\\ \text{COL index}\end{pmatrix} \times \begin{pmatrix}\text{Paasche}\\ \text{COL index}\end{pmatrix}} \; .$$

Example: Inserting the data from table 8.1 into the formula for the Fisher COL index gives the following:

(8.9) $$\text{Fisher COL index} = \sqrt{(170)\times(130)} = 148.66 \, .$$

Little substitution bias: The Fisher index has been shown to be very close to the exact COL index for a wide variety of possible utility functions.[9] It suffers from little or no substitution bias. However, it does require more data than either the Laspeyres or the Paasche index. In particular, it requires quantity data from both new and old periods.

Section 8.4: Change in one price

In this section, we apply the idea of compensating variation to a situation where only one price changes. This situation can be tackled using an indifference-curve diagram and computing the change in the cost of living as above, but it can also be tackled using a demand-curve diagram. However, to define the compensating variation precisely in this diagram, it turns out we must define a new kind of demand curve.

Noneconomists often compute the impact on consumers of a price change as simply the adjustment to income required to keep the old bundle affordable after the price change. Let Δp_1 denote the price change. Then we have

(8.10) $$\text{Income adjustment to keep old bundle affordable} = \left(\Delta p_1 \; q_1^{old}\right) .$$

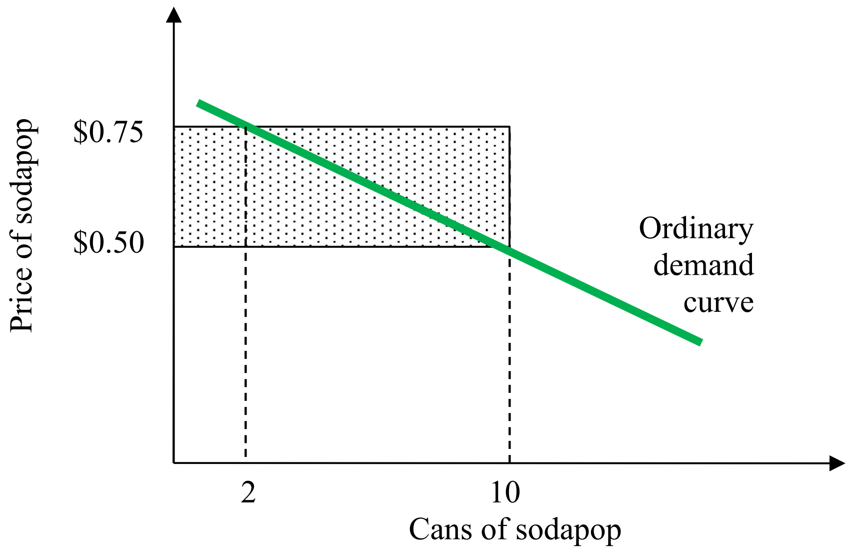
For example, suppose the consumer is buying 10 cans of sodapop per week, and the price of sodapop is \$0.50 per can. If the price rises by \$0.25, to \$0.75, then the required adjustment to income would be \$2.50. This calculation is depicted on a demand-curve graph in figure 8.5. The shaded area is the required adjustment to income.

Formula (8.10) implies that a *decrease* in price by the same amount would require an identical adjustment to income, except for sign. For example, if the price of sodapop *falls* by \$0.25, to \$0.25, then the required adjustment to income is *negative* \$2.50—one would need to take \$2.50 away from the consumer to keep the old bundle just affordable. This value is the shaded area in the demand-curve graph of figure 8.6.

[8] The Fisher index is technically the *geometric mean* of Laspeyres and Paasche indexes. If the Laspeyres and Paasche index are close to each other, the geometric mean is very close to the familiar *arithmetic mean*, (1/2)(Laspeyres COL index + Paasche COL index).

[9] W. Erwin Diewert, "Exact and Superlative Index Numbers," *Journal of Econometrics*, Vol. 4, No. 2 (1976), pp. 115-145.

Figure 8.5. Adjustment to income required to keep old bundle affordable after price increase.



But economists (such as you) will notice something funny about this approach. Note that an increase in the price of a good would surely cause the consumer to buy less of that good. According to figure 8.5, if the price rises from $0.50 to $0.75, then consumption of sodapop falls from 10 cans to 2 cans. Why did we choose the old quantity of consumption for this calculation (10 cans) instead of the new quantity after the price increase (2 cans) or some sort of average of the two?

Clearly, formula (8.10) is an overestimate of the income adjustment required to keep the consumer at the same level of utility, just as the Laspeyres COL index is an overestimate of the exact COL index. Both formula (8.10) and the Laspeyres COL index compute the Slutsky income adjustment, which keeps the old bundle affordable. The true compensating variation is the Hicks income adjustment, which is smaller and merely keeps the old *indifference curve* affordable. To compute the true compensating variation, we exploit the fact that for *very small* changes in price, the Hicks and Slutsky income adjustments are virtually identical (see chapter 6, section 6.5).

Approximate calculation. Let us recompute the income adjustment by summing the income adjustments for small changes in price. Return to the example of the rise in the price of sodapop. Instead of a single increase in price of $0.25, imagine 5 small price increases of five cents each. The first increase requires a Slutsky income adjustment of 10 x $0.05 = $0.50. This income adjustment is shown as the shaded rectangle in figure 8.7. So let the price rise by five cents and give the consumer $0.50 additional income.

Figure 8.6. Adjustment to income required to keep old bundle affordable after price decrease

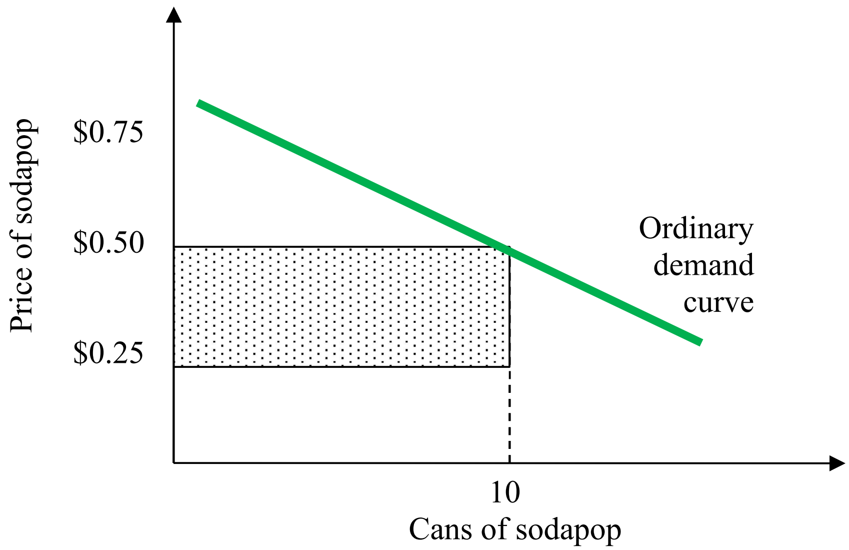
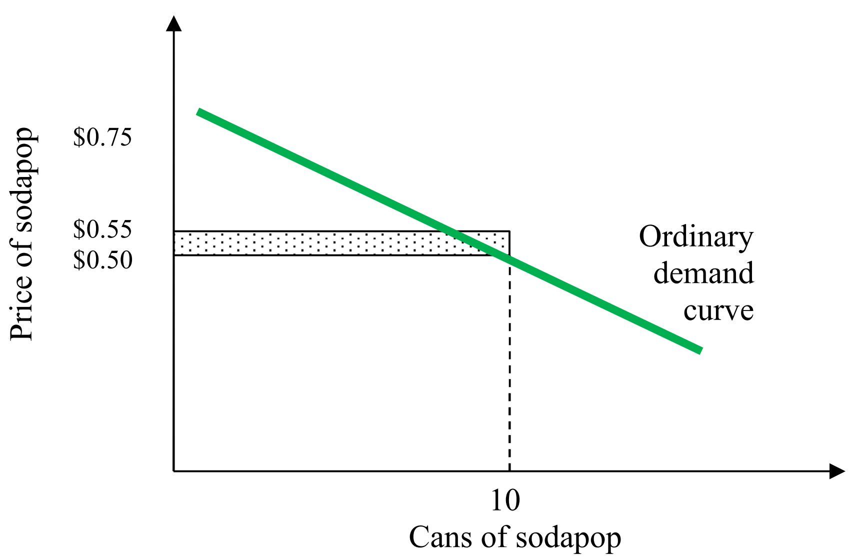


Figure 8.7. First step in computing true compensating variation



Now how many cans will the consumer choose each week? Fewer than 10 cans, surely, because although we have given the consumer enough income to offset the income effect of the price change, her or his purchases will still be decreased by a substitution effect. But more than the number of cans suggested by the ordinary demand curve, which reflects both the income and substitution effects. The ordinary demand curve does not permit any income adjustments.

Assume for this example that, after a price increase of $0.05 *and* an income adjustment of $0.50, the consumer now chooses 9.5 cans. Again let the price rise by another five cents, and now give the consumer a Slutsky income adjustment of 9.5 x $0.05 = $0.475. This second income adjustment is shown as the upper shaded rectangle in figure 8.8. After this second round, the consumer will choose still fewer cans—say 9.0 cans.

Continue raising price in five-cent increments and giving Slutsky income adjustments at every step, until the new price is reached. This yields a total of five shaded rectangles, shown in figure 8.9, whose combined area is a close approximation to the true compensating variation. Assuming in this example that the areas of the rectangles are, from lowest to highest, $0.50, $0.475, $0.45, $0.425, $0.40, then the compensating variation for the price increase in sodapop, for this consumer, would be $2.25. This is, of course, less than the $2.50 income adjustment calculated using formula 8.10 and depicted in figure 8.5.

Exact definition of compensating variation. This calculation of the compensating variation relied on the fact that the Hicks and Slutsky income adjustments are very close for small changes in price. The smaller the change in price, the closer they are to each other. So an even better approximation to the compensating variation would use even smaller changes in price. For sufficiently small changes in price, the stair-steps would appear to form a smooth ramp (see figure 8.10) The area between the ramp and the price axis, and between the two horizontal lines at the new and old prices, is the exact definition of compensating variation in income for a price increase. In this example, the exact compensating variation in income is the area of a trapezoid,[10] easily calculated in figure 8.10 to be $2.1875.[11]

Compensated demand curve. Note that the stair-steps (or ramp) do *not* fall on the ordinary demand curve. Along an ordinary demand curve, the consumer's income remains constant, but the consumer's utility changes—falling as the price increases. Here, by construction, the consumer's income rose at the same time the price was increased, so as to keep the consumer's utility constant. Put differently, the change in quantity demanded as price rose here reflected only the Hicks substitution effect, not the income effect. For this reason, the quantity did not fall as quickly as the ordinary demand curve would indicate. Instead, the stair-steps (or ramp) form a special kind of demand curve, called the *compensated demand curve* (see figure 8.11). The compensated demand curve is necessarily steeper (less elastic) than the ordinary demand curve (assuming the good is a normal good) because it does not include the income effect. [12]

---

[10] A trapezoid is called a "trapezium" in British English.

[11] Recall that the area of a trapezoid equals the height times the average of the parallel bases: $A = h\,(1/2)\,(b_1 + b_2)$.

[12] The ordinary demand curve is sometimes called the income-constant demand curve, or the Marshallian demand curve, after Alfred Marshall, who popularized it, (*Principles of Economics*, 8th edition, Philadelphia: Porcupine Press, 1920, pp. 78-83, 103-109). The compensated demand curve is sometimes called the utility-constant demand curve, or the Hicksian demand curve, after John R. Hicks (*Value and Capital*, 2nd edition, Oxford: Clarendon Press, 1946, pp. 38-41 and pp. 330-332).

Figure 8.8: Second step in computing true compensating variation

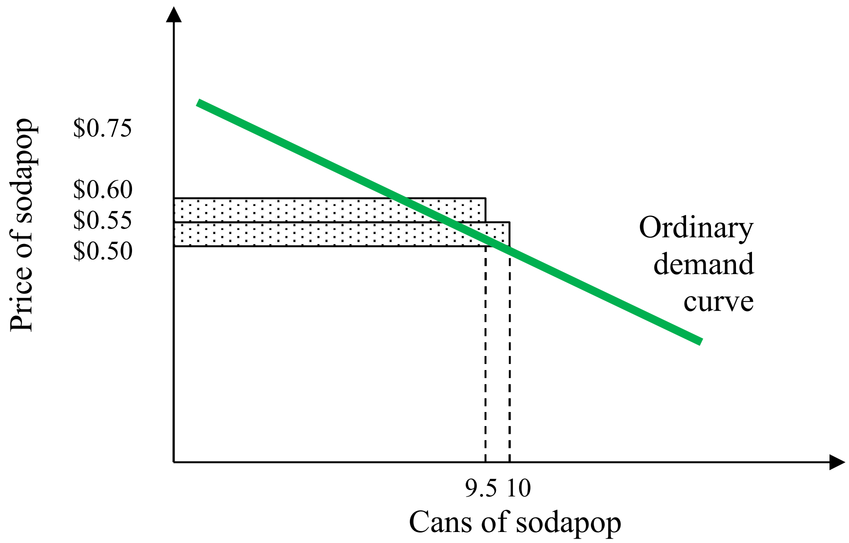


Figure 8.9: True compensating variation—approximation in five steps

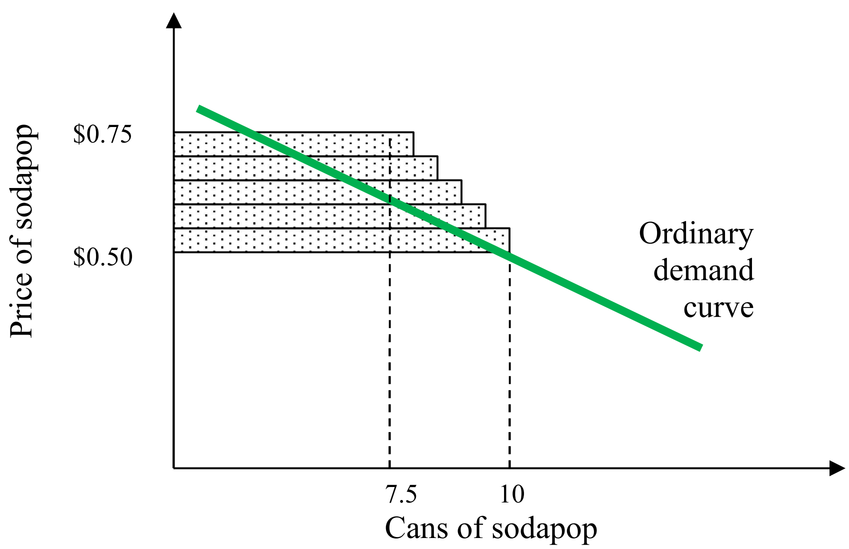


Figure 8.10: True compensating variation—closer approximation with tiny stairsteps

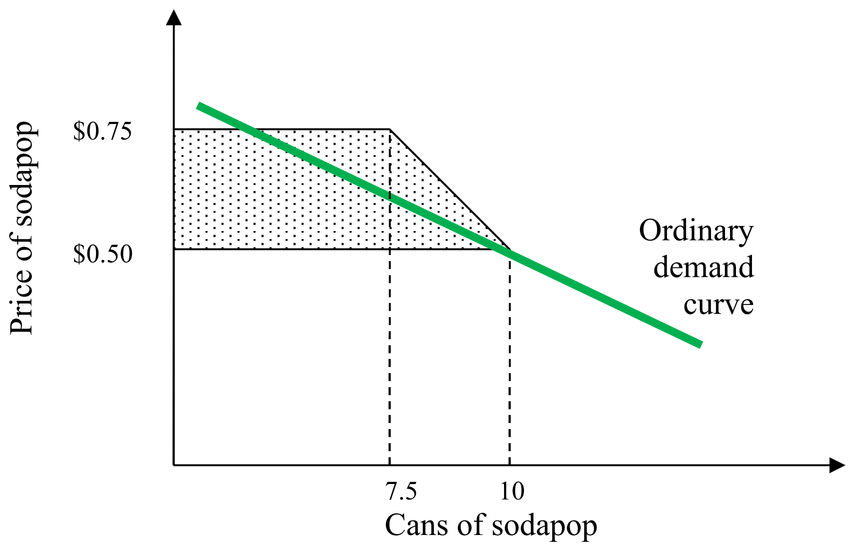
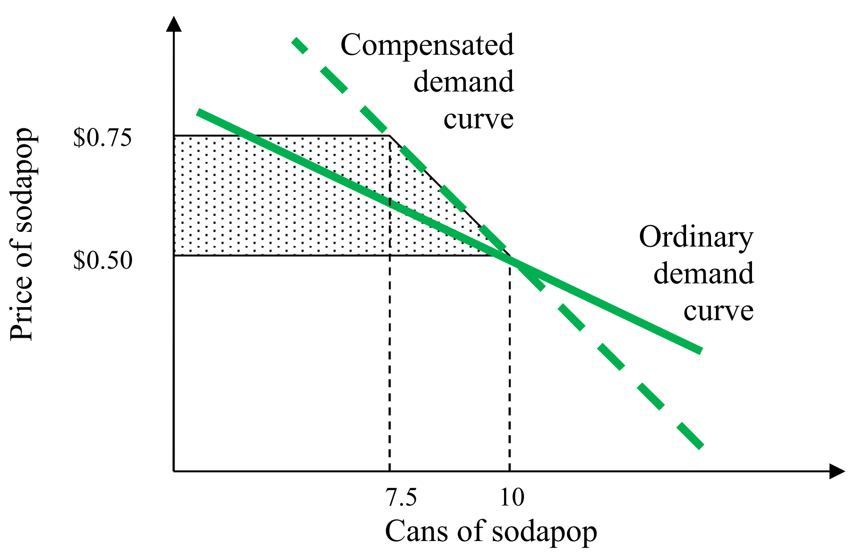


Figure 8.11: Compensating variation and compensated demand curve



Price decrease. So far, we have studied the effect of a price *increase.* How can we compute the compensating variation in income for a price *decrease?* The compensating variation for a price decrease is the amount of income that can be taken away from the consumer, simultaneous with the price decrease, while allowing the consumer to maintain the same level of utility as before the change.

To make the concept of compensating variation for a price decrease concrete, imagine a buying club or discount plan that allowed consumers to purchase some item (vegetables, wine, music recordings, telephone calls, etc.) at a price well below the usual price, after payment of a membership fee. The maximum membership fee a consumer would be willing to pay to join such a club or discount plan is precisely the compensating variation for the price decrease offered by the club or plan. A membership fee exactly equal to the compensating variation would leave the consumer just as well off as before joining the club or discount plan. If the club or discount plan charged any lower fee, it would leave the consumer strictly better off than before, and it would be eagerly accepted by the consumer. If the club or discount plan charged any higher fee, it would leave the consumer strictly worse off than before, and it would be rejected by the consumer.

Calculating the compensating variation for a price decrease is symmetric to the problem of a price increase, solved above. We must decrease price in very small increments, each time taking away from the consumer a Slutsky income adjustment. The sum of these adjustments is the compensating variation, and is again given by the area between the compensated demand curve and the vertical axis, and between the two horizontal lines at the new and old prices. In figure 8.12, the exact compensating variation in income for a decrease in price from $0.50 to $0.30 is the area of the shaded trapezoid, easily calculated to be $2.20.

Section 8.5: Change in consumer surplus

Suppose we only know a consumer's ordinary demand curve, not the compensated demand curve. In other words, suppose we only know how the quantity demanded responds to changes in price, not how it responds to changes in income. Can we approximate the compensating variation using the area under the ordinary demand curve instead of the area under the compensated demand curve? The answer is yes, although this approximate calculation goes by a different name, not "compensating variation."

The area between the demand curve, the price axis, and the horizontal line at the current price is called *total consumer surplus* (see figure 8.13). Roughly speaking, total consumer surplus measures the total value to a consumer of the units purchased, less the amount spent by the consumer on those units.[13] Calculating total consumer surplus for a consumer requires knowledge of the complete ordinary demand curve from its intercept on the price axis to the current price and quantity. This includes prices much higher than the current price—perhaps much higher than ever actually observed—so we should be wary of total consumer surplus calculations in practice. However, here all we need calculate are *changes* in consumer surplus. Calculations of *changes* in consumer surplus require only knowledge of the slope of the ordinary demand curve near the current price and are thus much more reliable.

[13] Alfred Marshall, *Principles of Economics*, 8th edition, Philadelphia: Porcupine Press, 1920, p. 103.

Figure 8.12: Compensating variation from a price decrease

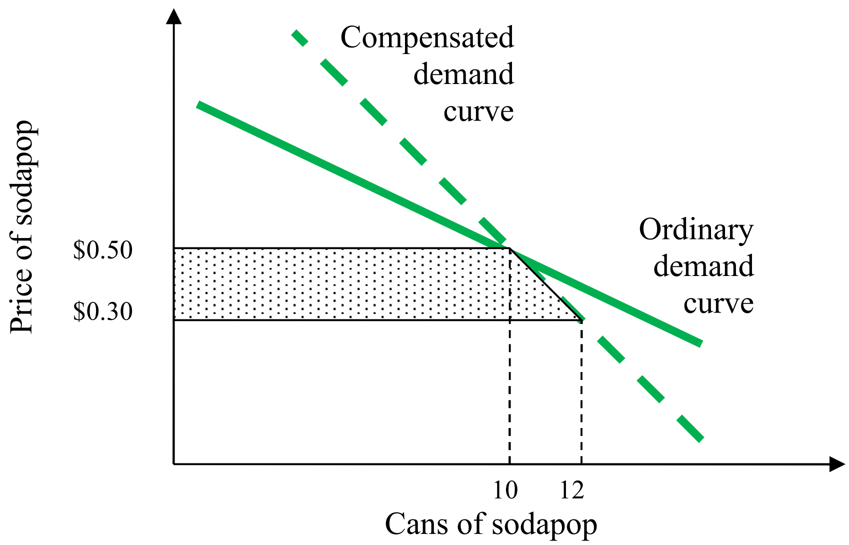
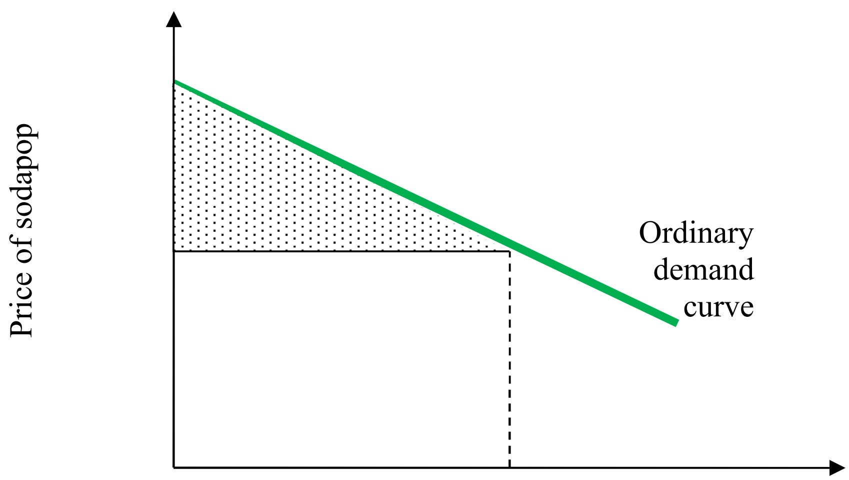


Figure 8.13: Total consumer surplus



Price increase. The change in consumer surplus from a price increase is the area of a trapezoid between the ordinary demand curve and the price axis, and between the two horizontal lines at the new and old prices. Continuing the example, figure 8.14 shows the change in consumer surplus from an increase in the price of sodapop from $0.50 to $0.75. Applying the formula for the area of a trapezoid to the numbers given in this figure shows a decrease in consumer surplus of $1.50. Note that this is smaller (in absolute value) than the compensating variation for the same price increase because the ordinary demand curve is flatter (more elastic) than the compensated demand curve.

Price decrease. Continuing the example, figure 8.15 shows the change in consumer surplus from a decrease in the price of sodapop from $0.50 to $0.30. Applying the formula for the area of a trapezoid to the numbers given in this figure shows an increase in consumer surplus of $2.40.

Example: Suppose when the price of electricity is $0.10 per kilowatt-hour, a typical consumer demands 700 kilowatt-hours, but when the price rises to $0.15 per kilowatt-hour, the same consumer demands only 500 kilowatt-hours. What is the loss of consumer surplus from the price increase? A quick sketch of the demand curve diagram shows that the loss is approximately the area of a trapezoid whose parallel bases are 700 and 500 kilowatt-hours, and whose height is $0.15−$0.10 = $0.05. This area is $30. This answer is exact if demand is a straight line, and approximate if demand is curved in some way.

Example: Suppose demand is given by $Q = 300 - 20\,P$. What is the gain in consumer surplus if price falls from $12 to $5? A quick sketch of the demand curve diagram shows that the gain is the area of a trapezoid whose parallel bases are 60 and 200, and whose height is $12−$5 = $7. This area is $910. This answer is exact because the demand was given to be a straight line.

Example: Bob currently rents about two DVDs per month when the rental rate is $3 each. If the rental rate were reduced to $0.50, Bob would probably rent ten DVDs per month. How much would Bob be willing to pay to join a "frequent renters club," if membership would lower his rental rate from $2 to $0.50? The answer is the gain in Bob's consumer surplus. A quick sketch of the demand curve diagram shows that the gain is approximately the area of a trapezoid whose parallel bases are 2 and 10, and whose height is $3−$0.50 = $2.50. This area is $15. Bob would be willing to pay a membership fee of $15 per month. This answer is exact if demand is a straight line, and approximate if demand is curved in some way.

If the demand curve is a straight line, the exact change in consumer surplus can be interpreted as the average of the income adjustment required to keep the old bundle affordable (equation 8.1), and the income adjustment required to keep the new bundle affordable. For example, in figure 8.5, the required income adjustment using formula (8.10) with q_1 equal to the old quantity was calculated above as $2.50. the required income adjustment using formula (8.10) with q_1 equal to the new quantity (2 cans) is clearly $0.50. The change in consumer surplus is the average of these two calculations: $1.50.

Figure 8.14: Change in consumer surplus from a price increase

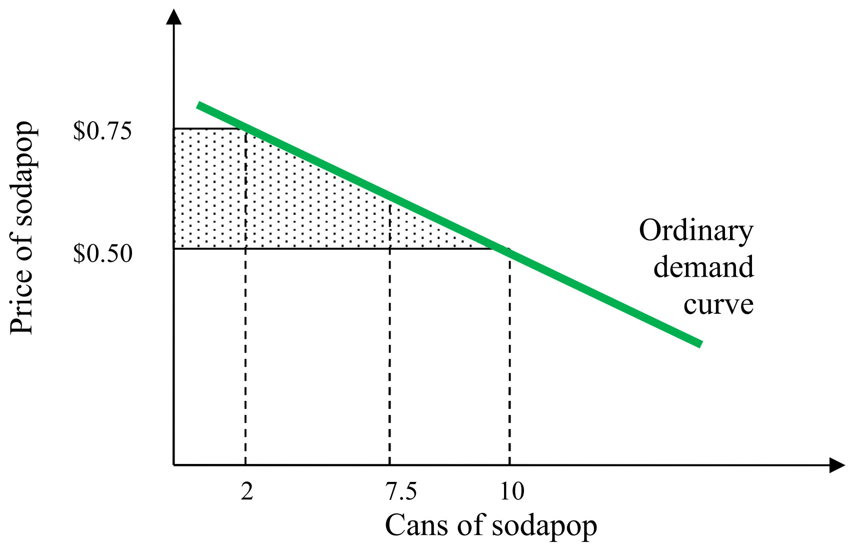
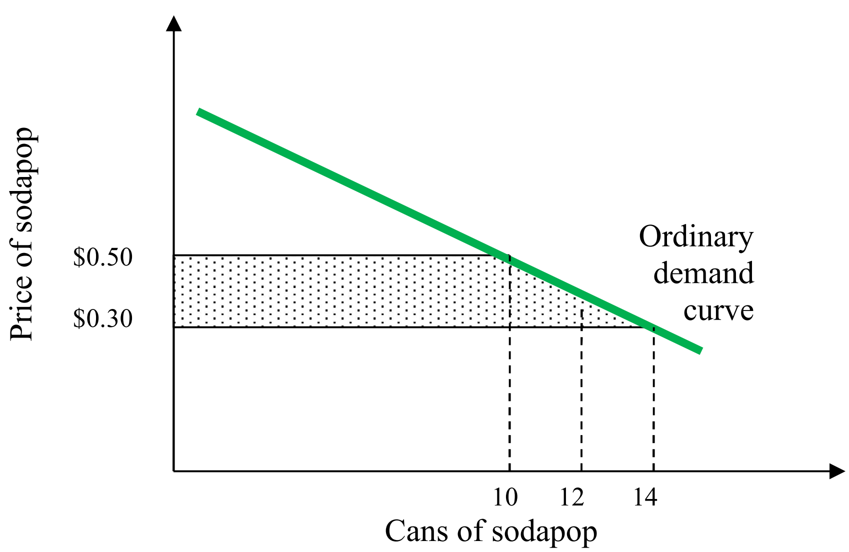


Figure 8.15. Change in consumer surplus from a price decrease



Consumer surplus versus compensating variation. Is the change in consumer surplus a good approximation to true compensating variation? Put differently, is the difference between the ordinary and compensated demand curves large enough to be of practical importance? The ordinary demand curve reflects both income and substitution effects of price changes, while the compensated demand curve reflects only the Hicks substitution effect. But income effects tend to be small for goods that occupy only a small fraction of a consumer's budget. So the two curves are often close and the change in consumer surplus is usually a very good approximation to compensating variation, especially for goods than occupy a small fraction of the budget.[14]

Figures 8.16 and 8.17 show how close the ordinary and compensated demand curves are for Cobb-Douglas utility functions. Figure 8.16 shows ordinary and compensated demand curves for a good that occupies one-half of the consumer's budget. The curves are somewhat close. Figure 8.17 shows ordinary and compensated demand curves for a good that occupies one-fifth of the consumer's budget. The curves are now almost indistinguishable, so the change in consumer surplus will be nearly identical to the true compensating variation.[15]

Section 8.6: Summary

The *compensating variation in income* is the amount of additional income we must give (or take away from) the consumer, in compensation for a change in prices, to allow the consumer to maintain the same level of utility as before the change. The *exact cost-of-living (COL) index* is the ratio of total required income at new prices (including the compensating variation), to income before the price change, usually multiplied by 100 for convenience. The exact COL index is difficult to calculate in practice because it requires knowledge of the consumer's indifference curve or utility function. Instead, a *Laspeyres COL index* using old quantities or a *Paasche COL index* using new quantities is often used. However, the *Fisher COL index* (the square root of the product of the Laspeyres and Paasche indexes) is usually much more accurate than either of these two. When only one price changes, the compensating variation equals the area of the trapezoid between the *compensated demand curve*, the price axis, and horizontal lines at the old and new prices. In practice, this area is usually very close to the corresponding area for the ordinary demand curve, which is called the *change in consumer surplus*.

[14] For a rigorous mathematical demonstration, see Robert D. Willig, "Consumer's Surplus Without Apology," *American Economic Review*, Vol. 66, No. 4 (September 1976), pp. 589-597.
[15] Figure 7.16 assumes $U=q_1q_2$ and $p_2=5$, and sets $I=100$ for the ordinary demand curve while setting $U=100$ for the compensated demand curve. Figure 7.17 assumes $U=q_1q_2^4$, and $p_2=5$, and sets $I=100$ for the ordinary demand curve while setting $U=262144$ for the compensated demand curve.

Figure 8.16: Ordinary demand versus compensated demand for good occupying one-half of consumer's budget

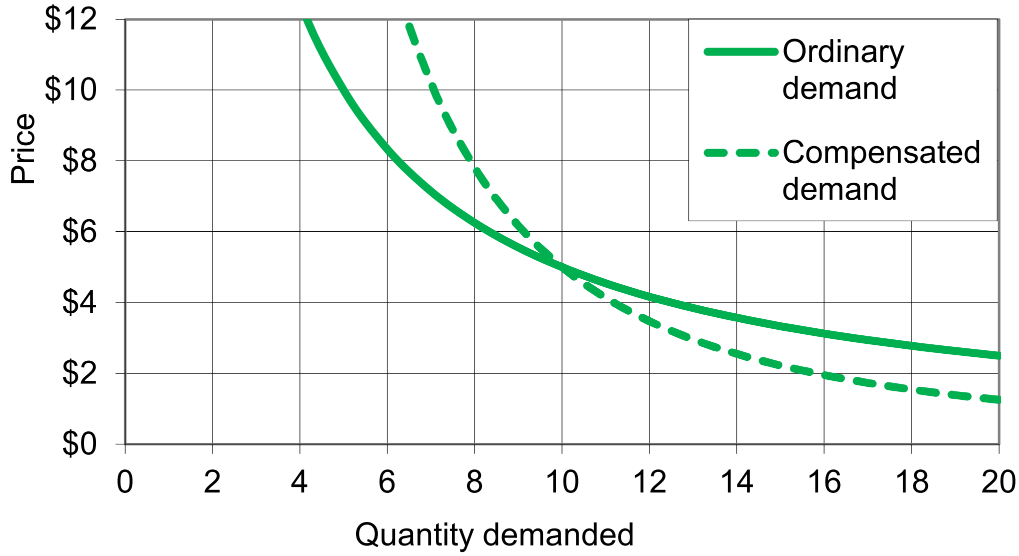
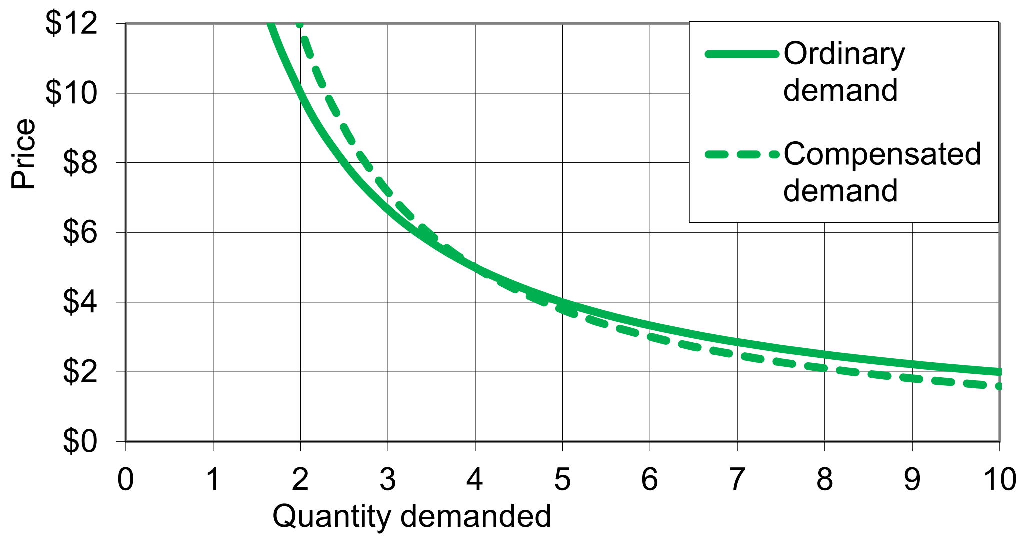


Figure 8.17: Ordinary demand versus compensated demand for good occupying one-fifth of consumer's budget



Problems

(8.1) [COL indexes] Suppose a consumer has utility function $U = q_1 q_2$. This consumer initially enjoys income = $160 and faces prices of $1 for good #1 and $4 for good #2.

a. Give an equation for the consumer's budget line.
b. Give an equation for the consumer's tangency condition.
c. Solve for the consumer's utility-maximizing choice of q_1 and q_2 and show that the consumer's utility is 1600 utils.

Now suppose the prices of good #1 and good #2 both rise to $5.

d. Without a change of income, will the consumer experience an increase in utility or a decrease in utility? Why?
e. Find the level of income at the new prices that is required to attain the old level of utility. [Hint: Write an equation for the *new* tangency condition. Then set $1600 = q_1 q_2$. Solve these two equations jointly to find the required new quantities q_1 and q_2. Then use the new prices to find the required new level of income.]
f. Use your answer to part (e) to compute the exact COL index.
g. Use your answer to part (c) to compute the Laspeyres COL index.
h. Use your answer to part (e) to compute the Paasche COL index.
i. Use your answer to parts (g) and (h) to compute the Fisher COL index.
j. Which index—Laspeyres, Paasche, or Fisher—is closest to the exact COL index?

(8.2) [COL indexes] Suppose a consumer's spending patterns are as follows.

	Quantity of food	Quantity of clothing	Price of food	Price of clothing
Old period	60 units	40 units	$2	$2
New period	40 units	60 units	$4	$2

Note that, from the old period to the new period, this consumer has shifted toward clothing and away from food, in response to the drop in the relative price of clothing.

a. Compute the Laspeyres index of the consumer's cost of living in the new period.
b. Compute the Paasche index of the consumer's cost of living in the new period.
c. Compute the Fisher index of the consumer's cost of living in the new period. [Hint: Part (c) requires a calculator.]

(8.3) [COL indexes] Suppose a consumer's spending patterns are as follows.

	Quantity of energy	Quantity of other goods	Price of energy	Price of other goods
Old period	150 units	200 units	$2	$1
New period	200 units	200 units	$3	$3

a. Compute the Laspeyres index of the consumer's cost of living in the new period.
b. Compute the Paasche index of the consumer's cost of living in the new period.
c. Compute the Fisher index of the consumer's cost of living in the new period. [Hint: Part (c) requires a calculator.]

(8.4) [COL indexes] Suppose a consumer views two goods as perfect complements and therefore has L-shaped indifference curves whose vertices lie along an income expansion path which is a straight line through the origin. Explain why the exact, Laspeyres, Paasche, and Fisher COL indexes must be identical, using an indifference-curve graph if possible.

(8.5) [COL indexes] Suppose all prices increase proportionately, so that $p_i^{new} = a\, p_i^{old}$ for all goods (indexed by i), where a is some fixed constant greater than one. Prove algebraically that Laspeyres, Paasche, and Fisher COL indexes must be identical.

(8.6) [COL indexes] Why is the following a silly proposal for a cost-of-living index?

$$\text{Proposed COL index} = \frac{p_1^{new} q_1^{new} + p_2^{new} q_2^{new}}{p_1^{old} q_1^{old} + p_2^{old} q_2^{old}} \times 100$$

(8.7) [Compensating variation, consumer surplus] Determine whether each statement is true or false, and explain your answer.

a. The change in cost of the old bundle is the same (except for sign) if the price of one good rises by a dollar or falls by a dollar.
b. The change in the compensating variation is the same (except for sign) if the price of one good rises by a dollar or falls by a dollar.
c. The change in consumer surplus is the same (except for sign) if the price of one good rises by a dollar or falls by a dollar.

(8.8) [Compensating variation] Determine whether each statement is true or false, and explain your answer. Assume all demand curves are straight lines.

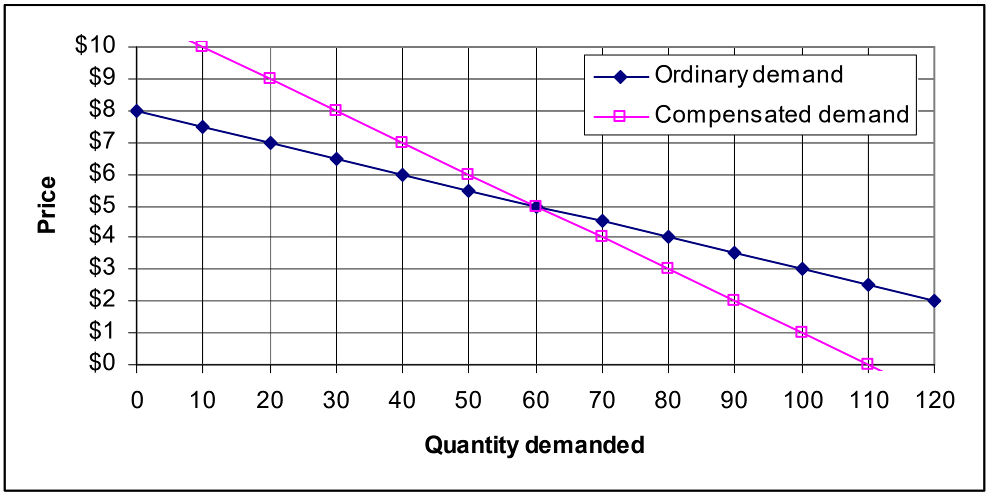
a. Suppose a family now buys 10 gallons of milk per month and the price is $2 per gallon. If the price rises next month from $2 to $5 per gallon, then this family will be just as well off as before if it is given an extra $30 per month in income.
b. Suppose a consumer currently pays for 100 minutes per month of telephone service. Then the same consumer would be willing to pay up to $3 per month (but no more) for a calling plan that would lower the price of phone service by $0.03 (3 cents) per minute.

(8.9) [Compensated demand curve]

a. Suppose a good is a normal good. Then which is steeper, the ordinary demand curve or the compensated demand curve? Why?
b. Suppose a good is an inferior good. Then which is steeper, the ordinary demand curve or the compensated demand curve? Why? [Hint: What effect would an income adjustment have on quantity demanded of an inferior good?]
c. Suppose a good is neither a normal good nor an inferior good (that is, income has no effect on demand). Then which is steeper, the ordinary demand curve or the compensated demand curve? Why?

(8.10) [Compensated demand curve] Consider the peculiar case where the consumer views q_1 and q_2 as perfect complements. Recall that the consumer's indifference curve is L-shaped, and that the consumer always chooses a bundle at the vertex of the indifference curve. In this special situation, what does the compensated demand curve look like? [Hint: If utility must remain constant, can the quantity change?]

The next two problems refer to the following graph.



(8.11) [Cost change, compensating variation, and consumer surplus] In the graph above, suppose the price of this good rises from \$5 to \$7.

a. Is the consumer better off or worse off as a result of the price change?
b. Compute the change in cost of the consumer's old bundle.
c. Compute the compensating variation in income.
d. Compute the change in consumer surplus.

(8.12) [Cost change, compensating variation, and consumer surplus] In the graph above, suppose the price of this good falls from \$5 to \$2.

a. Is the consumer better off or worse off as a result of the price change?
b. Compute the change in cost of the consumer's old bundle.
c. Compute the compensating variation in income.
d. Compute the change in consumer surplus.

(8.13) [Consumer surplus] Suppose the ordinary demand for movie rentals by a typical consumer is given by the following function: $Q = 20 - 5P$, where Q denotes the number of movies rented per month and P denotes the rental rate for a movie.

a. Sketch the demand curve. What are its intercepts on the P and Q axes?
b. Compute the number of movies rented and the total consumer surplus enjoyed if the rental rate is \$1 per movie.
c. Are consumers better off or worse off if the rental rate rises from \$1 to \$3?
d. Compute the change in consumer surplus if the rental rate rises from \$1 to \$3.
e. Would the compensating variation from this price change likely be larger or smaller than the change in consumer surplus? Explain your answer.

(8.14) [Consumer surplus] Suppose the ordinary demand for tomatoes by a typical consumer is given by the following function: $Q = 24 - 6P$, where Q denotes the quantity of tomatoes purchased (in pounds per month) and P denotes the price of tomatoes.

a. What are its intercepts of this demand curve on the P and Q axes? Sketch the demand curve (if submitting this problem on paper).
b. Compute the quantity purchased and the total consumer surplus enjoyed if the price of tomatoes is \$2 per pound. [Hint: Consumer surplus is the area of a triangle.]
c. Are consumers better off or worse off if the price falls from \$2 to \$1?
d. Compute the change in consumer surplus if the price falls from \$2 to \$1. [Hint: The change in consumer surplus is the area of a trapezoid.]
e. Would the compensating variation from this price change likely be larger or smaller than the change in consumer surplus? Explain your answer.

(8.15) [Change in consumer surplus] When the price of milk was \$2.00 per gallon, Household X purchased twenty gallons a month. After the price rose to \$3.50 per gallon, Household X purchased only twelve gallons a month.

a. Compute the change in the cost of Household X's old bundle—that is, the income adjustment required to keep the old bundle affordable.
b. Compute the change in Household X's consumer surplus, assuming demand is a straight line. [Hint: First sketch the relevant trapezoid and find its dimensions. Then compute its area.]
c. Did consumer surplus increase or decrease?

(8.16) [Change in consumer surplus] When ice-cream cones were priced at $1.00, Amy bought 10 per month. Now that they are priced at $1.50, she buys only 4 per month.

a. Compute the change in the cost of Amy's old bundle—that is, the income adjustment required to keep the old bundle affordable.
b. Compute the change in Amy's consumer surplus, assuming Amy's demand curve is a straight line. [Hint: First sketch the relevant trapezoid and find its dimensions. Then compute its area.]
c. Suppose the ice-cream store begins selling monthly "Ice Cream Lovers Club" cards, allowing consumers to continue to purchase ice cream cones at the old price of $1.00. (Without a card, a consumer must pay the new price of $1.50.) What is the maximum amount that Amy would pay for a monthly club membership card?

(8.17) [Change in consumer surplus] Suppose the price of movie admissions rises from $5 to $9. As a result, Fred cuts back on going to the movies from 4 times a month to 3 times a month.

a. Compute the change in the cost of Fred's old bundle.
b. Compute the change in Fred's consumer surplus, assuming Fred's demand curve is a straight line.
c. Suppose the movie theatre begins selling monthly "Movie Club" cards allowing consumers to continue to purchase individual tickets at the old price of $5.00. (Without a card, a consumer must pay the new price of $9.00.) What is the maximum amount that Fred would pay for a monthly club membership card?

[end of problem set]