LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

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Part 2: Consumers and demand

Chapter 7: Substitution and income effects

Section 7.1: Decomposing the effects of a price change

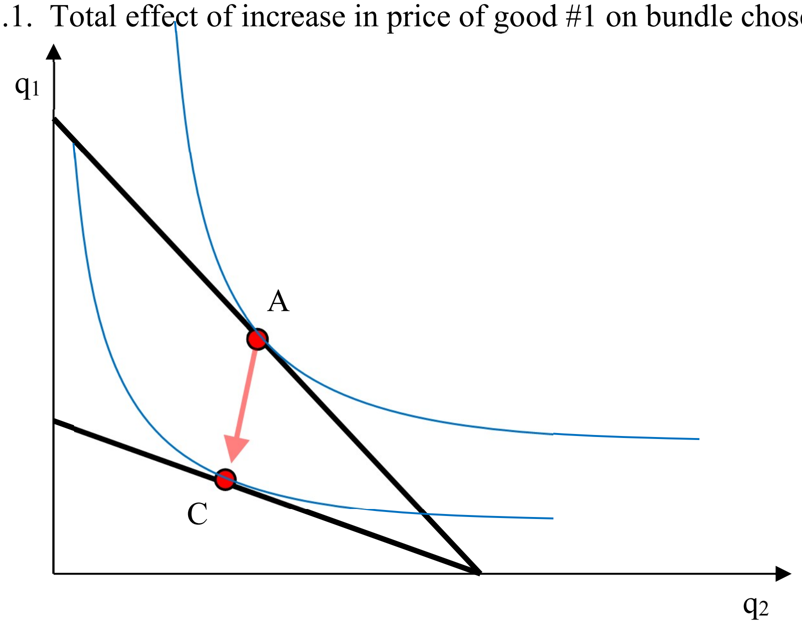
Proving the Law of Demand. For essentially all goods, a negative relationship—the Law of Demand—exists between the price of the good and the quantity demanded. While Giffen goods are theoretically possible given the three assumptions of transitivity, monotonicity, and diminishing MRSC, they are extremely rare, if they exist at all. It is thus natural to wonder if there is some common feature of preferences that generates the Law of Demand. There is indeed, as we shall show below. If a consumer's preferences satisfy the three assumptions and in addition the good is a normal good—that is, the consumer purchases more units of a good as her or his income rises—then it turns out that the Law of Demand must hold for that good. But to prove this result, we must decompose the total effect of the price change into two parts: a substitution effect and an income effect.

The total effect of a price change. If the price of a good increases, then the budget line rotates in, while anchored on the axis of the good whose price does not change. For example, if the price of good #1 increases, the budget line remains anchored at the same intercept on the axis for good #2, but it becomes flatter, since its slope is given by the formula $-p_2/p_1$. The consumer will now be forced down to a lower indifference curve, moving from bundle A to bundle C in figure 7.1. If we know the consumer's demand function $q_1^*=q_1^*(p_1,p_2,I)$, we can use the partial derivative to calculate the approximate change in quantity demanded for any small price change Δp_1 using the usual formula:

(7.1) total effect = $\Delta q_1^{TOT} \approx \Delta p_1 \left(\frac{\partial q_1^*}{\partial p_1}\right)$

Note that, in general, the value of the partial derivative may depend on the values of all prices and income.

Figure 7.1. Total effect of increase in price of good #1 on bundle chosen



Example: Suppose John Q. Consumer has the following demand function for food: $q_1^* = I/(4p_1)$. Assume he enjoys an income of \$6000 and initially faces a price of food equal to \$10, and therefore chooses 150 units of food. Now suppose the price of food rises by one dollar to \$11. The exact quantity of food chosen at the new price can be calculated from the demand function as 136 units (to the nearest unit) for a change in quantity demanded of −14. But the approximation using the partial derivative is pretty good. The formula for the partial derivative is $\partial q_1^*/\partial p_1 = -I/(4p_1^2)$, whose value is −15 at the original income and price. Therefore, the approximation formula (7.1) yields $\Delta q_1 \approx -15$. So the total effect of the one-dollar increase in price is a decrease of approximately 15 units in quantity demanded.

Decomposing the total effect. There are two consequences for a consumer when a price rises, as in figure 7.1. First, the slope of the budget line falls (in absolute value) as good #1 becomes more expensive relative to good #2. A shift in relative prices typically causes any consumer to shift relative quantities in the opposite direction—that is, to substitute the good whose price is falling (in relative terms) for the good whose price is rising. Therefore, this consequence is called the *substitution effect.* Second, the consumer's budget set shrinks. The old bundle is no longer affordable. This consequence is similar to a fall in income and is called the *income effect*. In the remainder of this chapter, we examine these two effects.

Section 7.2: The substitution effect[1]

Definition of the substitution effect. Consider a consumer faced with the solid budget line shown in figure 7.2 and choosing bundle A. Imagine what would happen if the price of good #1 increased but the consumer were given enough income to keep the old bundle affordable at the new prices. The effect can be described graphically by drawing a new hypothetical budget line, shown as the dotted line in figure 7.2, generated by *pivoting* the old budget line around the old chosen bundle. The new hypothetical budget line is flatter because the formula for slope is $-p_2/p_1$ and p_1 has increased. The new hypothetical budget line still passes through bundle A because, by assumption, the consumer has been given exactly enough income, in compensation for the price increase, to afford the same bundle as before. The substitution effect is defined as the consumer's response to this new, hypothetical budget line.

Example: In a previous example, John Q. Consumer enjoyed an income of $6000, faced a price of food (good #1) of $10, and (let us assume) a price of other goods of $1. His initial budget line is given by $\$6000 = 10\ q_1 + 1\ q_2$. We assume that, faced with this budget line, he chose to consume 150 units of food. If the price of good #1 increases to $11, what hypothetical budget line is required to keep the old bundle affordable? Clearly income must increase to $6150 to keep the old bundle affordable, so the equation for the new hypothetical budget line must be $\$6150 = 11\ q_1 + 1\ q_2$. John Q. Consumer's response to this new budget line is the substitution effect of the increase in the price of food.

Why the substitution effect cannot be positive. Given this new hypothetical budget line, it is unlikely that the consumer will again choose bundle A. If the indifference curve passing through bundle A is a smooth curve, it is tangent to the original budget line but not to the new, flatter, hypothetical budget line. In other words, the consumer's MRSC at bundle A is greater than (the absolute value of) the slope of the new budget line. The consumer can reach a higher indifference curve by purchasing less of good #1 (and more of good #2). It follows that the substitution effect must necessarily be negative: an *increase* in the price of a good causes a *decrease* in the quantity chosen of the same good, along this hypothetical budget line.

The sign of the substitution effect can also be seen without drawing any indifference curves. If bundle A was indeed the consumer's preferred bundle given the original budget line, it must have been preferred to any other bundle on or inside that original budget line. In particular, it must have been preferred to all bundles on the new hypothetical budget line to the left of bundle A (see figure 7.2). So given the new hypothetical budget line, the consumer will never choose any of these bundles, which contain more of good #1 than bundle A. Thus, the substitution effect of an increase in the price of a good cannot cause an increase in the quantity chosen of the same good.

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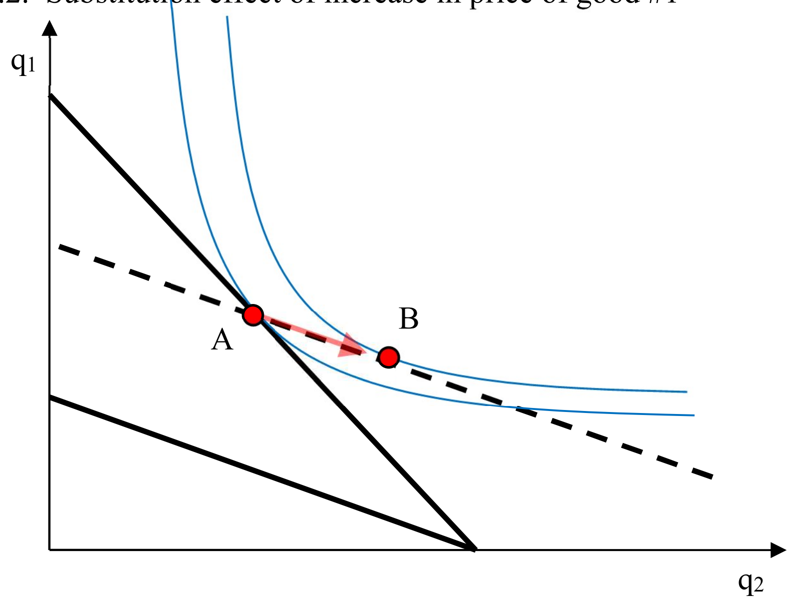
[1] The geometric interpretation of the Slutsky substitution effect used here is taken from Hal R. Varian, *Intermediate Microeconomics: A Modern Approach*, New York: W.W. Norton and Company, 2010, chapter 8.

Figure 7.2. Substitution effect of increase in price of good #1



Although the substitution effect cannot be positive, its size can vary, depending on the curvature of the indifference curves. Suppose good #1 and good #2 can readily be substituted for each other, like two soft drinks. Then the indifference curves will be relatively straight. We must then travel a long distance to reach the tangency to the new hypothetical budget line (see figure 7.3). The substitution effect in this case is large. Alternatively, suppose the indifference curves are tightly curved, as might be expected for goods used together, like flashlights and batteries. We must then travel only a short distance to reach the tangency to the new hypothetical budget line (see figure 7.4). In the extreme case of perfect complements, there is no change in the bundle chosen and therefore the substitution effect is zero (see figure 7.5). However, in every case, the new hypothetical bundle B must lie at or below bundle A, which means the quantity chosen of good #1 must not increase.

Section 7.3: The income effect

Definition of the income effect. We have defined part of the total effect of a price change on the consumer's choice of bundle as the substitution effect: the consumer's response to the change in prices, with income adjusted to keep the old bundle affordable. This response is shown as the movement from bundle A to hypothetical bundle B, the tangency point on the new hypothetical budget line in figure 7.6 . The remaining part is the movement between hypothetical bundle B and the consumer's choice on the new, real budget line below, shown as bundle C in the figure. Note that the new, real budget line is parallel to the hypothetical budget line because the two budget lines reflect the same prices but different levels of income. The hypothetical budget line includes the income added in defining the substitution effect, while the new real budget line does not. Hence the consumer's response to this change is called the "income effect."

Figure 7.3. Large substitution effect of increase in price of good #1 due to relatively straight indifference curves

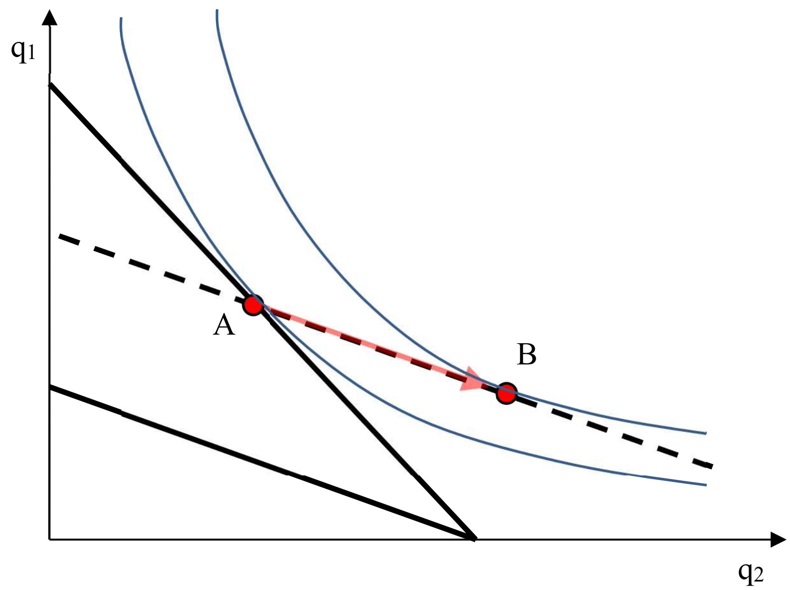


Figure 7.4. Small substitution effect of increase in price of good #1 due to sharply curved indifference curves

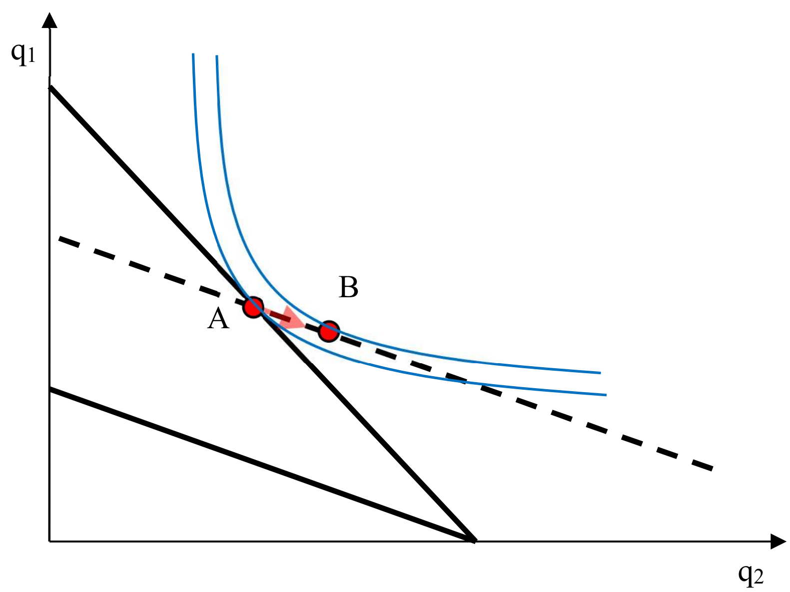


Figure 7.5. Zero substitution effect of increase in price of good #1 when goods are perfect complements

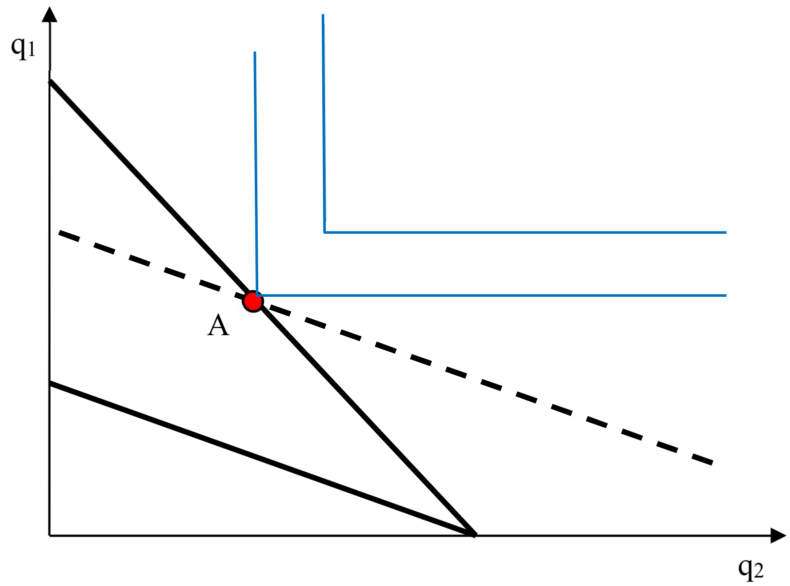
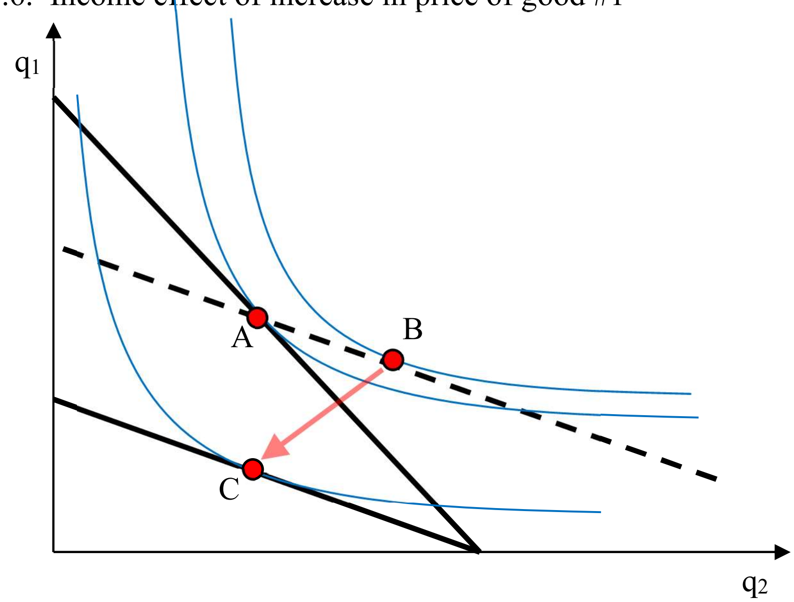


Figure 7.6. Income effect of increase in price of good #1



Why the income effect is negative for normal goods. The income effect is simply the consumer's response to giving back the income adjustment that she or he hypothetically received in defining the substitution effect. For normal goods, a simple loss of income always causes a decrease in the quantity purchased. Thus, if good #1 is a normal good (as most goods are) then bundle C must lie below hypothetical bundle B (see figure 7.6).

Calculating the income effect. For any given change in price Δp_1, the adjustment to income required to keep the old bundle affordable is given by $(\Delta p_1\, q_1)$ where q_1 denotes the quantity of good #1 chosen as part of the old bundle. For example, if the consumer were buying 10 cans of sodapop per week, and the price of sodapop rose by $0.25, then the required adjustment to income would be $2.50. If the partial derivative of the demand for a good with respect to income is known, then the income effect can be calculated by multiplying the negative of the hypothetical adjustment to income (negative because the adjustment is being *taken away*) times this partial derivative. Equation (7.2) shows the approximate change in the quantity of good #1 chosen due to the income effect alone.

(7.2) income effect $\approx -(q_1\, \Delta p_1)\left(\frac{\partial q_1^*}{\partial I}\right)$

Equation (7.2) shows that the income effect is negative if Δp_1 and $(\partial q_1^*/\partial I)$ are both positive—that is, if the price of good #1 has increased and good #1 is a normal good. It also shows that the size of the income effect depends on the size of the partial derivative—that is, how strongly the quantity purchased responds to changes in income—and on the number of units originally purchased—that is, q_1 .

Example: Suppose the average consumer uses 2,500 kilowatt-hours of electricity per month and that $\partial q_1^*/\partial I$ for electricity = 0.2 . Now suppose the price of electricity rises by $0.05 per kilowatt-hour. The adjustment to income required to keep the old bundle affordable is therefore $2500 \times 0.05 = \$125$. The income effect of the price change is $-\$125 \times 0.2 = -25$ kilowatt hours, that is, a decrease in electricity usage of 25 kilowatt hours per month.

Example: Suppose the average consumer buys 8 gallons of milk per month and that $\partial q_1^*/\partial I$ for milk = 0.025 . Now suppose the price of milk rises by $1.50 per gallon. The adjustment to income required to keep the old bundle affordable is therefore $8 \times 1.50 = \$12$. The income effect of the price change is $-\$12 \times 0.025 = -0.3$ gallons, that is, a decrease in milk purchases of 0.3 gallons per month.

Example: Consider again John Q. Consumer, who initially chose 150 units of food. As the price of food rose from $10 to $11, we calculated that the adjustment to income required to keep the old bundle affordable was $150. Now given the demand function for food $q_1^* = I/(4p_1)$, the partial derivative with respect to income is $\partial q_1^*/\partial I = 1/(4p_1)$, whose value is 0.025 at the original price of $p_1 = \$10$. Therefore, using equation (7.2), the income effect of the price rise is a 3.75-unit decrease in the quantity of food demanded.

Section 7.4: The Slutsky equation

Decomposing the total effect. Figure 7.6 shows that the total effect of a price change can be decomposed into an effect due to a *pivoting* of the old budget line around the bundle originally chosen and an effect due to a *shift* of the budget line down to its final position. The first is the substitution effect, shown in figure 7.6 as a movement from original bundle A to hypothetical bundle B. The second is the income effect, shown in figure 7.6 as a movement from hypothetical bundle B to final bundle C. Summarized as an equation, then, we have the following.

(7.3) Total effect = income effect + substitution effect

If Δp_1 denotes the change in the price of good #1, this equation can be written in terms of derivatives of the demand function as follows.

$$\Delta q_1^{TOT} \approx -(q_1\, \Delta p_1)\left(\frac{\partial q_1^*}{\partial I}\right) + \Delta q_1^{SUB} \tag{7.4}$$

Here, the notation Δq_1^{SUB} is used to denote the substitution effect due to the pivoting of the budget line. (This effect is sometimes called the "compensated effect" because it reflects the consumer's response to price when the consumer is simultaneously given income to compensate for any increase in price.) Alternatively, the equation can be expressed as rates of change in quantity demanded (per dollar of price change) by dividing all terms by Δp_1 .

$$\frac{\Delta q_1^{TOT}}{\Delta p_1} \approx -q_1\left(\frac{\partial q_1^*}{\partial I}\right) + \frac{\Delta q_1^{SUB}}{\Delta p_1} \tag{7.5}$$

Finally, as the change in price Δp_1 approaches zero, the equation becomes

$$\left(\frac{\partial q_1^*}{\partial p_1}\right)^{TOT} = -q_1\left(\frac{\partial q_1^*}{\partial I}\right) + \left(\frac{\partial q_1^*}{\partial p_1}\right)^{SUB} \tag{7.6}$$

This decomposition—whether expressed as (7.3), (7.4), (7.5), or (7.6) — is known as the *Slutsky equation.*[2]

Calculating the substitution effect. A practical formula for calculating the substitution effect can now be found by simply rearranging (7.4): substitution effect equals total effect minus income effect:

$$\Delta q_1^{SUB} \approx \Delta q_1^{TOT} + (q_1\, \Delta p_1)\left(\frac{\partial q_1^*}{\partial I}\right). \tag{7.7}$$

Example: In a previous example concerning John Q. Consumer, the total effect of the price change in food was calculated as a decrease in food purchases by approximately 15 units. In another previous example, the income effect of the price change was calculated as a decrease in food purchases by approximately 3.75 units. Thus the substitution effect of the price change is found by subtraction to have been a decrease by approximately 11.25 units.

Example: Suppose a consumer's monthly demand for donuts is given (approximately) by $q = 10 - 10\,p + 0.02\,I$. Monthly income is originally $I = \$1{,}000$ and the price of donuts is $p = \$0.40$. Substituting this price and income into the demand equation shows that the consumer purchases 26 donuts per month. Notice that for this demand curve, $\partial q/\partial p = -10$ and $\partial q/\partial I = 0.02$. Suppose the price of donuts rises to $p = \$1.00$. Let us apply equation (7.4) to compute the total effect, the income effect, and the substitution effect of this price change. Clearly, $\Delta p = 0.60$, so the total effect of the price change is

[2] John R. Hicks, *Value and Capital*, 2nd edition, Oxford: Clarendon Press, 1946, p. 309.

$\Delta q_1 \approx \Delta p_1 \dfrac{\partial q_1 *}{\partial p_1} = 0.60 \times (-10) = -6$, that is, a decrease of 6 donuts. The income effect of the price change is $-(\Delta p_1\, q_1) \times \dfrac{\partial q_1 *}{\partial I} = -(0.60 \times 26) \times 0.02 = -0.312$, that is, a decrease of 0.312 donuts. The substitution effect is the difference, a decrease of 6 - 0.312 = 5.688 donuts.

Example: Suppose a consumer has been consuming 10 gallons of gasoline per week at a price of $3.00 per gallon. Then the price of gasoline rises to $3.50 per gallon. Simultaneously, the consumer enjoys an increase in income of $5 per week. Consider the combined effect of these two simultaneous changes. Will the consumer increase, decrease, or keep constant her or his consumption of gasoline? Is the consumer better off or worse off than before the simultaneous change in price and income? To answer these questions, notice that the increase in income here exactly equals the income adjustment required to keep the old bundle affordable. So the combined effect of these two simultaneous changes is a *pure Slutsky substitution effect.* Of course, the substitution effect is still negative, so the consumer will still *decrease* her or his consumption of gasoline, despite the increase in income. Figure (7.2) shows that the consumer is *slightly better off* than before the simultaneous change in price and income.

Proving the Law of Demand. We have shown that the substitution effect of a price increase cannot be an increase in the quantity purchased of the same good, and will generally be a decrease. We have also shown that the income effect of a price increase will be a decrease in the quantity purchased of the same good, if that good is a normal good (that is, if $\partial q_1*/\partial I$ is positive). It follows from the Slutsky equation that the total effect of a price increase will be an unambiguous decrease in the quantity purchased of the same good, if that good is a normal good. Thus, the Law of Demand follows from the utility-maximizing model of the consumer.

Note how substitution and income effects work in the same direction for normal goods. If the price rises, both effects cause a decrease in the quantity purchased. Conversely, if the price falls, both effects cause an increase in the quantity purchased (see figure 7.7). We say that both effects are negative because the sign of the price change is opposite that of its two effects on quantity purchased.

However, for inferior goods like second-hand clothes and mass transit, $\partial q_1*/\partial I$ is negative. As the budget set shrinks, the consumer buys *more* of an inferior good (see figure 5.3 from previous chapter). Thus the income effect must be negative, so the substitution and income effects work in opposite direction. In particular, if the price rises, the income effect is positive in this case, partly offsetting the negative substitution effect (see figure 7.8). But the Law of Demand still holds if the income effect is not too strong.

Giffen goods. What must happen for the demand curve to slope *up?* For the total effect of an increase in price to be an *increase* in the quantity demanded, the good must be extremely inferior—that is, the income effect must work in the opposite direction of the substitution effect and complete overwhelm it (see figure 7.9). Thus a Giffen good is a special, extreme case of an inferior good. While theoretically possible, Giffen goods are surely extremely rare and perhaps nonexistent in the real world.

Figure 7.7. Substitution and income effects of *decrease* in price of good

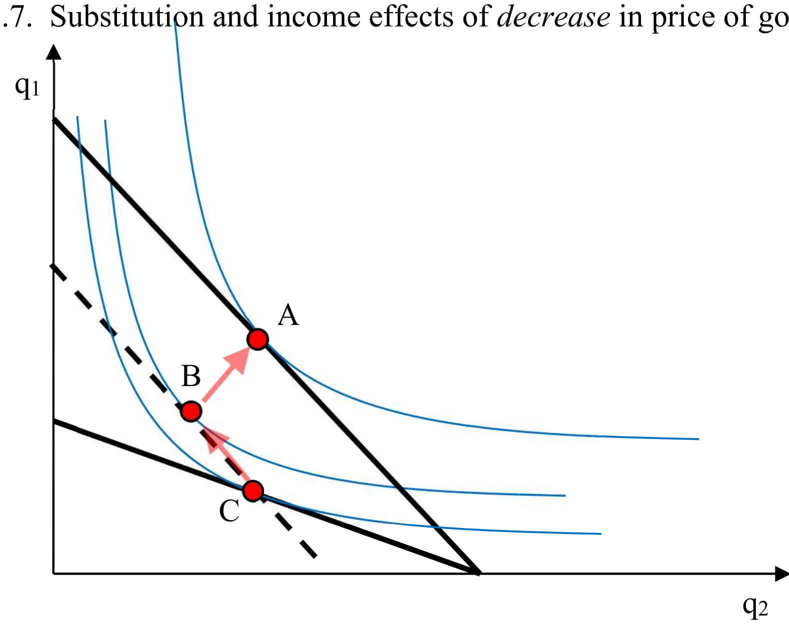


Figure 7.8. Income effect of increase in price of good #1 for an inferior good

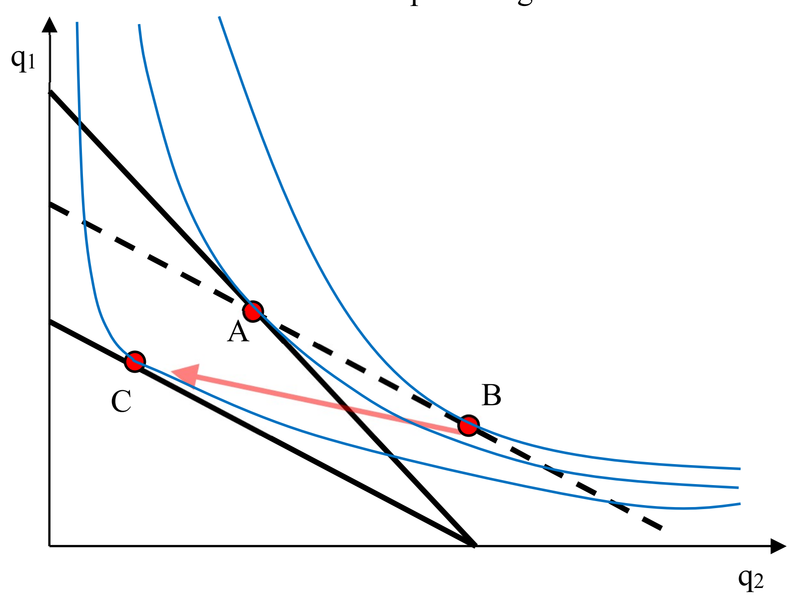
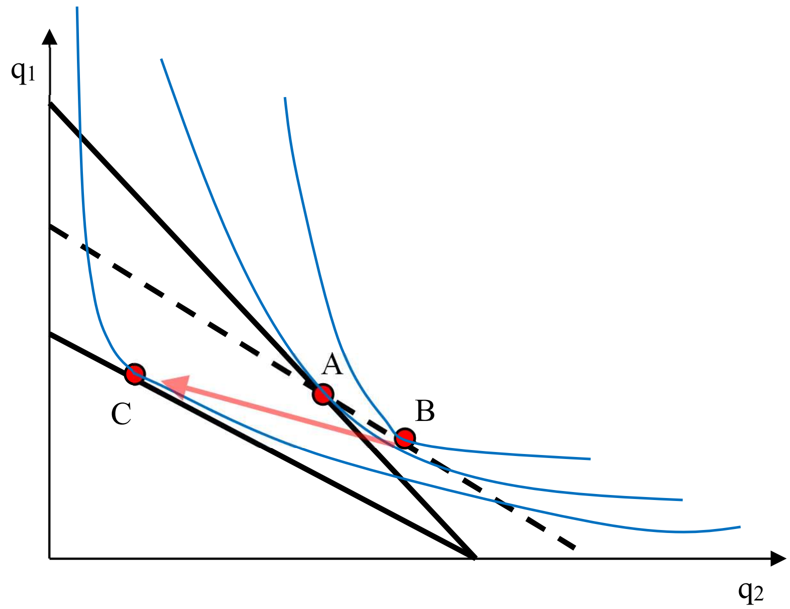


Figure 7.9. Substitution and income effects of increase in price of good #1 for a Giffen good



Slutsky equation in elasticity form. The Slutsky relationship can be expressed in terms of elasticities instead of slopes. Multiply both sides of equation (7.6) above by (p_1/q_1) to get the following.

(7.8) $$\left(\frac{\partial q_1^*}{\partial p_1}\right)^{TOT}\left(\frac{\boldsymbol{p_1}}{\boldsymbol{q_1}}\right) = -q_1\left(\frac{\boldsymbol{p_1}}{\boldsymbol{q_1}}\right)\left(\frac{\partial q_1^*}{\partial I}\right) + \left(\frac{\partial q_1^*}{\partial p_1}\right)^{SUB}\left(\frac{\boldsymbol{p_1}}{\boldsymbol{q_1}}\right)$$

Note that the left side of equation (7.8) is the total price elasticity of demand. The second term on the right side is the "substitution price elasticity," more commonly called the "compensated price elasticity" of demand, so hereafter denoted ε^{COMP}. But the first term on the right side is still rather messy. To simplify further, multiply the first term by (I/I) to get the following.

(7.9) $$\varepsilon = -\left(\frac{q_1 p_1}{I}\right)\left[\left(\frac{\partial q_1^*}{\partial I}\right)\left(\frac{I}{q_1}\right)\right] + \varepsilon^{COMP}$$

Note that the item in brackets is just the income elasticity of demand, which we will hereafter denote η. Also note that $(q_1\, p_1)$ equals spending on good 1. Let $S_1 = q_1 p_1/I$ denote the spending *share* of good 1 (that is, spending on good 1 expressed as a fraction of total income). Making these substitutions gives the Slutsky equation expressed very compactly in elasticity form:

(7.10) $$\varepsilon \;=\; -\,S_1\,\eta \;+\; \varepsilon^{COMP}$$

Section 7.5: The Hicks substitution effect.

Definition of the Hicks substitution effect. An alternative definition of the substitution effect due to Hicks is often used.[3] The Hicks substitution effect constructs the hypothetical budget line in a slightly different way. Instead of pivoting the budget line around the original bundle chosen, we *roll* the budget line around the original indifference curve on which the original bundle lies. The hypothetical budget line is again parallel to the new final budget line, but it is still tangent to the original indifference curve. Given this hypothetical budget line, the consumer would move from bundle A to hypothetical bundle B in figure 7.10. This movement from bundle A to bundle B is called the *Hicks substitution effect*.

Like the Slutsky substitution effect, the Hicks substitution effect is generally negative. It is obvious graphically that if indifference curves are smooth and show diminishing marginal rates of substitution, then the new tangency point B must be below and to the right of the original bundle A. It is also obvious graphically that the size of the Hicks substitution effect depends on the curvature of the indifference curves.

Comparing the two substitution effects. In general, the Hicks substitution effect is slightly larger (in absolute value) than the Slutsky substitution effect. This is because the adjustment to income required to keep the old bundle affordable is generally *greater* than the adjustment to income required to keep the consumer on the old indifference curve, as shown in figure 7.11. Put differently, the Slutsky hypothetical budget line will almost always be higher than the Hicks hypothetical budget line. In figure 7.10, the movement from bundle A to bundle B_S represents the Slutsky substitution effect, while the movement from bundle A to bundle B_H represents the Hicks substitution effect of the same price change. However, the two substitution effects are virtually identical for small changes in price.[4]

Example. In an earlier example, we calculated that John Q. Consumer required an income adjustment to keep the old bundle affordable of $150. However, figure 7.11 shows that this adjustment is slightly more than enough to allow the consumer to attain the same indifference curve as before.

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[3] J.R. Hicks, *Value and Capital: An Inquiry Into Some Fundamental Principles of Economic Theory*, 2nd edition, Oxford, United Kingdom: Oxford University Press, 1946, pages 31-32.

[4] For *infinitesimal* changes in price, the income adjustment required to keep the consumer on the same indifference curve is the same as the income adjustment required to keep the old bundle affordable—namely, the change in price times the original quantity chosen. (This fact is implied by Shephard's lemma, a well-known result in economic theory. See Hal R. Varian, *Microeconomic Analysis*, New York: W. W. Norton, 1978, p. 32.) Since the required income adjustments are identical, the Slutsky and Hicks income effects are identical, so the Slutsky and Hicks substitution effects must also be identical.

Figure 7.10. Hicks substitution effect of increase in price of good #1

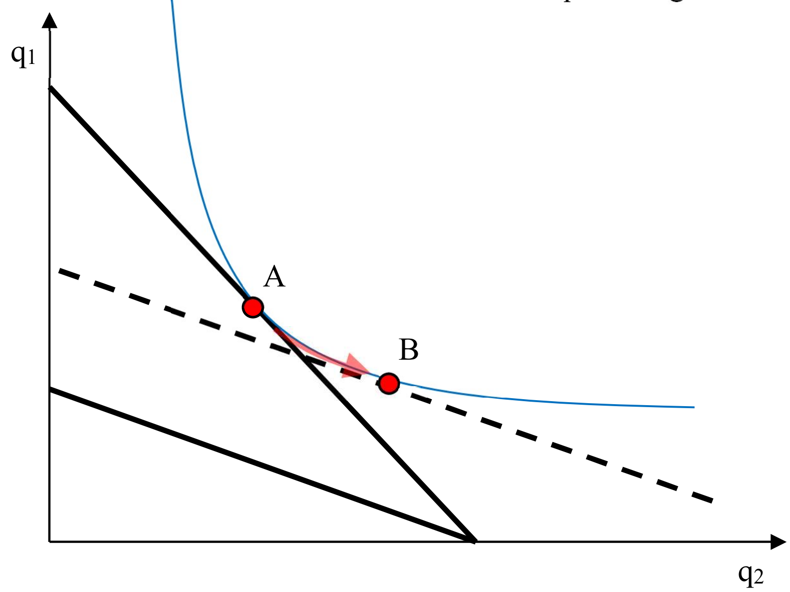
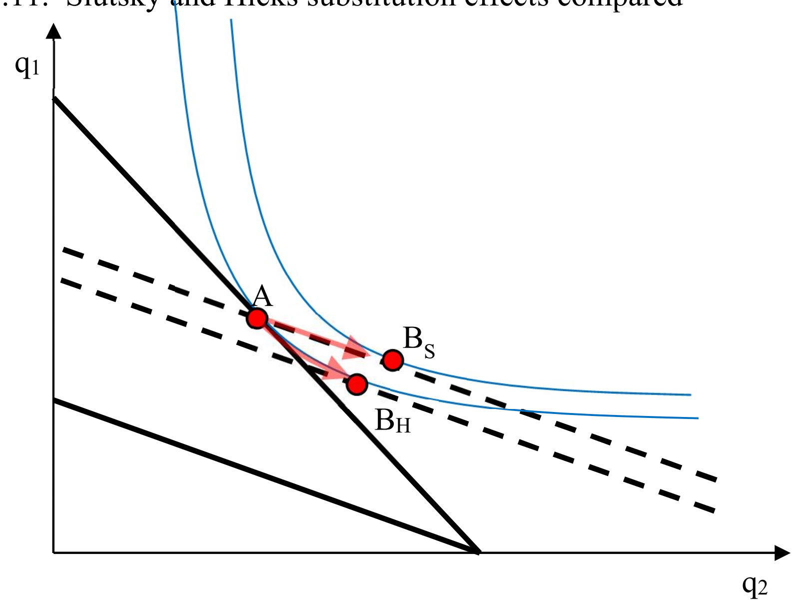


Figure 7.11. Slutsky and Hicks substitution effects compared



Section 7.6: Summary

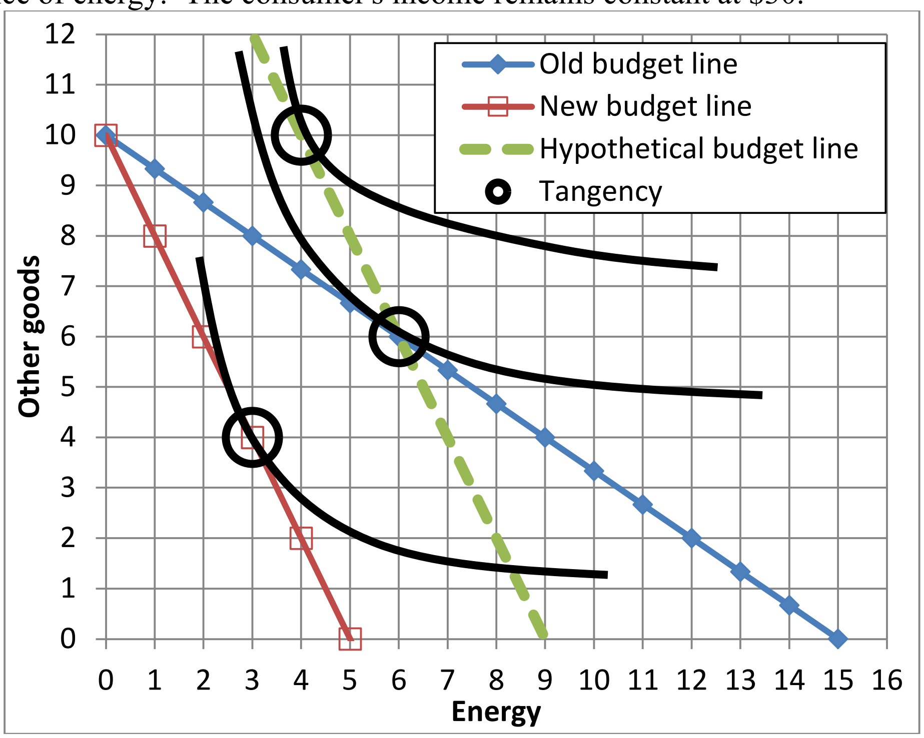
The total effect of a price change Δp_1 on the quantity demanded by a consumer can be decomposed into a substitution effect and an income effect. The substitution effect is defined as a the response to the price change combined with a hypothetical change in income that leaves the original bundle just affordable. This substitution effect measures the effect of a change in relative prices on the consumer's choice. The income effect is defined as a movement between the hypothetical budget line and the final, real budget line. It measures the effect of a change in the size of the budget set on the consumer's choice. The income effect may be calculated approximately as $(-\Delta p_1\ q_1)\ (\partial q_1^*/\partial I)$. The substitution effect may be calculated as $(\Delta p_1)\ (\partial q_1^*/\partial p_1)$ minus the income effect. For normal goods, the income effect of an increase in price is a decrease in the quantity chosen, so income and substitution effects work in the same direction and the Law of Demand must hold. The Law of Demand still holds for inferior goods $(\partial q_1^*/\partial I < 0)$ provided the income effect is not too strong. A Giffen good $(\partial q_1^*/\partial p_1 > 0)$ must be extremely inferior.

Problems

(7.1) [Total effect of price change] Suppose a consumer has the demand function $q_1^* = I/p_1 - (p_2/p_1)$. Suppose initially that income is $1000, the price of good #1 is $20, and the price of good #2 is $5.

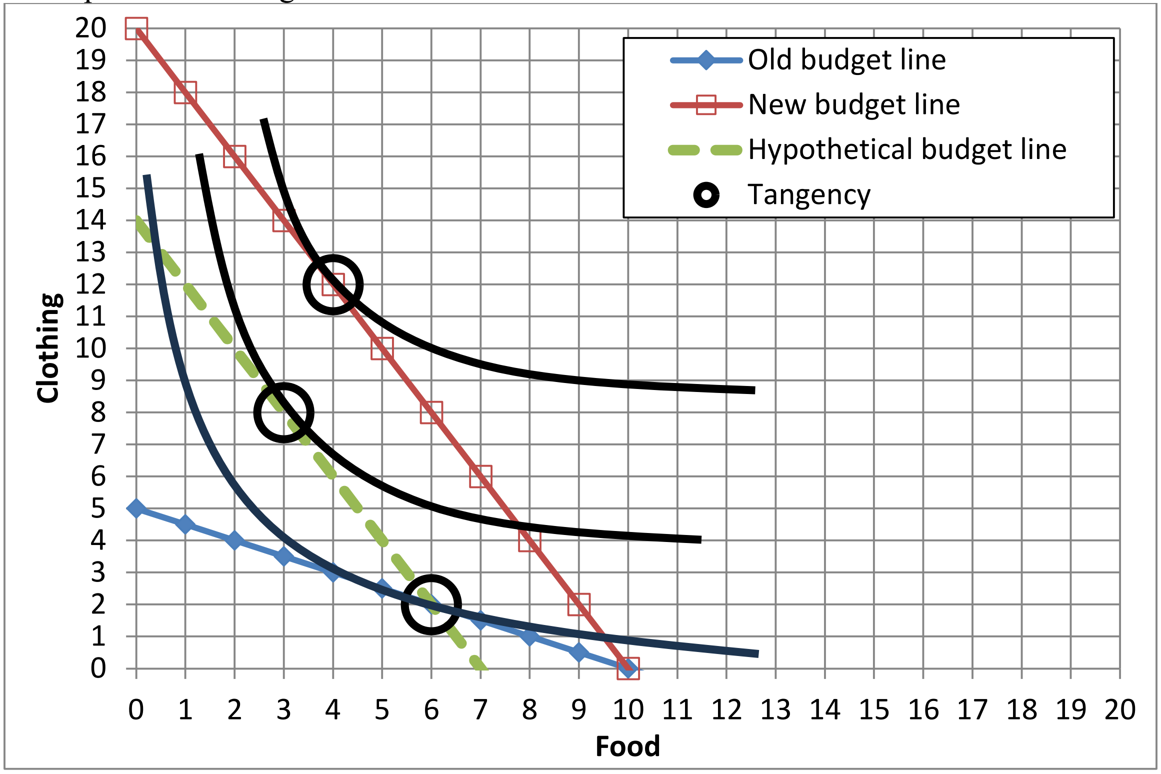
a. Calculate exactly the change in quantity demanded as the price of good #1 rises from $20 to $21.
b. Find a formula for the partial derivative of q_1^* with respect to p_1.
c. Compute the value of the partial derivative of q_1^* with respect to p_1 when income is $1000, the price of good #1 is $20, and the price of good #2 is $5.
d. Use the approximation formula (7.1) to calculate the change in quantity demanded as the price of good #1 rises from $20 to $21.

(7.2) [Slutsky substitution effect] The graph below shows a consumer's response to a rise in the price of energy. The consumer's income remains constant at $30.



a. What was the old price of energy, according to the old budget line?
b. How much energy was purchased with the old budget line?
c. What is the new price of energy, according to the new budget line?
d. Did the Slutsky substitution effect of the energy price change cause the consumer to buy *more* or *less* energy? How much more or less?
e. Did the income effect of the energy price change cause the consumer to buy *more* or *less* energy? How much more or less?
f. Did the total effect of the energy price change cause the consumer to buy *more* or *less* energy? How much more or less?

(7.3) [Slutsky substitution effect] The graph below shows a consumer's response to a fall in the price of clothing. The consumer's income remains constant at $20.



a. What was the old price of clothing, according to the old budget line?
b. How much clothing was purchased with the old budget line?
c. What is the new price of clothing, according to the new budget line?
d. Did the Slutsky substitution effect of the clothing price change cause the consumer to buy *more* or *less* clothing? How much more or less?
e. Did the income effect of the clothing price change cause the consumer to buy *more* or *less* clothing? How much more or less?
f. Did the total effect of the clothing price change cause the consumer to buy *more* or *less* clothing? How much more or less?

(7.4) [Slutsky equation] Suppose a consumer's monthly demand for telephone calls in minutes is given approximately[5] by $q_1 = 50 - 1000\, p_1 + 0.5\, I$. Monthly income is originally I=$1,000 and the price of telephone minutes is p_1=$0.05. Then the price of telephone minutes rises to p_1=$0.10. Compute the following:

a. the original amount purchased (q_1).
b. the total effect (Δq_1) of the price change.
c. the income adjustment required to keep the old bundle affordable.
d. the income effect of the price change.
e. the substitution effect of the price change.

[5] This cannot be the exact demand function because it is not homogeneous of degree zero in income and prices.

(7.5) [Slutsky equation] Suppose at a particular point, the partial derivatives of a consumer's weekly demand for gasoline take the following values. The partial derivative with respect to the price of gasoline is $\partial q^*/\partial p = -4$. The partial derivative with respect to the consumer's (weekly) income is $\partial q^*/\partial I = 0.05$. The consumer currently buys 20 gallons of gasoline per week.

a. Is gasoline a normal good or an inferior good for this consumer? Why?
b. Is gasoline an ordinary good or a Giffen good for this consumer? Why?
c. Compute the approximate total change in the amount of gasoline purchased if the price rose by \$0.50 (fifty cents).
d. Compute the income-effect component of this change.
e. Compute the substitution-effect component of this change.

(7.6) [Slutsky equation] Suppose the price of gasoline rose by fifty cents per gallon, but the government awarded tax credits to compensate for the increase. In particular, if the average consumer previously bought 500 gallons per year, then each consumer would be given a tax credit equal to \$250. Would consumption of gasoline by the average consumer increase, decrease, or stay the same? Explain your answer using an indifference curve diagram, if possible.

(7.7) [Slutsky equation] Suppose the government is concerned that poor people are having difficulty paying their electric power bills. Assume the price of electricity is currently \$0.10 per kilowatt hour and the typical poor family uses about 2000 kilowatt-hours per month. Thus, the typical poor family pays about \$200 per month for electricity. The government is considering two alternative programs to help poor people, programs which might seem equivalent to non-economists.

- *Lump-sum payment:* Poor families would continue to pay the rate of \$0.10 per kilowatt hour, but the government would mail a check for \$100 per month to all poor families that could be applied toward their utility bills.
- *Rate subsidy:* Poor families would enjoy a reduced electricity rate of \$0.05 per kilowatt hour and the government would pay electricity companies the difference.

Now consider two alternative formulas for the change in electricity use by poor families.

(i) $\Delta q_1 = -0.05 \left.\frac{\partial q^*}{\partial p}\right|_{sub} + (100) \times \frac{\partial q_1^*}{\partial I}$. (ii) $\Delta q_1 = (100) \times \frac{\partial q_1^*}{\partial I}$.

We want to know which program will have a greater effect on electricity consumption. To find the answer, we use the economic theory of the consumer. Here, the formula for the demand function for electricity $q_1^*(p_1,I)$ is unknown, but we may assume electricity is a normal good.

a. Is formula (i) positive or negative? Why?
b. Is formula (ii) positive or negative? Why?
c. Which program would cause the change in electricity consumption given by formula (i)? Why? [Hint: See equation 7.4.]
d. Which program would cause the change in electricity consumption given by formula (ii)? Why?
e. Which program would cause the larger increase in electricity usage by poor families—the lump-sum payment or the rate subsidy? Why?
f. Which program would be more costly for the government? Why?

(7.8) [Slutsky equation] Suppose the price of gasoline is $3 per gallon and at this price, the average person consumes 300 gallons per year. Two alternative government programs are proposed to help gasoline consumers.

- *Gasoline subsidy:* the government pays gasoline consumers $0.50 for every gallon consumed.
- *Lump-sum payment:* the government gives every gasoline consumer $150 regardless of how much gasoline is consumed.

The formula for the demand function for gasoline $q_1*(p_1,I)$ is unknown, but we may assume gasoline is a normal good.

a. Write an algebraic expression for the income effect of the gasoline subsidy. [Hint: use equation (7.2)].
b. Write an algebraic expression for the total effect of the lump-sum payment [hint: the total effect of a lump sum payment is $\Delta q_1 = \begin{pmatrix} \text{lump} \\ \text{sum} \end{pmatrix} \times \frac{\partial q_1 *}{\partial I}$.]
c. Show that the two expressions are equal.
d. Which program will cause consumers to use more gasoline? Why? [Hint: Use equation (7.4) and note the sign (+ or -) of the substitution effect.]

(7.9) [Income and substitution effects] The price of housing in some parts of the United States has risen very rapidly in the last thirty years. Newcomers are therefore choosing to buy smaller houses. But existing homeowners are *not* moving into smaller houses quite so quickly. Can the theory of the consumer tell us why?

a. Which group—newcomers or existing homeowners—experiences both an income and a substitution effect from a rise in the price of housing?
b. Which group—newcomers or existing homeowners—experiences only a substitution effect from a rise in the price of housing? Why? [Hint: The Slutsky substitution effect is the effect of rotating the budget constraint while keeping the old bundle still affordable.]
c. Which group should therefore have a stronger demand response to a rise the price of housing, according to demand theory? Explain your answer using an indifference curve, if possible.

(7.10) [Slutsky equation in elasticity form] Suppose the price elasticity of demand for food is -0.3 and the income elasticity is 0.2. Assume a consumer spends 15 percent of her or his income on food (that is $S_{food} = qp/I = 0.15$).

a. Compute the compensated elasticity (ε^{comp}) of demand for food.

Suppose the price of food rises by 10 percent and nothing else changes.

b. Will the quantity of food purchased increase or decrease? By how much (in percent)?

Now suppose the price of food still rises by 10%, but the government wants to make sure the consumer can still afford to buy enough food. Suppose the government gave the consumer an income boost (perhaps a tax rebate) equal to 10% times the consumer's prior spending on food. Thus the income boost equals 0.10qp, where q and p denote the person's old quantity and price of food.

c. Would the quantity of food purchased by this consumer still change in the face of the 10 percent rise in price? If so, by how much (in percent)? Explain your reasoning.

(7.11) [Slutsky equation in elasticity form] Suppose the price elasticity of demand for energy is -0.9, energy's share in total spending is 0.1, and the income elasticity of demand for energy is 0.6. Now suppose that the price of energy increases by 10%.

a. First, suppose nothing else changes. Will the quantity demanded of energy increase or decrease? By how much?
b. Alternatively, suppose the government cushions the blow of higher energy prices by giving everyone a cash transfer equal to 10 percent of last year's spending on energy. Will the quantity demanded of energy increase or decrease? By how much? [Hint: first find the compensated elasticity of demand for energy.]

[end of problem set]