LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

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Part 2: Consumers and demand

Chapter 6: Demand elasticities

Problems

(6.1) [Price elasticity] Suppose a consumer always spends a total of $50 on lattes every month, no matter what the price and no matter what her income.

a. Let q denote the quantity of lattes and let p denote the price of lattes, and write the above assumption as an equation.
b. Find a formula for this person's demand function $q = f(p)$.
c. Compute this person's price elasticity of demand.
d. Compute this person's income elasticity of demand.

(6.2) [Price elasticity] Suppose a demand function is a simple linear function: $p = a - b\,q$, where a and b are positive constants.

a. Prove that the price elasticity of demand must equal $p/(p-a)$.
b. What is the value of this elasticity at the intercept of the demand curve on the quantity axis?
c. What is the value of this elasticity at the intercept of the demand curve on the price axis?
d. What is the value of this elasticity at the midpoint of the demand curve?

(6.3) [Price and income elasticities] Assume a consumer always spends 25% of his income on housing, no matter what the price of housing and no matter what his income.

a. Let q denote the quantity of housing, let p denote the price of housing, and let I denote the consumer's income, and write the above assumption as an equation.
b. Find a formula for this person's demand function $q = f(p,I)$.
c. Compute this person's price elasticity of demand.
d. Compute this person's income elasticity of demand.

(6.4) [Price and income elasticities] Suppose the consumer has the particular utility function $U = q_1 q_2^2$ and faces budget constraint $I = p_1 q_1 + p_2 q_2$.

a. Find the consumer's demand function for good 1.
b. Is the consumer's own-price elasticity of demand for good #1 constant? If so, what is its value?
c. Is the consumer's cross-price elasticity of demand for good #1 with respect to the price of good #2 constant? If so, what is its value?
d. Is the consumer's income elasticity of demand for good #1 constant? If so, what is its value?

(6.5) [Price elasticity of demand] Indicate for each of the following demand functions whether the function has a constant own-price elasticity of demand. If the price elasticity is constant, give its value.

a. $q_1^* = I / (2p_1)$.
b. $q_1^* = 20 - 2p_1$.
c. $q_1^* = I / (p_1 + p_1^2/p_2)$.
d. $q_1^* = (2I)/(3p_1) - 10p_2/p_1$.
e. $q_1^* = 5\, p_1^{-0.7}\, p_2^{0.2}\, I^{0.5}$.

(6.6) [Price elasticity and revenue] Determine whether the following statement is true, false, or uncertain, and justify your answer using the concept of elasticity. "If College X raises tuition, it will get more tuition revenue."

To answer the next problems, use the following definitions:

Price elasticity of demand = (% change in quantity) ÷ (% change in price),

Spending or revenue[1] = price × quantity.

The second definition implies the following approximation formula:

% change in spending ≈ (% change in price) + (% change in quantity)

(6.7) [Price elasticity and revenue] Refer to the definitions in the box above. Suppose a company believes that the price elasticity of demand for its product is –1.5, and consider what would happen if it decreased its price by 2%. Assume income and other prices do not change.

a. Is demand elastic, inelastic, or unitary-elastic?
b. Will the quantity sold increase or decrease? By how much?
c. Will the total amount of revenue generated by the product increase or decrease? By approximately how much?

[1] Note that the dollar amount of spending by buyers must equal the amount of revenue received by sellers.

(6.8) [Price elasticity and revenue] Refer to the definitions in the box above. The price elasticity of demand for bread in the United States has been estimated[2] to be about -0.4. Consider what would happen if the price of bread increased by 5%. Assume income and other prices do not change.

a. According to this estimate, is demand elastic, inelastic, or unitary-elastic?
b. Will the quantity sold increase or decrease? By how much?
c. Will the total amount of money spent by consumers on bread increase or decrease? By approximately how much?

(6.9) [Price elasticity and revenue] Refer to the definitions in the box above. The price elasticity of demand for steak in the United States has been estimated[3] to be about -0.7. Consider what would happen if the price of steak decreased by 10%. Assume income and other prices do not change.

a. According to this estimate, is demand elastic, inelastic, or unitary-elastic?
b. Will the quantity sold increase or decrease? By how much?
c. Will the total amount of money spent by consumers on steak increase or decrease? By approximately how much?

(6.10) [Price elasticity and revenue] Refer to the definitions in the box above. The price elasticity of demand for fresh tomatoes in the United States has been estimated[4] to be about -0.8. Consider what would happen if the price of fresh tomatoes decreased by 5%. Assume income and other prices do not change.

a. According to this estimate, is demand elastic, inelastic, or unitary-elastic?
b. Will the quantity sold increase or decrease? By how much?
c. Will the total amount of money spent by consumers on fresh tomatoes increase or decrease? By approximately how much?

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[2] Source: USDA Economic Research Service (http://www.ers.usda.gov/data-products/commodity-and-food-elasticities.aspx , accessed August 1, 2012) citing K.S. Huang, and B. Lin., "Estimation of Food Demand and Nutrient Elasticities from Household Survey Data," Food and Rural Economic Division, Economic Research Service, US Department of Agriculture, Technical Bulletin, Number 1887 (August 2000). Marshallian own-price elasticity.

[3] Source: USDA Economic Research Service (http://www.ers.usda.gov/data-products/commodity-and-food-elasticities.aspx , accessed August 1, 2012) citing O. Capps Jr., "Utilizing Scanner Data to Estimate Retail Demand Functions for Meat Products," *American Journal of Agriculture Economics*, Vol. 75 (August 1989), pp. 750-760.

[4] Source: USDA Economic Research Service (http://www.ers.usda.gov/data-products/commodity-and-food-elasticities.aspx , accessed August 1, 2012) citing S.T. Yen, B. Lin, J. M. Harris, and N. Ballenger. "Demand for Differentiated Vegetables," paper presented at AAEA annual meeting, Denver, CO, 1-4 August 2004. Marshallian own-price elasticity.

(6.11) [Price elasticity and revenue] Refer to the definitions in the box above. The price elasticity of demand for butter in the United States has been estimated[5] to be about -0.2. Consider what would happen if the price of butter increased by 5%. Assume income and other prices do not change.

a. According to this estimate, is demand elastic, inelastic, or unitary-elastic?
b. Will the quantity sold increase or decrease? By how much?
c. Will the total amount of money spent by consumers on butter increase or decrease? By approximately how much?

(6.12) [Income elasticity of demand] Determine whether the following statement is true, false, or uncertain, and justify your answer.

"If a person's income elasticity of demand for clothing is one, then as the person's income rises, the fraction of income spent on clothing remains constant."

(6.13) [Computing income elasticity of demand] The U.S. government's Consumer Expenditure Survey reports the following figures. [6]

	Low-income consumers	High-income consumers
Total annual expenditures	$30 thousand	$50 thousand
Expenditure on eggs	$38	$42
Expenditure on car rentals	$200	$440

Assume that total annual expenditures equal consumer income. Assume that all consumers face the same prices; this implies that the percent change in the quantity of a good is equal to the percent change in the expenditure on that good.

a. Compute the income elasticity of demand for eggs using the arc-elasticity formula. [Hint: See Lecture Notes section 1.7.]
b. Compute the income elasticity of demand for car rentals using the arc-elasticity formula.
c. Compute the income elasticity of demand for eggs using the difference-in-logarithms formula. [Hint: See Lecture Notes section 1.7.]
d. Compute the income elasticity of demand for car rentals using the difference-in-logarithms formula.

[5] Source: USDA Economic Research Service (http://www.ers.usda.gov/data-products/commodity-and-food-elasticities.aspx , accessed August 1, 2012) citing K. Huang, "US Demand for Food: A Complete System of Price and Income Effects," United States Department Of Agriculture, Economic Research Service. Technical Bulletin 1714. 1986.

[6] Consumer Expenditure Survey, 2007. Table 2. Income before taxes: Average annual expenditures and characteristics. Figures compare columns labeled "$20,000 to $29,999" and "$50,000 to $69,999". (Figures have been rounded for this question.)

(6.14) [Income elasticity of demand] Indicate for each of the following demand functions whether the function has a constant income elasticity of demand. If the income elasticity is constant, give its value.

a. $q_1^* = I / (2p_1)$.

b. $q_1^* = 20 - 2p_1$.

c. $q_1^* = I / (p_1 + p_1^2/p_2)$.

d. $q_1^* = (2I)/(3p_1) - 10p_2/p_1$.

e. $q_1^* = 5\, p_1^{-0.7}\, p_2^{0.2}\, I^{0.5}$.

To answer the next problems, use the following definitions:

Income elasticity of demand = (% change in quantity) ÷ (% change in income),

Spending share = price × quantity ÷ income (or total spending).

The second definition implies the following approximation formula, assuming that price is held constant:

% change in budget share ≈ (% change in quantity) - (% change in income)

(6.15) [Income elasticity and budget share] Refer to the definitions in the box above. Suppose the income elasticity of demand for travel is 2.5. Now suppose income rises by 2%.

a. According to the information given above, is travel an inferior good, a necessary good, or a superior (or luxury) good?
b. Will the amount of travel demanded increase or decrease? By how much?
c. Will spending on travel, as a fraction of a consumer's total budget, increase or decrease? By approximately how much?

(6.16) [Income elasticity and budget share] Refer to the definitions in the box above. The income elasticity of demand for orange juice in the United States has been estimated[7] to be about 2.1. Now suppose income rises by 10%.

a. According to this estimate, is orange juice an inferior good, a necessary good, or a superior (or luxury) good?
b. Will the amount of orange juice demanded increase or decrease? By how much?
c. Will spending on orange juice, as a fraction of a consumer's total budget, increase or decrease? By approximately how much?

[7] Source: USDA Economic Research Service (http://www.ers.usda.gov/data-products/commodity-and-food-elasticities.aspx , accessed August 1, 2012) citing M.G. Brown, "The Demand for Fruit Juices: Market Participation and Quantity Demanded," *Western Journal Of Agricultural Economics,* Vol. 11 (December 1986), pp. 179-183.

(6.17) [Income elasticity and budget share] Refer to the definitions in the box above. One study[8] estimated that the income elasticity of demand for beer in the United States was about -0.5. Now suppose income decreases by 6%.

a. According to this estimate, is beer an inferior good, a necessary good, or a superior (or luxury) good?
b. Will the amount of beer demanded increase or decrease? By how much?
c. Will spending on beer, as a fraction of a consumer's total budget, increase or decrease? By approximately how much?

(6.18) [Income elasticity and budget share] Refer to the definitions in the box above. One study[9] estimated that the income elasticity of demand for "beverages and tobacco" (as a group) in the United States was about 0.1. Now suppose income increases by 5%.

a. According to this estimate, are "beverages and tobacco" an inferior good, a necessary good, or a superior (or luxury) good?
b. Will the amount of "beverages and tobacco" demanded increase or decrease? By how much?
c. Will spending on "beverages and tobacco," as a fraction of a consumer's total budget, increase or decrease? By approximately how much?

(6.19) [Demand elasticities] Suppose a typical consumer is believed to have the following demand function for electricity: $q_1^* = 50\, p_1^{-0.6}\, p_2^{0.1}\, I^{0.5}$. Here, p_1 denotes the price of electricity, p_2 denotes the price of natural gas, and I denotes the consumer's income.

a. Is this function homogeneous of degree zero in income and prices? Why or why not?
b. Find the price elasticity of demand for electricity. Is electricity an ordinary good or a Giffen good? Why?
c. Find the income elasticity of demand for electricity. Is electricity a normal good or an inferior good? Why?
d. Find the cross price elasticity of demand for electricity with respect to the price of natural gas. Are electricity and natural gas substitutes, complements, or unrelated goods? Why?

[8] Source: USDA Economic Research Service (http://www.ers.usda.gov/data-products/commodity-and-food-elasticities.aspx , accessed August 1, 2012) citing C.A.Gallet and J.A. List, "Elasticities of Beer Demand Revisited," *Economic Letters,* vol. 61, pp. 67-71, 1998.

[9] Source: USDA Economic Research Service (http://www.ers.usda.gov/data-products/commodity-and-food-elasticities.aspx , accessed August 1, 2012) citing J. Seale Jr., A. Regmi, and J. Berstein., "International Evidence on Food Consumption Patterns," United States Department of Agriculture, Economic Research Service. Technical Bulletin Number 1904, 2003.

(6.20) [Income elasticity and spending shares] The U.S. Consumer Expenditure Survey reports the following spending shares for two groups of consumers. The income of group 2 is about 40% higher than the income of group 1.[10]

	Group 1 (low income)	*Group 2 (high income)*
Spending shares		
All food	0.151	0.132
Housing, including utilities, furniture, and appliances	0.333	0.312
Vehicle purchases	0.081	0.103
Health care	0.079	0.062
Entertainment	0.041	0.046
Total expenditures	$29 thousand	$42 thousand

Use the relationship between the income elasticity and spending shares to answer the following questions. Justify each answer.

a. Is food a necessary good or a luxury (or "superior") good? Why?
b. Is housing a necessary good or a luxury (or "superior") good? Why?
c. Are vehicles a necessary good or a luxury (or "superior") good? Why?
d. Is health care a necessary good or a luxury (or "superior") good? Why?
e. Is entertainment a necessary good or a luxury (or "superior") good? Why?

[10] U.S. Bureau of Labor Statistics, Consumer Expenditure Survey, 2002, Table 46. The two income groups are "$20,000 to $29,999" and "40,000 to $49.999." Average total expenditures in each group (which might be interpreted as permanent income) were $28,836 and $41,787.

(6.21) [Income elasticity and spending shares] Let Q denote the quantity chosen. Let I denote the consumer's income. Let P denote the price of the good. Let S denote the spending share of a particular good in a consumer's budget. Thus S equals (PQ)/I and therefore must be a fraction between zero and one. By the properties of logarithms, then,

$$\ln(S) = \ln(P) + \ln(Q) - \ln(I)$$

and similarly

$$\Delta\ln(S) = \Delta\ln(P) + \Delta\ln(Q) - \Delta\ln(I) \ .$$

(Recall that $\Delta\ln(X)$ = is a decimal equal to the percent change in X.)

a. Show that if the price does not change, then the percent change in S equals the percent change in Q minus the percent change in I.
b. Use the definition of the income elasticity (η = $\Delta\ln(Q) / \Delta\ln(I)$) to prove that
$$\eta = \frac{\Delta\ln(S)}{\Delta\ln(I)} + 1$$
c. Compute the income elasticity of demand for food using the data in the previous problem.
d. Compute the income elasticity of demand for housing using the data in the previous problem.
e. Compute the income elasticity of demand for vehicles using the data in the previous problem.
f. Compute the income elasticity of demand for health care using the data in the previous problem.
g. Compute the income elasticity of demand for entertainment using the data in the previous problem.

(6.22) [Cross-price elasticity of demand] The cross-price elasticity of demand for butter in the United States, with respect to the price of margarine, has been estimated[11] to be about +0.048.

a. According to this estimate, are butter and margarine substitutes, complements, or unrelated goods?
b. If the price of margarine increases by 10%, will the quantity of butter sold increase or decrease? By how much? (Assume income and other prices do not change.)

[11] Source: USDA Economic Research Service (http://www.ers.usda.gov/data-products/commodity-and-food-elasticities.aspx , accessed August 1, 2012) citing K. Huang, "US Demand for Food: A Complete System of Price and Income Effects," United States Department Of Agriculture, Economic Research Service. Technical Bulletin 1714. 1986.

(6.23) [Elasticities with Cobb-Douglas utility] Suppose a consumer has the utility function $U(q_1,q_2) = U(q_1,q_2) = q_1^a q_2^b$, where a and b are positive constants.

a. Find an expression for the consumer's marginal rate of substitution of good #2 for good #1.

As usual, let I denote the consumer's income and let p_1 and p_2 denote the prices of goods #1 and #2.

b. Find the consumer's demand function for good #1.
c. Is the consumer's own-price elasticity of demand for good #1 constant? If so, what is its value?
d. Would you use this utility function to model the demand for food? Why or why not?
e. Is the consumer's income elasticity of demand for good #1 constant? If so, what is its value?
f. Would you use this utility function to model the demand for vehicle purchases or health care? Why or why not?

(6.24) [Elasticities with Cobb-Douglas utility] Nobel Prize winner George Stigler is alleged to have said, "All elasticities are one." Was he joking or was he for real? Suppose a consumer has the utility function $U(q_1,q_2) = U(q_1,q_2) = q_1^a q_2^b$, where a and b are positive constants.

a. Find the consumer's demand functions $q_1^*(p_1, p_2, I)$ and $q_1^*(p_1, p_2, I)$.
b. Find the consumer's price elasticities of demand for the two goods.
c. Find the consumer's income elasticities of demand for the two goods.
d. The elasticity of substitution in consumption is defined as $\sigma = \frac{d \ln (q_1/q_2)}{d\ ln(MRS)}$ or equivalently, $\sigma = \frac{d \ln (q_1^*/q_2^*)}{d\ ln(p_2/p_1)}$. Find the consumer's elasticity of substitution.

[end of problem set]