LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

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Part 2: Consumers and demand

Chapter 4: Budgets and choice

Section 4.1: The budget constraint

Affordable bundles. Consumers choose the most preferred bundle they can afford. Consumer decision-making consists of choosing affordable combinations of goods and services that best meet the consumer's needs and wants. Suppose there are n different goods and services available for sale. Any possible combination or bundle of goods can be described by the n-tuple that lists all these quantities $(q_1, q_2, ..., q_n)$. The amount spent on this bundle is calculated by multiplying each quantity q_i by the corresponding price p_i, to give

(4.1) $\quad$ Spending $= q_1 p_1 + q_2 p_2 + ... + q_n p_n$.

Not all bundles are affordable. A particular consumer can spend no more than her or his income. Let I denote income. Affordable bundles are those that satisfy the *budget constraint*

(4.2) $\quad I \geq q_1 p_1 + q_2 p_2 + ... + q_n p_n$.

The *budget set* consists of all bundles that satisfy this inequality constraint. Bundles in the budget set are affordable, with possibly some income left over.

Plotting the budget line. In reality, n is a very large number for a typical consumer in a developed country. An ordinary supermarket, alone, offers tens of thousands of goods. However, most of the key issues of consumer behavior can be seen with only two goods. So suppose a consumer chooses between just two goods and therefore faces a budget constraint given by

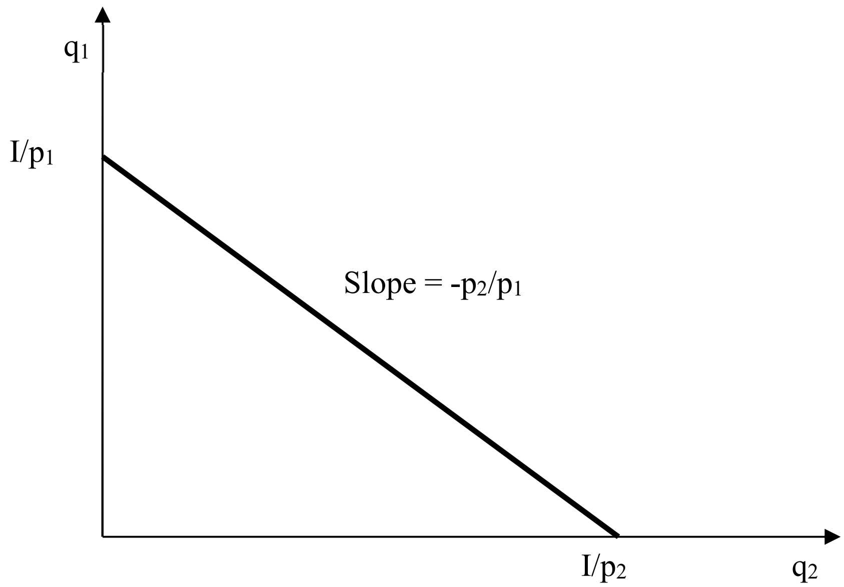
(4.3) $\quad I \geq q_1 p_1 + q_2 p_2$.

The *budget line* results from replacing the inequality with an equality.

(4.4) $\quad I = q_1 p_1 + q_2 p_2$.

Let us plot this line on a graph with q_1 on the vertical axis and q_2 on the horizontal axis (see figure 4.1). We assume the consumer has no control over income I or prices p_1 and p_2 so these may be treated as fixed constants. Using equation (4.4), the intercepts are easy to find. By definition the intercept on the q_1 axis is the value of q_1 when q_2 equals zero—that is, (I/p_1). Similarly, the intercept on the q_2 axis is the value of q_2 when q_1 equals zero—that is, (I/p_2).

Figure 4.1. A budget line for two goods



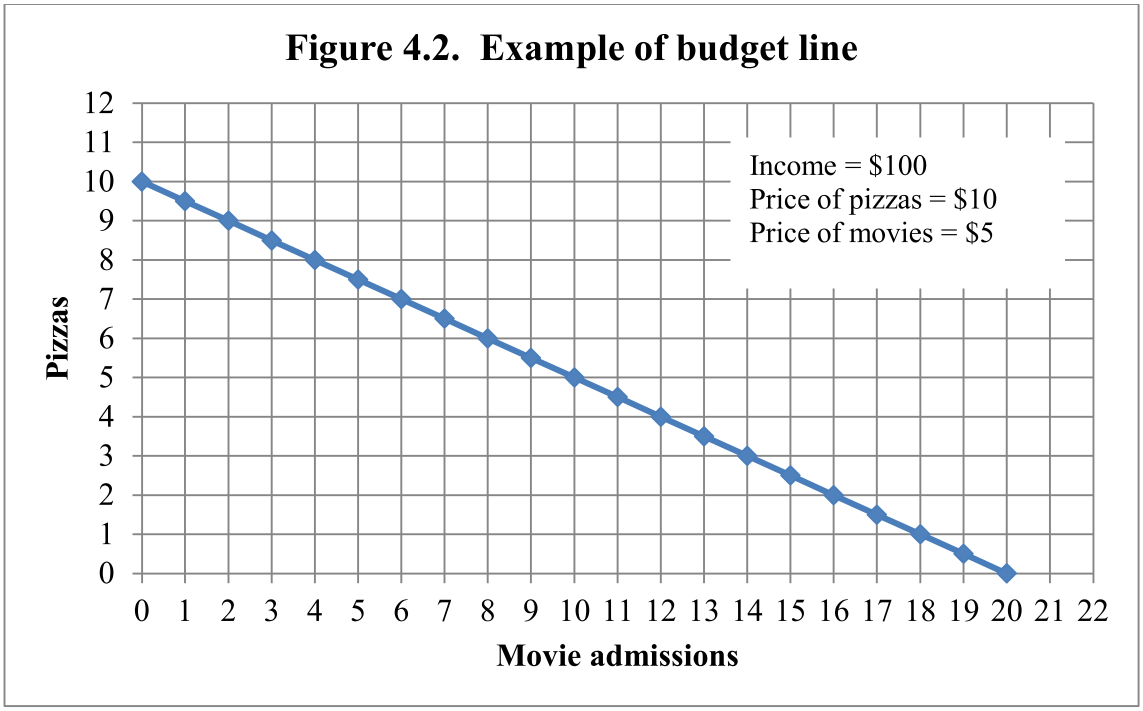
Slope of budget line. Solving for q_1 gives the slope-intercept form:

(4.5) $$q_1 = \frac{I}{p_1} - \frac{p_2}{p_1} q_2 \ .$$

Remember that I, p_1, and p_2 are fixed constants. Equation (4.5) shows that the budget line has slope $dq_1/dq_2 = (-p_2/p_1)$. Note that p_2 is in the numerator and p_1 is in the denominator of the slope. The negative sign indicates a tradeoff. Given fixed income I, the more of one good the consumer chooses, the less she or he must choose of the other good. The value of the slope is the rate of possible tradeoff, which obviously depends on the prices. To get one more unit of good #2, the consumer must give up $(-p_2/p_1)$ units of good #1.

Figure 4.2 shows an example of a budget line for a consumer choosing between pizzas and movie admissions. The consumer's income is \$100, the price of pizzas is \$10, and the price of movie admissions is \$5. Note that the intercepts and slope satisfy the formulas derived above.

Example: Suppose a consumer has \$100 to spend on energy and other goods. Let q_1 denote the quantity of energy and q_2 denote the quantity of other goods. Suppose the price of energy is \$5 and the price of other goods is \$4. Find the equation for the budget line, the intercepts, and the slope when energy is on the vertical axis. Following (4.4), the equation is $100 = q_1\,5 + q_2\,4$. The intercept on the energy axis is $100/5 = 20$. The intercept on the "other goods" axis is $100/4 = 25$. The slope with energy is on the vertical axis is $-4/5$.



Example: Suppose that food costs $2 per pound and electricity costs $0.10 per kilowatt hour. Assume the consumer spends all her or his income on these two items. If the consumer wants to purchase one more pound of food, how many kilowatt-hours of electricity must she or he give up? If electricity is on the vertical axis and food is on the horizontal axis, then the slope of the budget line is

$$\frac{\Delta \text{quantity of electricity}}{\Delta \text{quantity of food}} = -\frac{\text{price of food}}{\text{price of electicity}} = -\frac{2}{0.10} = -20$$

. So if the consumer wants one more pound of food (Δquantity of food = 1), then this equation shows that the change in the quantity of electricity must be -20. The consumer must give up 20 kilowatt-hours of electricity to get one more pound of food.

Affordable or not? All bundles under the budget line--that is, between the budget line and the origin--are affordable with some income left over. All bundles outside the budget line are not affordable.

Example: Suppose a consumer has $20 to spend at the state fair, buying corn dogs and midway rides. Corn dogs cost $4 and midway rides cost $2. Suppose bundle A consists of 3 corn dogs and 4 midway rides, bundle B consists of 3 corn dogs and 8 midway rides, and bundle C consists of 2 corn dogs and 4 midway rides. Which bundles lie under the budget line and which lie outside the budget line? Bundle A costs $20, so it lies exactly on the budget line. Bundle B costs $28, so it lies outside the budget line. Bundle C costs $16, so it lies under the budget line.

Changes in the budget line. If income (I) or prices (p_1 and p_2) change, the budget line changes.

Consider first a change in income. If income increases and prices remain constant, then the budget line shifts up and out, away from the origin, in parallel fashion. The formulas for the intercepts (I/p_1 and I/p_2) show that the intercepts increase proportionally with any increase in income. On the other hand, the slope ($-p_2/p_1$) does not change. Figure 4.3 shows how the budget line plotted in the previous figure shifts as income rises from \$100 to \$150.

Now consider a change in the price of good #1. If the price of good #1 increases, then the slope of the budget line ($-p_2/p_1$) becomes smaller (in absolute value) and the intercept on the axis labeled " q_1 " decreases, but the intercept on the axis labeled " q_2 " does not change. Put differently, the budget line rotates while its q_2-intercept remains fixed. Figure 4.4 shows how the line rotates as the price of pizzas rises to \$20.

Now consider a change in the price of good #2. If the price of good #2 increases, then the slope of the budget line ($-p_2/p_1$) becomes larger (in absolute value) and the intercept on the axis labeled " q_2 " decreases, but the intercept on the axis labeled " q_1 " does not change. Again, the budget line rotates while its q_1-intercept remains fixed. Figure 4.5 shows how the line roates as the price of movie admissions rises to \$10.

Finally, consider a simultaneous change in income and prices. If income and prices rise at the same rate--say, a fifty percent increase--then the slope ($-p_2/p_1$) and intercepts (I/p_1 and I/p_2) remain unchanged. Thus *pure inflation* (defined as proportionate increases in income and all prices) leaves the budget line unchanged.

Section 4.2: Choice

The most preferred affordable bundle. We assume the consumer chooses her or his most preferred affordable bundle. In graphical terms, the consumer chooses the bundle on the highest indifference curve the consumer can reach from her or his budget set. In mathematical terms, the consumer chooses the bundle that maximizes her or his utility, subject to the budget constraint.

Where is that most-preferred bundle likely to lie? It surely will not lie inside the budget line, as does point A in figure 4.6. The assumption of monotonicity implies that points on the budget line above and to the right of point A are more preferred. So the most preferred bundle must lie *on* the budget line, satisfying equation (4.4) or (4.5) exactly. But the most preferred bundle will not lie at an intersection of the budget line and an indifference curve, as do points B and C . Points on the budget line between B and C lie above the indifference curve, and so are preferred to either B or C .

If the indifference curve has *kinks* (the case of perfect complements is an example) then the most preferred bundle might lie at a kink point, as in figure 4.7. Or the most preferred bundle might lie on at a *corner* of the budget set, where the budget line meets an axis, as in figure 4.8. At a corner solution, the bundle includes zero amounts of one good or the other.

Figure 4.3. Parallel shift when income rises to $150

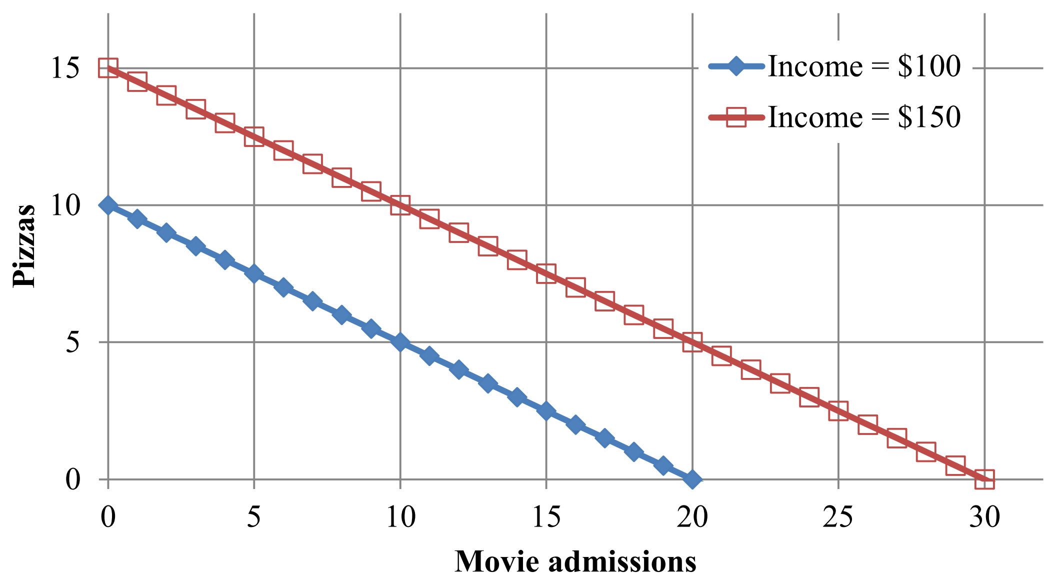
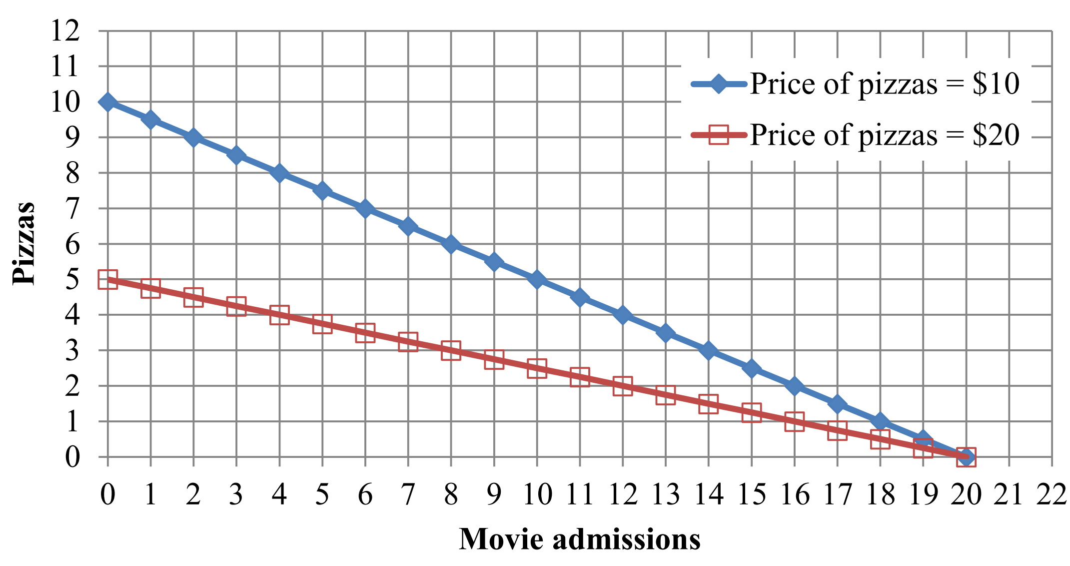


Figure 4.4. Rotation when price of pizzas rises to $20



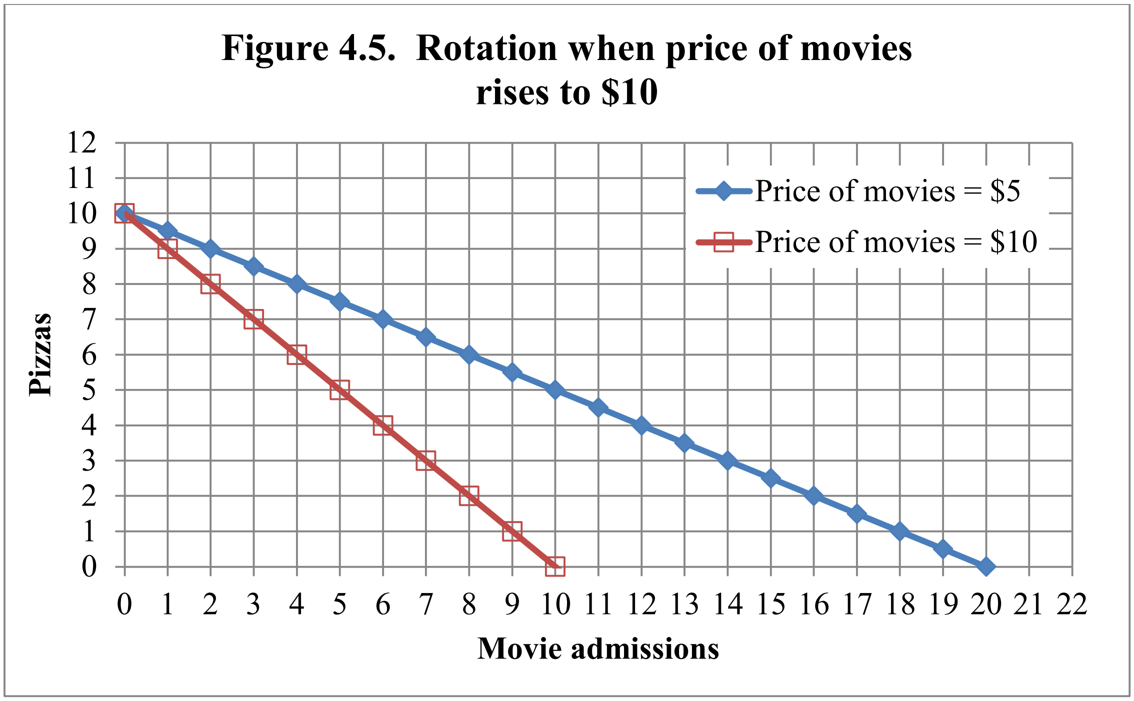


Figure 4.6. Wrong choices

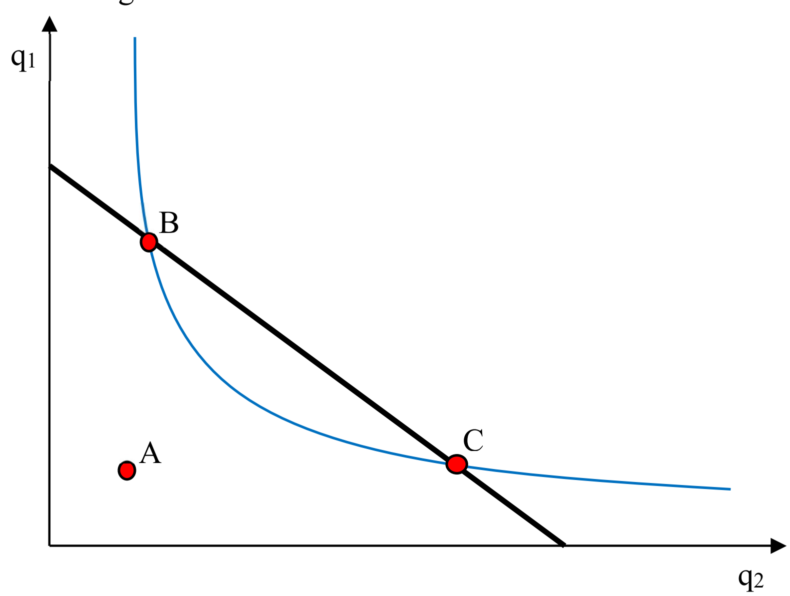


Figure 4.7. Choice at kink point of indifference curve

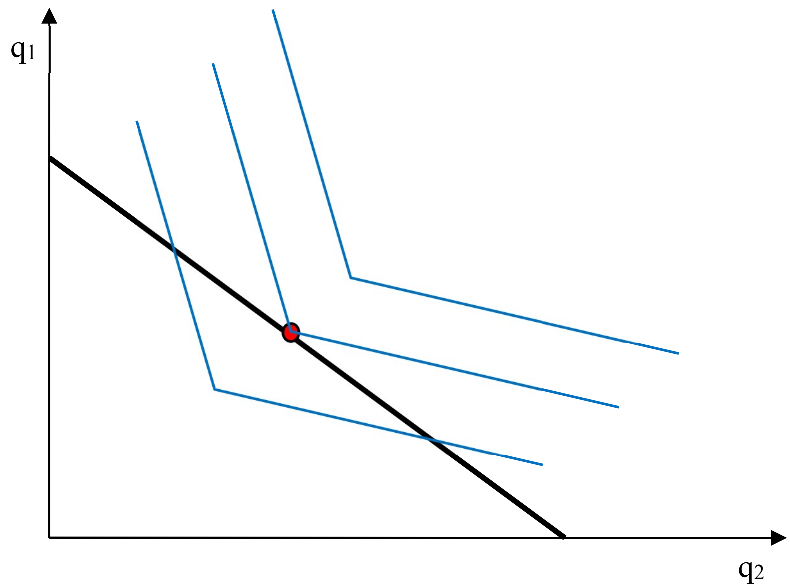
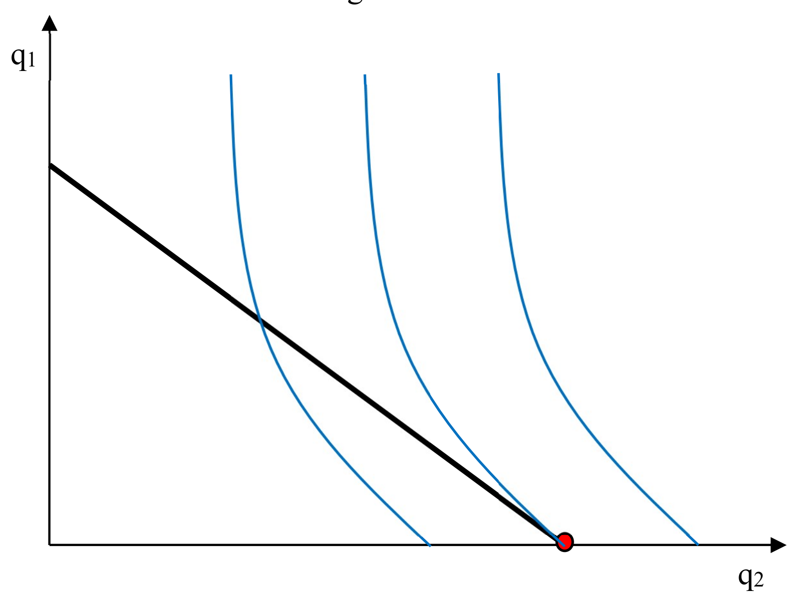
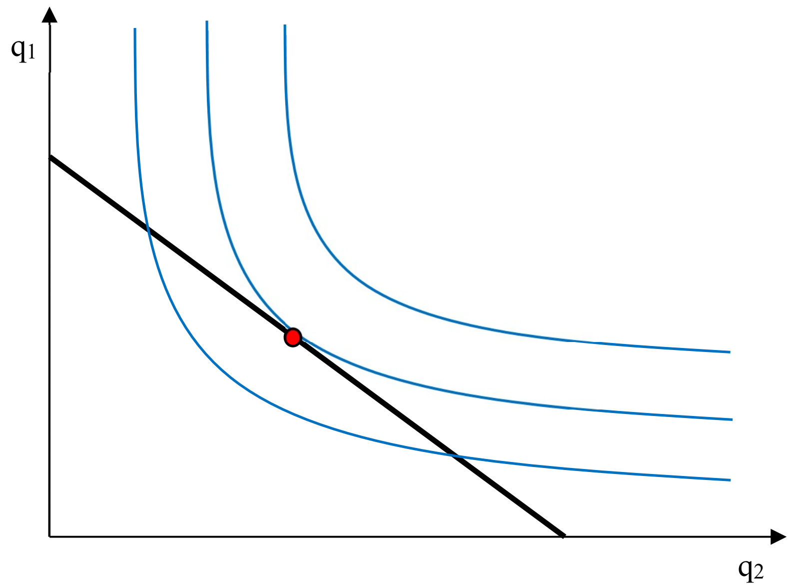


Figure 4.8. Choice at corner of budget set



The tangency condition. If the most preferred bundle lies neither at a kink point nor a corner, it must lie at a *tangency* between the budget line and the highest indifference curve the consumer can reach, as in figure 4.9. At a tangency, the slope of the indifference curve is exactly equal to the slope of the budget line.

Figure 4.9. Choice at tangency between budget line and indifference curve



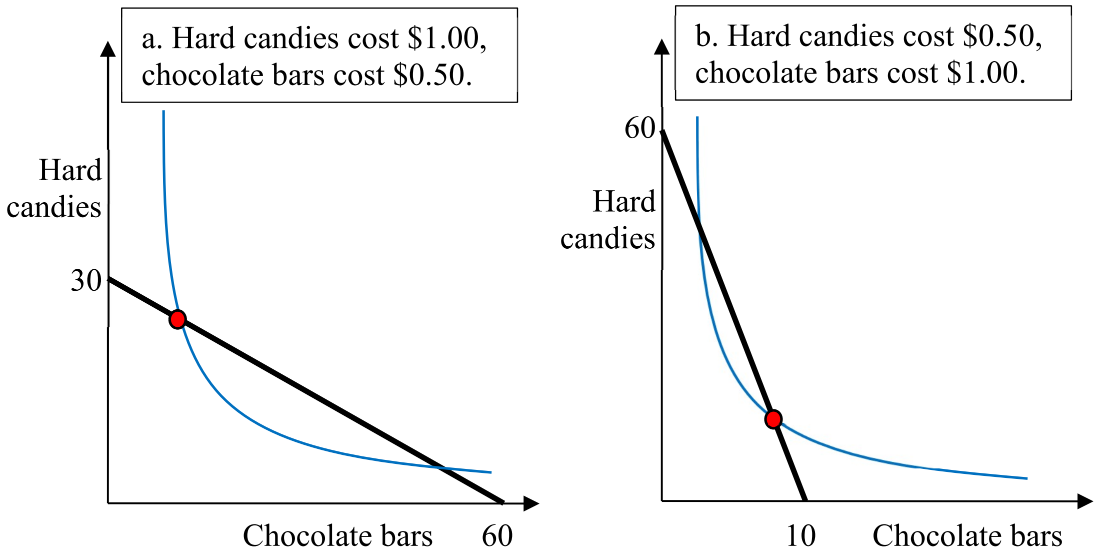
Now, ignoring the minus sign, the slope of the indifference curve is the marginal rate of substitution in consumption (MRSC) of good 2 for good 1, which was earlier shown to equal the ratio of the marginal utilities of good 2 and good 1. Meanwhile, again ignoring the minus sign, the slope of the budget line was earlier shown to equal the ratio of the prices of good 2 and good 1. It follows that the *tangency condition* is described by the following equation.

(4.6) $$\frac{MU_2}{MU_1} = \frac{p_2}{p_1}$$

This condition makes sense because if the equality did not hold, the consumer could find a more preferred bundle. For example, suppose good #1 represented hard candies and good #2 represented chocolate bars and the consumer's MRSC at her or his current bundle were two. Thus this consumer would be willing to give up as many as two hard candies to get one more chocolate bar. Now if the price of hard candies were one dollar and the price of chocolate bars were fifty cents, as in figure 4.10 panel a, the ratio of their prices would be one-half and the consumer need only give up one-half of a hard candy to get one more chocolate bar. Clearly this consumer could find a more preferred bundle, without exceeding her or his budget, by exchanging hard candies for chocolate bars.

Alternatively, suppose hard candies cost fifty cents and chocolate bars cost three dollars, as in figure 4.10 panel b. Now an MRSC of two also means this consumer would be willing to give up as much as one-half of a chocolate bar to get one more hard candy. But at this price, the consumer need only give up one-sixth of a chocolate bar to get one more hard candy. Again, clearly this consumer could find a more preferred bundle, without exceeding her or his budget, by exchanging chocolate bars for hard candies.

Figure 4.10. Hard candies versus chocolate bars



Only when the consumer's MRSC equals its price ratio will the consumer find no opportunities to reach a more preferred bundle.

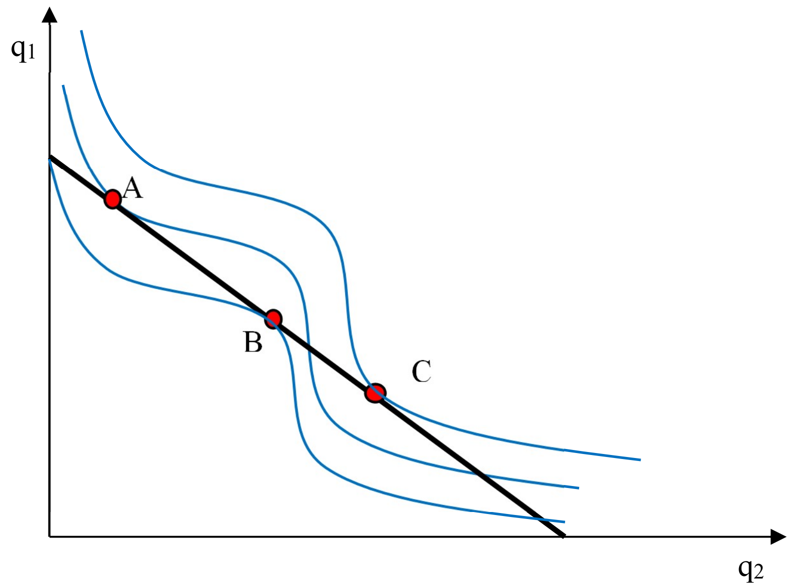
Example: Suppose a consumer has the multiplicative utility function $U(q_1,q_2) = q_1\ q_2$, where q_1 denotes the price of pizzas and q_2 denotes the quantity of movie admissions. If the price of pizzas is $p_1 = \$10$ and the price of movie admissions is $p_2 = \$5$, then the tangency condition is $MU_2/MU_1 = q_1/q_2 = 5/10 = 1/2$.

Example: Suppose a consumer has the Cobb-Douglas utility function $U(q_1,q_2) = q_1^3 q_2^2$. The MRS of good 2 for good 1 is $MU_2/MU_1 = (2q_1^3 q_2)/(3q_1^2 q^2) = (2q_1)/(3q_2)$. If $p_1 = 3$ and $p_2 = 4$, then the tangency condition is $(2q_1)/(3q_2) = 4/3$.

It must be admitted that, under peculiar situations, the tangency condition could hold for bundles that are not the most preferred, as in figure 4.11. Here, bundles A and B meet the tangency condition, but clearly bundle C is preferred to both because it lies on a higher indifference curve. However, this ambiguous situation arises only because these indifference curves curl back and forth. If we maintain the assumption of diminishing marginal rates of substitution, such indifference curves are impossible.

Calculating a consumer's choice. Suppose we want to calculate how much of each good a consumer will choose, given the consumer's utility function and income and the prices in the market. To calculate the consumer's choice graphically, we could plot the budget line and as many indifference curves as possible on graph paper. The consumer's choice would be given by the point on the budget line touching the highest attainable indifference curve, as in figure 4.9. But this graphical method is likely to be cumbersome and inaccurate.

Figure 4.11. Sometimes tangency points are not the best choice



An easier, more accurate way uses algebra and calculus. We substitute the income and prices into the budget line (equation (4.4)), find formulas for the marginal utilities and substitute them into the tangency condition (equation (4.6)), and then solve the two equations together. This method is straightforward if the MRSC is not too complicated.

Example: Suppose a consumer has the multiplicative utility function $U(q_1,q_2) = q_1\, q_2$, where q_1 denotes the quantity of pizzas and q_2 denotes the quantity of movie admissions. The price of pizzas is $p_1 = \$10$ and the price of movie admissions is $p_2 = \$5$. This consumer has an income of \$200. Given these assumptions, the budget line is given by $10q_1+5q_2=200$, and the tangency condition is given by $MU_2/MU_1 = q_1/q_2 = 5/10$. Solving budget line and the tangency condition together gives $q_1{*} = 10$ pizzas and $q_2{*} = 20$ movie admissions.

Example: Suppose a consumer with the same multiplicative utility function $U(q_1,q_2) = q_1\, q_2$ has an income of \$180 and faces a price of pizzas of $p_1 = \$5$, and a price of movie admissions of $p_2 = \$6$. Given these assumptions, the budget line is given by $5q_1+6q_2=180$, and the tangency condition is given by $MU_2/MU_1 = q_1/q_2 = 6/5$. Solving budget line and the tangency condition together gives $q_1{*} = 18$ pizzas and $q_2{*} = 15$ movie admissions. This example shows that a change in prices and income prompts the consumer to make a new choice.

Example: Suppose a consumer has the Cobb-Douglas utility function $U(q_1,q_2) = q_1^2 q_2^3$. Suppose this consumer faces prices of $p_1=\$10$ and $p_2=\$5$ and has an income of \$200. Given these assumptions, the budget line is given by $10q_1+5q_2=200$, and the tangency condition is given by $MU_2/MU_1 = (3q_1)/(2q_2) = 5/10$. Solving budget line and the tangency condition together gives $q_1{*} = 8$ pizzas and $q_2{*} = 24$ movie admissions. This example shows that consumers facing the same budget line will make different choices if they have different preferences.

Example: Suppose a consumer has the utility function $U(q_1,q_2) = (-1/q_1) + (-2/q_2)$, where q_1 denotes the quantity of video rentals and q_2 denotes the quantity of concert tickets. The price of video rentals is \$2 , the price of concert tickets is \$9 , and the consumer has an income of \$120. Given these assumptions, the consumer's budget line is given by $2q_1+9q_2=120$ and the tangency condition is given by $MU_2/MU_1 = 2\,(q_1/q_2)^2 = 9/2$. Dividing both sides of the tangency condition by 2 and multiplying both sides by q_2^2 yields $q_1^2 = (9/4)q_2^2$. Taking square roots of both sides yields $q_1 = 1.5\,q_2$. Substituting this into the budget line and solving yields $q_2^* = 10$ concert tickets. Substituting this into either the budget line or the tangency condition yields $q_1^* = 15$ video rentals.

Broader implications of the tangency condition. If every consumer faces the same prices then every consumer's budget line will have the same slope, even though intercepts will vary because of differences in income. Differences in income and differences in preferences will lead consumers to make different choices. Nevertheless, each consumer will choose a bundle where her or his MRSC equals the price ratio p_2/p_1 . It follows that all consumers will have equal marginal rates of substitution in consumption, and the rates at which consumers are willing to trade one good for another will be equal. This suggests that there are no potential gains from trade among consumers. We will investigate this issue further when we study general equilibrium.

Section 4.3: Summary

The budget line shows all bundles of goods that are just affordable given the consumer's income. The most preferred bundle that a consumer can afford will lie on this line. If that bundle does not lie at a kink point of the indifference curve nor at a corner of the budget set, then it will lie at a point of tangency between the budget line and the highest indifference curve the consumer can reach. Given the consumer's utility function, the consumers income, and the prices of goods, one can find the consumer's most preferred affordable bundle by solving the budget line and the tangency condition together.

Problems

(4.1) [Budget line] Suppose a consumer has $50 to spend on hamburgers and mini-pizzas this month. Hamburgers cost $3 and mini-pizzas cost $4. Consider this consumer's budget line.

a. Let q_1 denote the number of hamburgers and q_2 denote the number of mini-pizzas. Give an equation for the consumer's budget line. [Hint: income = spending.]
b. Which of the following bundles are just affordable? Which are affordable with money left over? Which are not affordable?
   Bundle (i), consisting of 5 hamburgers and 5 mini-pizzas.
   Bundle (ii) consisting of 10 hamburgers and 5 mini-pizzas.
   Bundle (iii) consisting of 6 hamburgers and 8 mini-pizzas.
   Bundle (iv) consisting of 8 hamburgers and 8 mini-pizzas.

(4.2) [Budget line] Suppose a consumer has $80 to spend on movie tickets and muffins this month. Movie tickets cost $5 and muffins cost $4. Consider this consumer's budget line.

a. Let q_1 denote the number of movie tickets and q_2 denote the number of muffins. Give an equation for the consumer's budget line. [Hint: income = spending.]
b. Compute the intercept of the budget line on the "movie ticket" axis.
c. Compute the intercept of the budget line on the "muffin" axis.
d. Find the slope of the budget line when movie tickets are on the vertical axis and muffins are on the horizontal axis.

(4.3) [Intertemporal budget line] Suppose the consumer has $500 to spend on consumption now (c_1) or consumption next year (c_2). Any money not spent now can be deposited in a bank account, accumulating interest at an annual rate of 3%, and withdrawn for spending next year. Consider the budget line defining the tradeoff between consumption now and consumption next year.

a. Compute the intercept of the budget line on the c_1 axis.
b. Compute the intercept of the budget line on the c_2 axis.
c. Give an equation for the consumer's intertemporal budget line. [Hint: Any format is acceptable—you do not need to put 500 on the left side of the equation. However, the only variables should be c_1 and c_2.]
d. Find the slope of the budget line when c_1 is on the vertical axis and c_2 is on the horizontal axis. This is the opportunity cost of $1 of consumption tomorrow (c_2) in terms of foregone consumption today (c_1). [Hint: Use your answer to part (c) above. Solve for c_1 and find the coefficient of c_2.]

(4.4) [Intertemporal budget line] Suppose a consumer has no income now, but expects $600 in income next year. The consumer can borrow at an annual interest rate of 20%. Let c_1 denote consumption spending now and c_2 denote consumption spending next year.

a. Given the consumer's expected income next year, what is the maximum amount the consumer can borrow now? This is the intercept of the consumer's budget line on the c_1 axis. [Hint: Amount borrowed times 1.20 equals amount to be repaid.]
b. If the consumer consumed nothing now, and therefore borrowed nothing, how much could the consumer spend next year? This is the intercept of the consumer's budget line on the c_2 axis.
c. Give an equation for the consumer's intertemporal budget line. [Hint: Any format is acceptable—you do not need to put $600 on the left side of the equation. However, the only variables should be c_1 and c_2.
d. Find the slope of the budget line when c_1 is on the vertical axis and c_2 is on the horizontal axis. This is the opportunity cost of $1 of consumption tomorrow (c_2) in terms of foregone consumption today (c_1). [Hint: Use your answer to part (c) above. Solve for c_1 and find the coefficient of c_2.]

(4.5) [Budget line] Consider the impact on a consumer's budget constraint of each scenario below. Indicate whether the impact is a *parallel shift* in the budget line, a *rotation* of the budget line, or *no change* in the budget line. Also indicate whether the budget line moves *closer* to the origin or *farther away* from the origin.

a. Income increases by 20%.
b. The price of one good increases by 20%.
c. The price of both goods increase by 20%.
d. The price of one good increases by 20% and income simultaneously increases by 20%.
e. The prices of both goods increase by 20% and income simultaneously increases by 20%.

(4.6) [Kinked budget line] Suppose oranges cost $4 per pound for the first 5 pounds, but, due to a special discount program, additional oranges cost only $1 per pound. Assume that apples always cost $2 per pound and that the consumer has $30 income. Plot the consumer's budget constraint. [Hint: This budget constraint has a kink where the quantity of oranges equals 5 pounds.]

(4.7) [Kinked budget line] Suppose a consumer can enjoy a reduced price on food by paying an up-front annual membership fee at a discount food store. The consumer has a total income of $1000. Without a discount, the usual price of food is $2 per unit. If the consumer pays a fee of $200, then the price of food is only $1 per unit. The price of other goods is always $1 per unit.

a. Give an equation for the consumer's budget line *without* the discount. Sketch the budget line or describe it in words. Compute the intercepts. Compute the slope when food is on the horizontal axis.
b. Give an equation for the consumer's budget line *with* the discount. [Hint: Treat the membership fee as a loss of income.] Sketch the budget line or describe it in words. Compute the intercepts. Compute the slope when food is on the horizontal axis.

(4.8) [Kinked budget line] Suppose a consumer enjoy a "frequent-customer" discount on DVD rentals. The first five rentals in a month cost $5 each. Additional rentals cost only $3 each. The consumer has an entertainment budget of $100 per month and other entertainment goods cost $1 each. Note that this change in price after the fifth rental causes a kink in the budget line. Consider the graph of this budget line with DVD rentals on the horizontal axis.

a. What is the maximum number of other goods this consumer could afford, if they never rented DVDs?
b. What is the maximum number of video rentals this consumer could afford, if they spent their entire budget on DVD rentals?
c. Compute the coordinates of the kink point. [Hint: How many other goods could the consumer afford if they purchased five DVD rentals?]
d. Compute the slope of the budget line, with DVD rentals on the horizontal axis, when the consumer rents fewer than five DVDs.
e. Compute the slope of the budget line, with DVD rentals on the horizontal axis, when the consumer rents more than five DVDs.
f. Sketch the budget line or describe it in words.
g. From your sketch of the budget line, do you think anyone would ever choose to rent exactly five DVDs per month? Why or why not?

(4.9) [Choice] Suppose the price of beans is p_1=$5 and the price of potatoes is p_2=$2. Find equations for the tangency conditions for each of the following consumers. Which consumers will make the same choices if they have the same income? Explain your reasoning. [Hint: Compare the tangency conditions for these consumers.]

a. Anne, whose utility function is $U(q_1,q_2) = q_1^2 q_2$.
b. Bill, whose utility function is $U(q_1,q_2) = q_1^{2/3} q_2^{1/3}$.
c. Carol, whose utility function is $U(q_1,q_2) = (q_1-2)(q_2-1)$.

(4.10) [Choice] Suppose a consumer has \$150 to spend on food and clothing. Food costs \$4 per unit and clothing costs \$5 per unit. The consumer's utility function is $U(q_1,q_2) = q_1^2 q_2$, where q_1 denotes the quantity of food and q_2 denotes the quantity of clothing.

a. Give an equation for the consumer's budget line. [Hint: income = spending.]
b. Give a formula for the consumer's marginal rate of substitution in consumption (MRSC) of clothing for food. [Hint: This is the slope of the consumer's indifference curve when food is on the vertical axis and clothing is on the horizontal axis.]
c. Compute the quantities of food and clothing that this consumer will choose. Show your work and circle your final answers.

(4.11) [Choice] Suppose a consumer has \$310 to spend on energy and other goods. Energy costs \$3 per unit and other goods cost \$2 per unit. The consumer's utility function is $U(q_1,q_2) = q_1(q_2-5)$, where q_1 denotes the quantity of energy and q_2 denotes the quantity of other goods.

a. Give an equation for the consumer's budget line. [Hint: income = spending.]
b. Give a formula for the consumer's marginal rate of substitution in consumption (MRSC) of other goods for energy. [Hint: This is the slope of the consumer's indifference curve when energy is on the vertical axis and other goods are on the horizontal axis.]
c. Compute the quantities of energy and other goods that this consumer will choose. Show your work and circle your final answers.

(4.12) [Choice] Suppose the consumer in the previous problem enjoys an increase in income to \$370. There is no change in prices. Compute the quantities of energy and other goods that this consumer will now choose. Show your work and circle your final answers.

(4.13) [Choice] Suppose a consumer has \$210 to spend on health care and other goods. Health care costs \$9 per unit and other goods cost \$8 per unit. The consumer's utility function is $U(q_1,q_2) = -(5/q_1) - (10/q_2)$, where q_1 denotes the quantity of health care and q_2 denotes the quantity of other goods.

a. Give an equation for the consumer's budget line. [Hint: income = spending.]
b. Give a formula for the consumer's marginal rate of substitution in consumption (MRSC) of other goods for health care. [Hint: This is the slope of the consumer's indifference curve when health care is on the vertical axis and other goods is on the horizontal axis.]
c. Compute the quantities of health care and other goods that this consumer will choose. Show your work and circle your final answers.

(4.14) [Choice] Suppose a consumer has \$60 to spend on food and other goods. Food costs \$2 per unit and other goods cost \$4 per unit. The consumer's utility function is $U(q_1,q_2) = q_1^{1/2} + q_2^{1/2}$, where q_1 denotes the quantity of food and q_2 denotes the quantity of other goods.

a. Give an equation for the consumer's budget line. [Hint: income = spending.]
b. Give a formula for the consumer's marginal rate of substitution in consumption (MRSC) of other goods for food. [Hint: This is the slope of the consumer's indifference curve when food is on the vertical axis and other goods is on the horizontal axis.]
c. Compute the quantities of food and other goods that this consumer will choose. Show your work and circle your final answers.

(4.15) [Choice with corner solutions] Suppose a consumer has income of \$60 to spend on sodapop. This consumer has utility function $U(q_1,q_2) = q_1 + 2q_2$, where q_1 denotes the number of small bottles of sodapop and q_2 denotes the number of large bottles of sodapop consumed.

a. Draw the consumer's indifference curves when $U = 20$, 30, or 40.
b. On the same graph, but in a different color, draw the consumer's budget line when $p_1 = \$2$ and $p_2 = \$3$. How many large bottles and how many small bottles will this consumer choose? Explain your reasoning. [Hint: Calculus is useless for this problem because the solution is not a tangency. Instead, just study your graph.]
c. On the same graph, but in a different color, draw the consumer's budget line when $p_1 = \$2$ and $p_2 = \$6$. How many large bottles and how many small bottles will this consumer choose? Explain your reasoning.

(4.16) [Intertemporal choice] Suppose the consumer has \$100 to spend on consumption now (c_1) or consumption next year (c_2). Any money not spent now can be deposited in a bank account, accumulating interest at an annual rate of 10%, and then withdrawn for spending next year. Therefore, the consumer's budget constraint is $c_2 = (100-c_1)\ 1.10$.

a. Find the slope of the consumer's budget line when c_1 is on the vertical axis and c_2 is on the horizontal axis. [Hint: Solve the budget line for c_1 and find the coefficient of c_2.]

Assume the consumer has the multiplicative utility function $U(c_1,c_2) = c_1\ c_2$.

b. Find the marginal rate of substitution of c_2 for c_1. [Hint: This is the slope of the consumer's indifference curve when c_1 is on the vertical axis c_2 is on the horizontal axis.]
c. Compute the quantities of c_1 and c_2 that this consumer will choose. Show your work and circle your final answers.

(4.17) [Choice, finance] Suppose an investor has the utility function $U(R,\sigma) = R - 0.03\sigma^2$. (Here, R denotes the expected rate of return (R) of their investment portfolio and σ denotes the risk associated with that portfolio, but this information is not necessary to solve this problem.)

a. Find a formula for the marginal rate of substitution in consumption (MRSC) of σ for R. [Hint: This is the absolute value of the slope of the indifference curve, when R is on the vertical axis and σ is on the horizontal axis.]

According to the Capital Asset Pricing Model, if there is a risk-free asset with a return of 4 percent, and if the market return is 10 percent with a standard deviation of 20 percent, then the investor faces a constraint of $R = 4 + (10\text{-}4)/20\ \sigma$ or $R = 4 + 0.3\ \sigma$.

b. Compute the slope of the constraint, $dR/d\sigma$.
c. Compute the values of R and σ that this investor will choose. [Hint: Set your answer to (a) equal to your answer to (b) and solve for σ. Then insert this into the constraint to find R.]

In reality, the investor cannot purchase R and σ directly. Instead, the investor purchases the risk-free asset and the market portfolio. The investor's R and σ are then weighted averages of the values for the two assets. In particular, let w equal the fraction of the investor's wealth invested in the risk-free asset, and thus (1-w) equal the fraction of the investor's wealth invested in the market portfolio. Then

$$R = 4\ w + 10\ (1\text{-}w) \quad \text{and} \quad \sigma = 0\ w + 20\ (1\text{-}w).$$

d. Compute w, the fraction of wealth that the investor will devote to the risk-free asset, and (1-w), the fraction that the investor will devote to the market portfolio. Show your work and circle your final answers.

(4.18) [Choice, finance] Suppose an investor has the utility function $U(R,\sigma) = R - 0.01\sigma^2$. (Here, R denotes the expected rate of return (R) of their investment portfolio and σ denotes the risk associated with that portfolio, but this information is not necessary to solve this problem.)

a. Find a formula for the marginal rate of substitution in consumption (MRSC) of σ for R. [Hint: This is the absolute value of the slope of the indifference curve, when R is on the vertical axis and σ is on the horizontal axis.]

According to the Capital Asset Pricing Model, if there is a risk-free asset with a return of 4 percent, and if the market return is 10 percent with a standard deviation of 20 percent, then the investor faces a constraint of $R = 4 + (10\text{-}4)/20\ \sigma = 4 + 0.3\ \sigma$.

b. Compute the slope of the constraint, $dR/d\sigma$.
c. Compute the values of R and σ that this investor will choose. [Hint: Set your answer to (a) equal to your answer to (b) and solve for σ. Then insert this into the constraint to find R.]

In reality, the investor cannot purchase R and σ directly. Instead, the investor purchases the risk-free asset and the market portfolio. The investor's R and σ are then weighted averages of the values for the two assets. In particular, let w equal the fraction of the investor's wealth invested in the risk-free asset, and thus $(1\text{-}w)$ equal the fraction of the investor's wealth invested in the market portfolio. Then

$$R = 4\ w + 10\ (1\text{-}w) \quad \text{and} \quad \sigma = 0\ w + 20\ (1\text{-}w).$$

d. Compute w, the fraction of wealth that the investor will devote to the risk-free asset, and $(1\text{-}w)$, the fraction that the investor will devote to the market portfolio. Show your work and circle your final answers.

(4.19) [Minimizing spending] Suppose a person has utility function $U(q_1,q_2) = q_1\, q_2^2 / 100^3$, where q_1 denotes housing (in square feet) and q_2 denotes units of food. Suppose housing costs \$1 per square foot, and food costs \$5 per unit. We want to find the minimum total spending required to raise this person's utility $U(q_1,q_2)$ to a given target level. This is a tangency, of course, but here the total spending is unknown rather than given.

a. Give a formula for the consumer's marginal rate of substitution in consumption (MRSC) of food for housing. [Hint: This is the slope of the consumer's indifference curve when housing is on the vertical axis and food is on the horizontal axis.]
b. Give the equation for the person's indifference curve when U equals 20 utils. [Hint: Just set the person's utility function equal to 20.]
c. Compute the quantities of housing (q_1) and food (q_2) that this person should choose to minimize spending while reaching $U = 20$ utils. Show your work and circle your final answers. [Set the MRSC equal to the price ratio. Solve this equation jointly with your answer to part (b).]
d. Compute the required total spending (in dollars) to reach $U = 20$ utils.

[end of problem set]