LECTURE NOTES ON MICROECONOMICS

ANALYZING MARKETS WITH BASIC CALCULUS

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Part 2: Consumers and demand

Chapter 3: Preferences and utility

Section 3.1: Preferences and indifference curves

Bundles. Consumers face a wide variety of types of goods and services—such as bread, electricity, sweatshirts, basketball tickets, restaurant meals, doctor visits, and so forth. In any given period, the consumer must choose a combination or bundle of quantities of each type of good. For example, a particular bundle might consist of 3 loaves of bread, 1500 kilowatt-hours of electricity, 2 sweatshirts, 4 basketball tickets, 3 restaurant meals, 2 doctor visits, and so forth. To represent bundles compactly, we will simply list the quantities. So the bundle just described would be represented as (3, 1500, 2, 4, 3, 2, …). More generally, if there are n goods, a bundle would be represented as $(q_1, q_2, \ldots, q_n)$, where q_1 denotes the quantity of the first good, q_2 denotes the quantity of the second good, and so forth until q_n denotes the quantity of the last good.

In developed countries, the number of types of goods is enormous. But most of the key issues of consumer behavior can be seen clearly with just two goods.[1] Moreover, the two-good case is easy to graph. For two goods, bundles can be represented simply as (q_1, q_2).

Key assumptions. Faced with a budget line, a consumer chooses the affordable bundle that she or he most prefers. While individuals' preferences differ according to personal taste, everyone's preferences appear to have some common features that might be loosely described as being "sensible" or "logically consistent." The following three assumptions are intended to capture these common features.

Assumption #1: Transitivity. If a consumer prefers bundle X to bundle Y, and prefers bundle Y to bundle Z, then that consumer prefers bundle X to bundle Z.

This assumption implies that a person can put all available bundles into a unique rank ordering, from most preferred to least preferred. The assumption of transitivity is essential if we believe consumers are capable of making "logically consistent" decisions.

[1] A common approach is to define one good as the good of interest (such as energy or health care) and lump all other goods into a "composite commodity." This approach was formally justified by J.R. Hicks, *Value and Capital: An Inquiry Into Some Fundamental Principles of Economic Theory*, 2nd edition, Oxford, United Kingdom: Oxford University Press, 1946, page 33.

Of course, some bundles might be equally preferred. For example, some consumer might find the following bundles equally attractive: bundle A consisting of 6 movie tickets and 3 restaurant meals, and bundle B consisting of 3 movie tickets and 4 restaurant meals.

A useful way to display preferences graphically is to draw curves connecting all equally-preferred bundles. (To do so requires some further technical assumptions, such as that fractional amounts of goods may be purchased. We will assume these all hold.) Such curves are called *indifference curves*, because the consumer is indifferent between any two bundles on the same curve. By contrast, if two bundles are on separate indifference curves, then the consumer prefers one to the other. Figure 3.1 shows an example of an indifference curve for the consumer of the previous paragraph.

Assumption #2: Monotonicity. If bundle X includes more of some good than bundle Y, and at least as much of all other goods, then any consumer prefers bundle X to bundle Y.

This assumption implies that "more is better." If two bundles are the same, except that one bundle contains more of a particular good than the other bundle, the consumer will surely prefer the first bundle according to this assumption. But is more really always better? Surely consumers do not want more garbage, more pollution, or more noise! Nevertheless consumers do not buy these items. If we confine our attention to goods that consumers actually buy, the assumption of monotonicity is reasonable.

Example: Consider the bundle A consisting of 10 units of food and 5 units of clothing: (10,5). Use the assumption of monotonicity to determine whether the following bundles are more or less preferred to bundle A: B=(11,5), C=(8,8), D=(12,7), E=(6,5), F=(6,3), G=(12, 3). Bundle B is *more preferred* because it has the same amount of clothing but more food. Bundle C *cannot be determined* without more information of the person's preferences because it has less food but more clothing. Bundle D is *more preferred* because it has more of both goods. Bundle E is *less preferred* because it has less food although the same amount of clothing. Bundle F is *less preferred* because it has less of both goods. Bundle G *cannot be detemined* without more information about the person's preferences because it has more food but less clothing.

Shape of indifference curves. The graphical interpretation of monotonicity is shown in figure 3.2. All bundles above and to the right of bundle A are preferred to bundle A, because they contain more of good #1 and/or more of good #2.

Note that if monotonicity holds, then figure 3.1 is incorrect. Bundle C contains more movie tickets and more restaurant meals than bundle B. Yet the bundles are alleged to be equally preferred, because they lie on the same indifference curve. Monotonicity implies that indifference curves *must slope down*, not up.

If monotonicity holds, then figure 3.3 is also incorrect. In this figure, bundle A is equally preferred by this consumer to bundle B, because they lie on the same indifference curve. Similarly, bundle B is equally preferred to bundle C, so it would seem that bundle A is *equally preferred* to bundle C. But bundle C contains more movie tickets and more restaurant meals than bundle A so bundle C must be *strictly preferred*, not equally preferred, to bundle A. Thus crossing indifference curves would imply a contradiction. Thus monotonicity implies that indifference curves *cannot cross*.

Figure 3.1. Example of an indifference curve

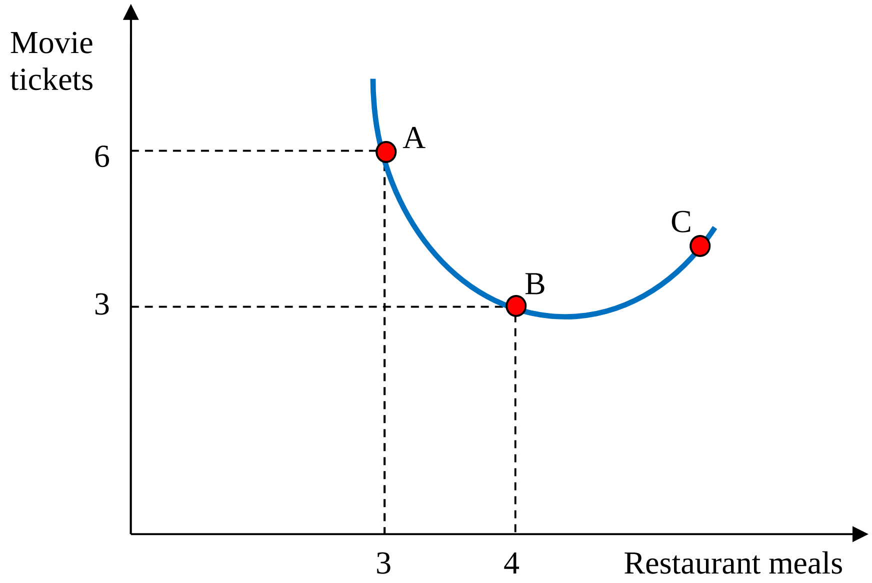


Figure 3.2. Implications of monotonicity

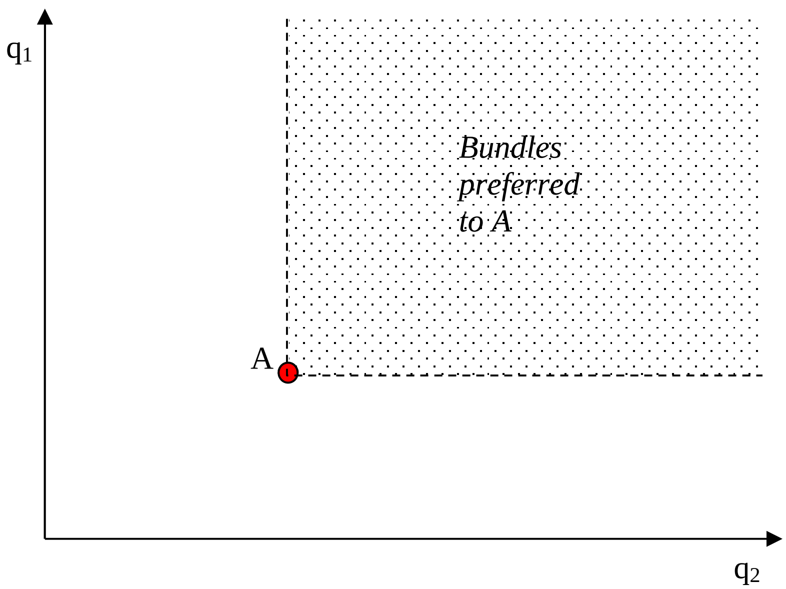
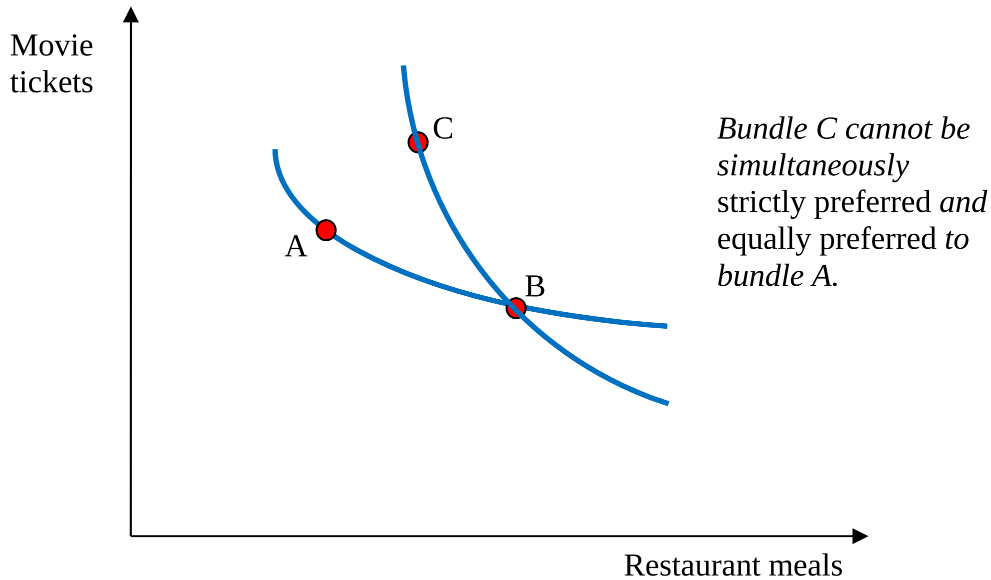


Figure 3.3. Why indifference curves cannot cross



To see the general implications of monotonicity, suppose bundle A is located on some indifference curve as in figure 3.4. Then all bundles above and to the right of the entire indifference curve--even if they happen to be below or to the left of bundle A --are preferred to bundle A. This is because all such bundles are above and to the right of *some* bundle on the indifference curve which is equally preferred to A .

Some special preferences. Suppose two goods are always equally desirable to a consumer. For example, suppose a consumer likes Coke and Pepsi equally and would willingly trade a can of one for a can of the other. Thus the consumer is indifferent between the following bundles: three cans of Coke only, two cans of Coke and one can of Pepsi, one can of Coke and two cans of Pepsi, and three cans of Pepsi. Plotting and connecting these bundles gives an indifference curve that forms a straight, downward-sloping line (see figure 3.5). In this example, the consumer is willing to substitute Coke for Pepsi at a constant rate (in this case, one-for-one). To this consumer, Coke and Pepsi are *perfect substitutes*.

By contrast, suppose a consumer requires two goods in fixed proportion. For example, consider left and right shoes. Extra left shoes have no value. Neither do extra right shoes. Thus the consumer is indifferent between the following bundles: three left shoes and two right shoes, two left shoes and two right shoes, and two left shoes and three right shoes. Plotting and connecting these bundles gives an indifference curve that forms an L-shaped curve (see figure 3.6). In this example, the consumer is unwilling to substitute left shoes for right shoes. To this consumer, left and right shoes are *perfect complements*.

Figure 3.4. General implications of monotonicity

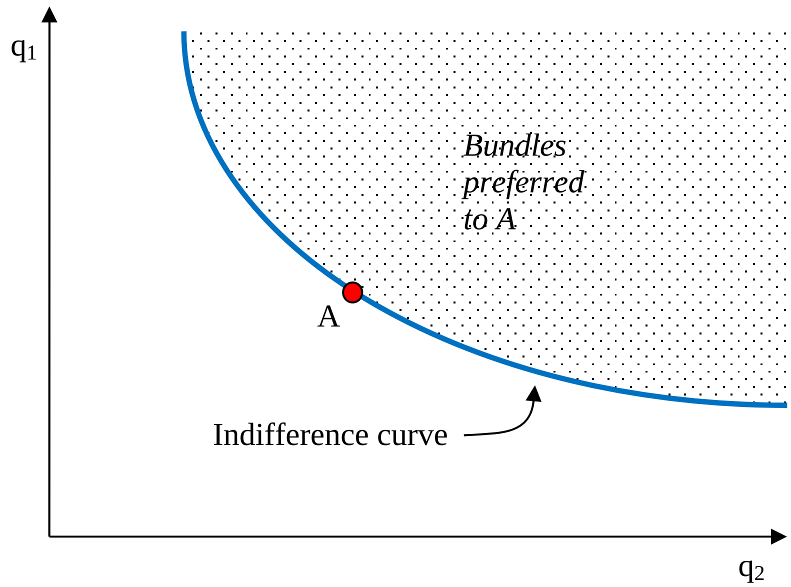


Figure 3.5. An indifference curve for perfect substitutes

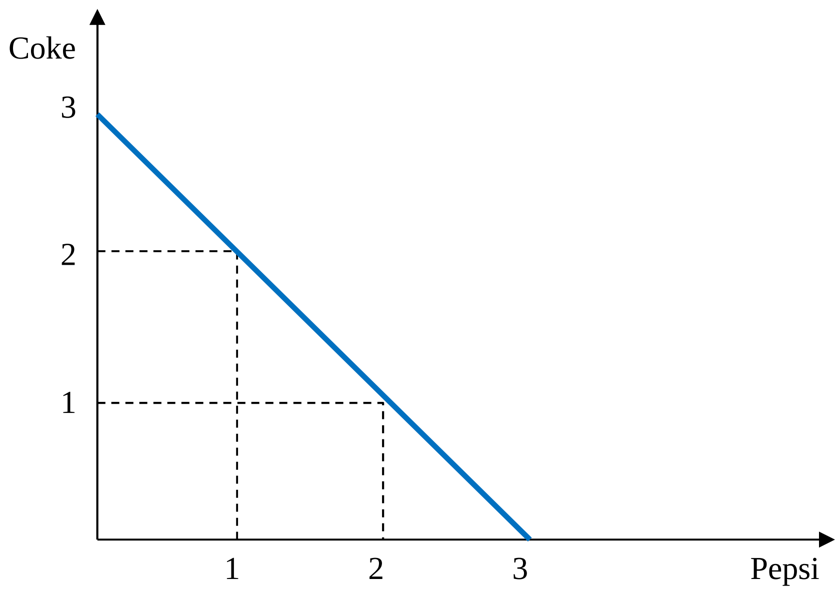
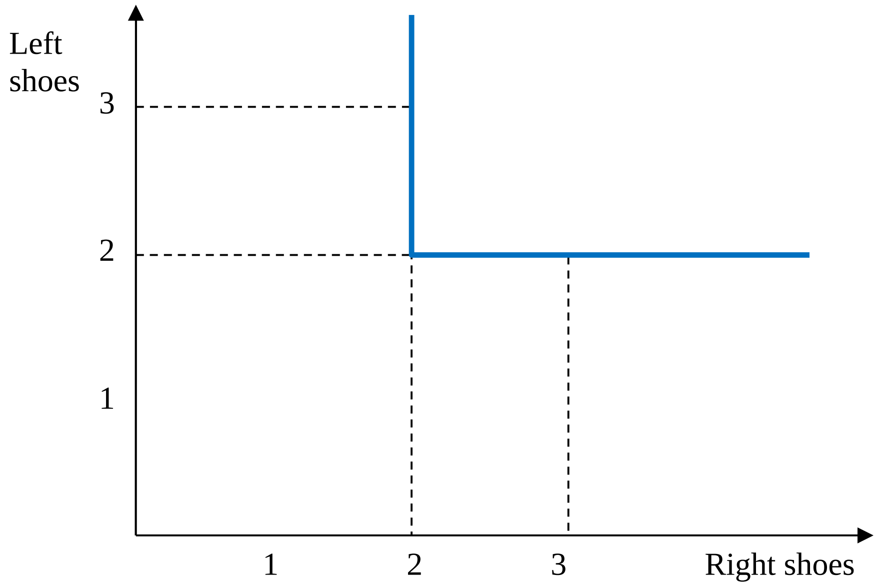


Figure 3.6. An indifference curve for perfect complements

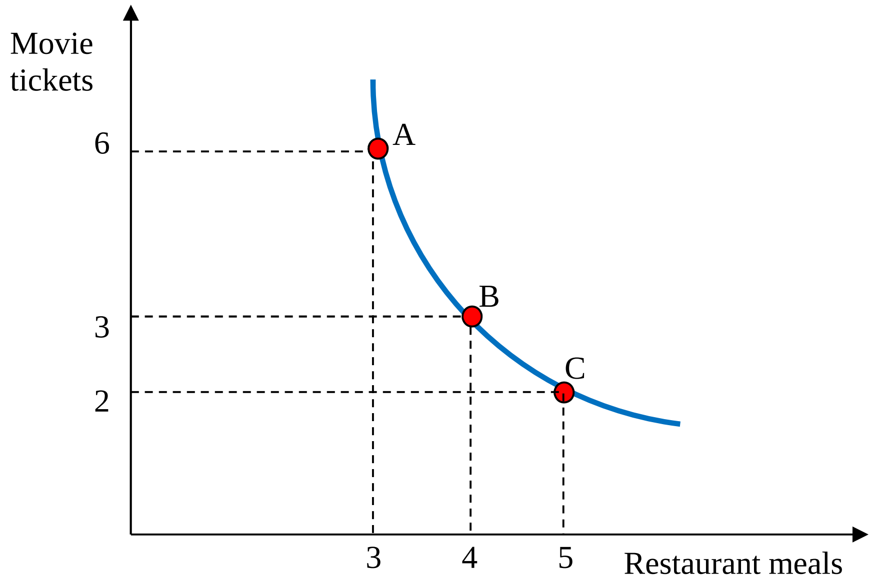


The slope of indifference curves. Any two points on the same indifference curve represent equally-preferred bundles. In general, the slope of the indifference curve between them shows the rate at which one good can be swapped for another without any loss of well-being, in the consumer's eyes. For example, suppose bundle A, consisting of 6 movie tickets and 3 restaurant meals, is equally preferred to bundle B, consisting of 3 movie tickets and 4 restaurant meals, as shown in figure 3.7. If the consumer already had bundle A, she or he would swap three movie tickets for one restaurant meal without any loss of well-being.

In general, the slope of the indifference curve ($\Delta q_1/\Delta q_2$, with good #1 on the vertical axis and good #2 on the horizontal axis) represents the rate at which the consumer will swap one good for another without any loss of well-being. If the slope is steep, the consumer is willing to give up many units of good #1 for one more unit of good #2. By implication, the consumer does not value good #1 very much compared to the good #2. In contrast, if the slope is flat, the consumer is only willing to give up very small amounts of good #1 for one more unit of good #2. By implication, the consumer values good #1 highly compared to good #2. The absolute value of this slope is called the *marginal rate of substitution in consumption* (MRSC). For example, the MRSC between bundles A and B in figure 3.7 is 3.

Assumption #3: Diminishing MRSC. As we move down any indifference curve, adding more of good #2 and taking away more of good #1, the MRSC diminishes. In graphical terms, as we move down any indifference curve, it gets flatter. So indifference curves are curved away from the origin.

Figure 3.7. The slope of an indifference curve

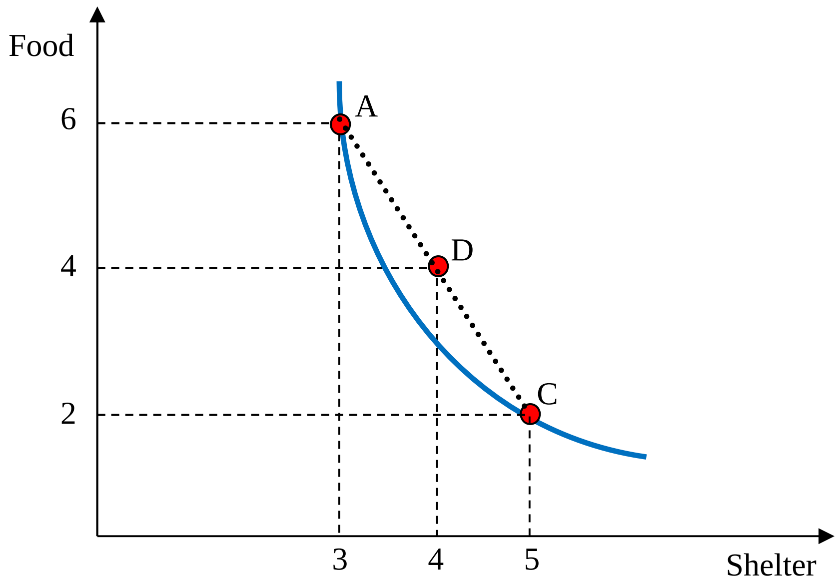


This assumption implies that, as a good becomes relatively scarcer in the consumer's bundle, it becomes relatively more valuable to the consumer. For example, suppose the consumer originally enjoying bundle A has swapped three movie tickets for one restaurant meal to arrive at bundle B. Now the consumer has only three movie tickets and will be more reluctant to give them up. In fact, figure 3.7 shows that the consumer would only be willing to give up one movie ticket to get a fifth restaurant meal. The MRSC between bundles B and C is only 1.

This assumption also implies that *averages are preferred to extremes*. The average of two bundles is the midpoint of the line connecting them. In figure 3.8, for example, the average of the two bundles $A=(6,3)$ and $C=(2,5)$ is $D=(4,4)$. (Here, the first number in each pair refers to the quantity of good #1 and the second number refers to the quantity of good #2.) Although bundles A and C lie on the same indifference curve, bundle D lies above it because of the indifference curve is curved away from the origin. In general, if two bundles are equally preferred, their average (midpoint) will be more preferred.

Example: Suppose bundle A has 3 units of health care and 20 units of other goods, bundle B has 5 units of health care and 10 units of other goods, and bundle C has 4 units of health care and 15 units of other goods. If bundles A and B are equally preferred, is bundle C more or less preferred than A and B? The assumption of monotonicity is no help here because bundle C has more health care but fewer other goods than bundle A. Similarly, bundle C has less health care but more other goods than bundle B. However, notice that bundle C is a simple average of bundles A and B because $(4,15)=\left(\tfrac{1}{2}(3+5),\tfrac{1}{2}(20+10)\right)$. If we may assume diminishing MRSC, then bundle C is preferred to A and B, because diminishing MRSC implies that averages are preferred to extremes.

Figure 3.8. Diminishing marginal rate of substitution in consumption



Section 3.2: Utility

Definition of utility. Nineteenth-century philosphers and economists defined utility as a person's *measurable* well-being. They did not know how to measure utility, but they were sure it would be eventually possible. Moreover, they assumed that any bundle confers a particular level of utility on the person that consumes it. Thus, any person should be able to put all available bundles in a unique rank ordering from highest utility to lowest utility. So the concept of utility has the same implications as the concept of preferences--saying that bundle A is preferred to bundle B is like saying that bundle A confers more utility than bundle B.

However, the nineteenth-century concept of utility implies something more than preferences. If utility is measurable (like weight or temperature) then it is a *cardinal* concept, whereas preferences are only an *ordinal* concept. It is meaningful to say "bundle A confers twice as much utility as bundle B," whereas it is not meaningful to say "bundle A is twice as preferred as bundle B." Similarly, it is meaningful to say "the gain in utility from switching from bundle A to bundle B is greater than the gain in utility from switching from bundle B to bundle C," whereas it is not meaningful to say "the gain in preference from switching from bundle A to bundle B is greater than the gain in preference from switching from bundle B to bundle C." This is because preference is just a rank ordering, not a measurable quantity.

The nineteenth-century cardinal concept of utility has fallen out of favor. No one has found a way to measure utility convincingly. Moreover, there is little need to do so. A consumer's choices can be completely predicted using the consumer's preference ordering.

Economists today still refer to "utility" but with caution. They use *utility functions*, which assign a number to each bundle, as a compact way of describing preferences. Nevertheless, economists recognize that utility functions provide too much information. The particular values that utility functions give to bundles are not meaningful except for their rank ordering.

Utility functions. A utility function shows the relative utility or well-being of any bundle to a particular consumer. The arguments of the function are the quantities of all the different goods in the bundle: $U(q_1,q_2,...,q_n)$. For example, some consumer whose possible bundles consist of only two different goods might have the multiplicative utility function $U(q_1,q_2) = q_1q_2$, where q_1 stands for the number of movie tickets and q_2 stands for the number of restaurant meals. For this consumer, a bundle consisting of four movie tickets and five restaurant meals (U=20) is more desirable than a bundle consisting of two movie tickets and eight restaurant meals (U=16).

If consumers have different preferences, those preferences must be represented by different utility functions. Economists have used all of the functions listed in table 2.3, and many others, to model various consumers' preferences for various goods.

Marginal utility functions are the partial derivatives of the utility function. They show the rate at which the consumer's utility increases as the amount of one good in the bundle increases. For example, if a consumer has utility function $U(q_1,q_2) = q_1q_2^2$, then the formula for the marginal utility of good #1 is $MU_1 = \partial U/\partial q_1 = q_2^2$, and the marginal utility of good #2 is $MU_2 = \partial U/\partial q_2 = 2q_1q_2$. Marginal utilities must be positive for the assumption of monotonicity to hold. Of course, the values of marginal utilities are not meaningful, for the reasons discussed above, but *ratios* of the values of marginal utilities are quite meaningful, as we shall soon see.

Utility functions and indifference curves. Bundles that are equally-preferred must yield equal amounts of utility for the consumer. Therefore indifference curves are level curves of the consumer's utility function. Given any utility function $U(q_1,q_2)$, the equation describing the indifference curves is given by

(3.1) $U(q_1,q_2) = \text{constant}.$

Here, the "constant" is larger for higher indifference curves connecting more-preferred bundles, and smaller for lower indifference curves connecting less-preferred bundles. For example, the equation describing the indifference curves for the multiplicative utility function are given by

(3.2) $q_1q_2 = \text{constant},$

which happens to be the equation for a rectangular hyperbola. For this utility function, the following bundles lie all on the same indifference curve and are therefore equally preferred: (1,12), (6,2), (4,3).

Example: Given the utility function $U(q_1,q_2) = q_1^3\ q_2^2$, find formulas for the marginal utilities MU_1 and MU_2 , and determine whether the assumption of monotonicity holds. The marginal utilities are $MU_1 = \partial U/\partial x_1 = 3\ q_1^2\ q_2^2$ and $MU_2 = \partial U/\partial x_1 = 2\ q_1^3\ q_2^1$. These marginal utilities are both positive (provided q_1 and q_2 are positive) so the assumption of monotonicity *does* hold.

Slope of indifference curves. The absolute value of the slope of an indifference curve (the MRSC) can be found just as any MRS is found—as the ratio of the partial derivatives. Thus the general formula for the MRSC of good #2 for good #1 is given by the following:

$$(3.3) \qquad MRSC = \frac{MU_2}{MU_1} = \frac{\partial U / \partial q_2}{\partial U / \partial q_1}.$$

Example: Suppose a consumer has the utility function $U(q_1,q_2) = (q_1q_2)^{1/2}$. Find the equation for the indifference curve that passes through the bundle $(q_1,q_2) = (12,3)$, find the formula for the MRSC corresponding to this utility function, and compute the value of the MRSC at the bundle $(q_1,q_2) = (12,3)$. Now at the bundle $(q_1,q_2) = (12,3)$, the consumer's utility is $(12\times3)^{1/2} = 6$, so the equation for the indifference curve that passes through that bundle is simply $6 = (q_1q_2)^{1/2}$. The formula for the MRSC is given by $\frac{MU_2}{MU_1} = \frac{(1/2)(q_1q_2)^{-1/2}q_1}{(1/2)(q_1q_2)^{-1/2}q_2} = \frac{q_1}{q_2}$. The value of the MRSC at the bundle $(q_1,q_2) = (12,3)$ is 4.

Example: Given the utility function $U(q_1,q_2) = -2q_1^{-2} - 3q_2^{-2}$, find a formula for the MRSC and determine whether the MRSC diminishes as q_1 decreases and q_2 increases. The MRSC is given by $\frac{MU_2}{MU_1} = \frac{6q_2^{-3}}{4q_1^{-3}} = 1.5\left(\frac{q_1}{q_2}\right)^3$. As q_1 decreases and q_2 increases, the expression in parentheses gets smaller. Therefore, since the exponent 3 is positive, the whole MRSC diminishes as q_1 decreases and q_2 increases.

Example: Given the utility function $U(q_1,q_2) = (q_1-5)^2\ (q_2-3)^3$, find a formula for the MRSC and determine whether the MRSC diminishes as q_1 decreases and q_2 increases. (Assume that $q_1 > 5$ and $q_2 > 3$.) The MRSC is given by $\frac{MU_2}{MU_1} = \frac{(q_1-5)^2 3(q_2-3)^2}{2(q_1-5)(q_2-3)^3} = \frac{3(q_1-5)}{2(q_2-3)}$. As q_1 decreases and q_2 increases, the numerator shrinks and the denominator expands, so the whole MRSC diminishes.

Utility function not unique. It should be noted that many different utility functions can yield the same MRSC formula. For example, the utility functions $U(q_1,q_2) = (q_1q_2)^{1/2}$, $V(q_1,q_2) = (q_1q_2)$, and $W(q_1,q_2) = \ln(q_1q_2)$ all have the MRSC formula (q_1/q_2). We will see in the next chapter that the MRSC formula contains all the information needed to predict a consumer's choices. An implication is that many different utility functions predict exactly the same choices. Again we see that utility functions provide more information than is needed to predict consumer choices.

Section 3.3: Summary

While individuals' preferences differ according to personal taste, they are usually characterized by transitivity, monotonicity, and diminishing marginal rates of substitution. We can represent preferences graphically using indifference curves, which connect equally-preferred bundles of goods. Alternatively, we can represent preferences using a utility function, which shows the level of utility or well-being a consumer enjoys from any bundle of goods. These approaches are equivalent in that indifference curves are simply level curves of the utility function, whose slopes are given by the marginal rate of substitution in consumption (MRSC). The values that utility functions give to bundles are not meaningful except for their rank ordering. Moreover, many different utility functions can represent the same set of indifference curves and yield the same MRSC formula.

Problems

(3.1) [Preferences] Consider bundle X , consisting of 6 cans of beans and 4 boxes of cereal. Use just the assumption of monotonicity ("more is better") to determine whether each of the following bundles are

- *more preferred* to bundle X, or
- *less preferred* to bundle X, or
- preference *cannot be determined* without more information.

Briefly justify your answers.

a. Bundle A , consisting of 6 cans of beans and 3 boxes of cereal.
b. Bundle B , consisting of 7 cans of beans and 4 boxes of cereal.
c. Bundle C , consisting of 10 cans of beans and 2 boxes of cereal.
d. Bundle D , consisting of 4 cans of beans and 6 boxes of cereal.
e. Bundle E , consisting of 8 cans of beans and 5 boxes of cereal.

(3.2) [Preferences] Suppose the bundles X and Y are equally preferred, where bundle X consists of 5 units of energy and 8 units of food, while bundle Y consists of 11 units of energy and 6 units of food. Use the assumption of diminishing marginal rate of substitution in consumption (MRSC) to determine whether the following bundles are

- *more preferred* to either bundle X or bundle Y, or.
- *less preferred* to either bundle X or bundle Y.

Briefly explain your answers. [Hint: You may find it useful to plot bundles X, Y, A, and B carefully on a graph.]

a. Bundle A , consisting of 8 units of energy and 7 units of food.
b. Bundle B , consisting of 17 units of energy and 4 units of food.

(3.3) [Utility functions] Suppose a person has the utility function $U(q_1, q_2) = q_1 q_2^{1/2}$, where q_1 denotes the quantity of food the person enjoys and q_2 denotes the quantity of clothing. Rank the following bundles from most preferred to least preferred.

a. Bundle A , consisting of 10 units of food and 16 units of clothing.
b. Bundle B , consisting of 7 units of food and 25 units of clothing.
c. Bundle C , consisting of 13 units of food and 9 units of clothing.

(3.4) [Utility functions] Suppose a person has the utility function $U(q_1, q_2) = 5q_1 + 3q_2$. Assume q_1 and q_2 are positive quantities.

a. Find formulas for the marginal utilities MU_1 and MU_2 .
b. Determine whether this utility function satisfies the assumption of monotonicity ("more is better"). Explain your reasoning. [Hint: Determine whether the marginal utilities are positive.]
c. Find a formula for the marginal rate of substitution in consumption (MRSC) of good 2 for good 1. [Hint: This is the absolute value of the slope of the indifference curve, when good 1 is on the vertical axis and good 2 is on the horizontal axis.]
d. Determine whether this utility function satisfies the assumption of diminishing MRSC. Explain your reasoning. [Hint: According to the formula for the MRSC, does it diminish as q_1 decreases and q_2 increases?]

(3.5) [Utility functions] Suppose a person has the utility function $U(q_1, q_2) = (q_1-5)\, q_2^2$. Assume $q_1 > 5$ and $q_2 > 0$.

a. Find formulas for the marginal utilities MU_1 and MU_2 .
b. Determine whether this utility function satisfies the assumption of monotonicity ("more is better"). Explain your reasoning. [Hint: Determine whether the marginal utilities are positive.]
c. Find a formula for the marginal rate of substitution in consumption (MRSC) of good 2 for good 1. [Hint: This is the absolute value of the slope of the indifference curve, when good 1 is on the vertical axis and good 2 is on the horizontal axis.]
d. Determine whether this utility function satisfies the assumption of diminishing MRSC. Explain your reasoning. [Hint: According to the formula for the MRSC, does it diminish as q_1 decreases and q_2 increases?]

(3.6) [Utility functions] Suppose a person has the utility function $U(q_1, q_2) = -(3/q_1) - (5/q_2)$. Assume q_1 and q_2 are positive quantities.

a. Find formulas for the marginal utilities MU_1 and MU_2 .
b. Determine whether this utility function satisfies the assumption of monotonicity ("more is better"). Explain your reasoning. [Hint: Determine whether the marginal utilities are positive.]
c. Find a formula for the marginal rate of substitution in consumption (MRSC) of good 2 for good 1. [Hint: This is the absolute value of the slope of the indifference curve, when good 1 is on the vertical axis and good 2 is on the horizontal axis.]
d. Determine whether this utility function satisfies the assumption of diminishing MRSC. Explain your reasoning. [Hint: According to the formula for the MRSC, does it diminish as q_1 decreases and q_2 increases?]

(3.7) [Utility functions] Suppose a person has the utility function $U(q_1, q_2) = 3\,q_1^{1/2} + 2\,q_2^{1/2}$. Assume q_1 and q_2 are positive quantities.

a. Find formulas for the marginal utilities MU_1 and MU_2 .
b. Determine whether this utility function satisfies the assumption of monotonicity ("more is better"). Explain your reasoning. [Hint: Determine whether the marginal utilities are positive.]
c. Find a formula for the marginal rate of substitution in consumption (MRSC) of good 2 for good 1. [Hint: This is the absolute value of the slope of the indifference curve, when good 1 is on the vertical axis and good 2 is on the horizontal axis.]
d. Determine whether this utility function satisfies the assumption of diminishing MRSC. Explain your reasoning. [Hint: According to the formula for the MRSC, does it diminish as q_1 decreases and q_2 increases?]

(3.8) [Utility functions] Consider the utility function $U(q_1, q_2) = q_1^{3/4}\, q_2^{1/4}$.

a. Find a formula for the marginal rate of substitution in consumption (MRSC) of good 2 for good 1.
b. Find three different utility functions that yield exactly the same MRSC formula as your answer to part (a). Check your answers by finding the MRSC formulas in each case.

(3.9) [Utility functions, finance] In portfolio theory, the utility of investors is often modeled as a function of the expected rate of return (R) of their investment portfolio and the risk associated with that portfolio. Risk is measured as standard deviation (σ). A typical utility function might be $U(R,\sigma) = R - 0.03\,\sigma^2$.

a. Find formulas for the marginal utilities MU_R and MU_σ .
b. We usually assume that "more is better" for a consumer, and therefore that the marginal utilities should be positive. Explain why it makes sense for MU_R to be positive and for MU_σ to be negative in this situation.
c. Find a formula for the marginal rate of substitution in consumption (MRSC) of σ for R. [Hint: This is the absolute value of the slope of the indifference curve, when R is on the vertical axis and σ is on the horizontal axis.]
d. Do the investor's indifference curves slope up or down in this situation?

(3.10) [Utility functions, finance] In portfolio theory, the utility of investors is often modeled as a function of the expected rate of return (R) of their investment portfolio and the risk associated with that portfolio. Risk is measured as standard deviation (σ). A typical utility function might be $U(R,\sigma) = R - 0.01\,\sigma^2$.

a. Find formulas for the marginal utilities MU_R and MU_σ .
b. We usually assume that "more is better" for a consumer, and therefore that the marginal utilities should be positive. Explain why it makes sense for MU_R to be positive and for MU_σ to be negative in this situation.
c. Find a formula for the marginal rate of substitution in consumption (MRSC) of σ for R. [Hint: This is the absolute value of the slope of the indifference curve, when R is on the vertical axis and σ is on the horizontal axis.]
d. Do the investor's indifference curves slope up or down in this situation?

[end of problem set]