

EXAMINATION 4 ANSWER KEY

Version A

I. Multiple choice

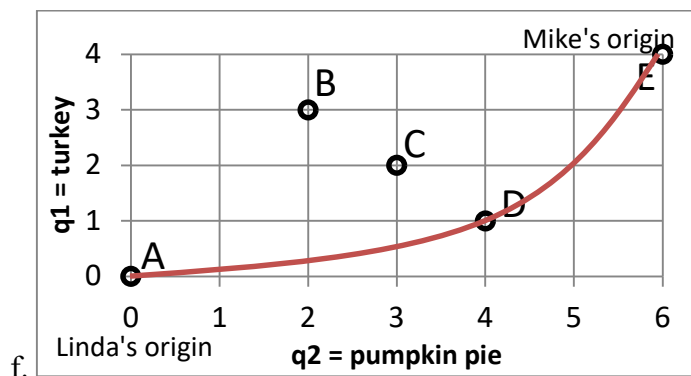
- (1)a. (2)e. (3)d. (4)b. (5)b. (6)b. (7)c. (8)d.

II. Short answer

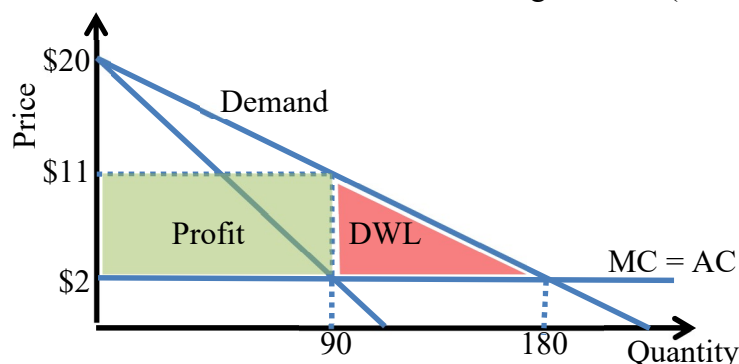
- (1) a. 3 units of other goods. b. $1/3$ energy. c. slope = -3.
d. $P_{\text{energy}} = \$18$, because slope of each consumer's budget line = $-P_{\text{energy}}/P_{\text{other}} = -3$.
- (2) a. $\$2.90 = 3.90 - (0.10 \times 10)$. b. increase. c. $\$0.90 = 2.90 - 2.00$.
- (3) a. $P_A = MC / (1 + [1/\epsilon_A]) = \16 .
b. $P_C = MC / (1 + [1/\epsilon_C]) = \10 .
- (4) a. $L = 1/|\epsilon| = 1/4$. b. $L = 1/(n |\epsilon|) = 1/20$.
- (5) a. \$5. b. 7 thousand. c. \$0 because $P=MC$.
d. $MR = 12 - 2Q$ ("Same intercept, twice the slope as demand").
e. Plot MR as a straight line with P-intercept = \$12, slope = $-2/\text{thousand}$.
f. \$8. g. 4 thousand. h. \$6 thousand.
- (6) Note: this game is similar to a "Prisoner's Dilemma."
a. Pareto optimal: no, yes, yes, yes.
b. Dominant-strategy equilibria: yes, no, no, no.
c. Nash equilibria in pure strategies: yes, no, no, no.

III. Problems

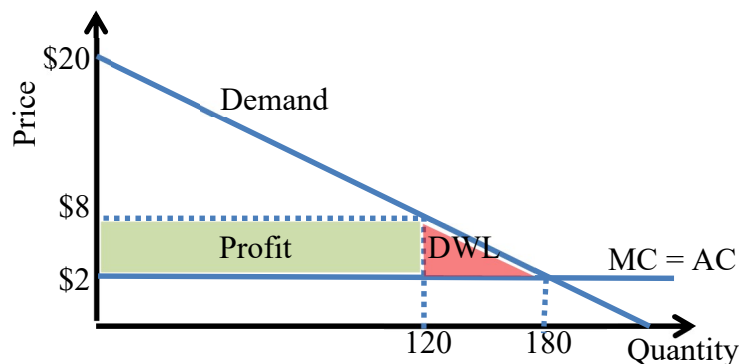
- (1) [Exchange efficiency] Note that Linda's $MRS_L = 3q_1/q_2$ and Mike's $MRS_M = q_1/(2q_2)$.
a. **Yes**, A is Pareto-efficient, because no one can be made better off without someone else being made worse off. Mike already has everything, so she cannot be made better off. Linda has nothing, so he cannot be made better off without taking some of Mike's turkey or pumpkin pie, which would make Mike worse off. Put simply, since Mike already has everything, any feasible change would make Mike worse off.
b. **No**, C is not Pareto-efficient, because $MRS_L = 9/2 \neq MRS_M = 1/8$.
c. **No**, D is not Pareto-efficient, because $MRS_L = 2 \neq MRS_M = 1/3$.
d. **Yes**, B is Pareto-efficient, because $MRS_L = 3/4 = MRS_M = 3/4$.
e. **Yes**, E is Pareto-efficient, because no one can be made better off without someone else being made worse off. Linda already has everything, so he cannot be made better off. Mike has nothing, so she cannot be made better off without taking some of Linda's turkey or pumpkin pie, which would make Linda worse off. Put simply, since Linda already has everything, any feasible change would make Linda worse off.



- (2) [Monopoly, profit maximization]
- $TR = P \times Q = 20Q - (Q^2/10)$.
 - $MR = dTR/dQ = 20 - (Q/5)$.
 - Set $MC = 2 = MR$ and solve to get $Q_M = 90$.
 - Substitute into demand function: $P_M = 20 - (90/10) = \$11$.
 - Profit = $TR - TC = (90 \times 11) - (90 \times 2) = 90 \times (11 - 2) = \810 (see green rectangle below.)
 - The efficient level of output lies where marginal cost intersects demand ("marginal cost pricing"). Find this quantity by setting $P = MC$, or $20 - (Q/10) = 2$, which yields $Q = 180$. Then evaluate DWL as the area of a triangle: **\$405** (see red triangle below).



- (3) [Cournot duopoly]
- $TR_1 = P q_1 = 20q_1 - (q_1^2/10) - (q_1q_2/10)$.
 - $MR_1 = \partial TR_1(q_1, q_2) / \partial q_1 = 20 - (2q_1/10) - (q_2/10)$.
 - Set $MR_1 = MC = \$2$ and solve to get $q_1^* = 90 - (q_2/2)$.
 - Since $q_1^* = q_2^*$, $q_1^* = 90 - (q_1^*/2)$. Solving yields $q_1^* = 60 = q_2^*$.
 - $Q^* = q_1^* + q_2^* = 120$. Substituting into demand equation: $P^* = 20 - (120/10) = \$8$.
 - Profit = $(P^* \times Q^*) - (AC \times Q^*) = (P^* - AC) \times Q^* = (8 - 2) \times 120 = \720 (see green rectangle below.).
 - The efficient level of output lies where marginal cost intersects demand ("marginal cost pricing"). Find this quantity by setting $P = MC$, or $P = 20 - (Q/10) = 2$ and solving to get $Q = 180$. Deadweight loss is the area between demand and marginal cost, from the Cournot equilibrium quantity $Q^* = 120$ to the efficient quantity = 180 (see below). This is the area of a triangle, equal to **\$405** (see red triangle below).



IV. Critical thinking

(1) Perfect price discrimination.

Profit-maximizing quantity: Now the firm can charge a different price for every unit sold, so it will set price equal to willingness-to-pay for that unit. To maximize profit, it will sell all units where willingness-to-pay is at least equal to marginal cost. So total quantity is at intersection of demand and marginal cost, which was found in problem (2) above as $Q = 180$.

Revenue: The highest price charged anyone is \$20, the intercept of the demand curve. The lowest price is \$2, at the intersection of demand and marginal cost. So total revenue is the area of a trapezoid whose parallel sides are \$20 and \$2, and whose height is 180:

$$TR = (20+2)/2 \times 180 = \mathbf{\$1980}.$$

Profit: Total cost is $TC = Q \times AC = 180 \times 2 = \360 . Profit = $TR - TC = \mathbf{\$1620}$. Note that profit is considerably higher with perfect price discrimination than with single-price monopoly.

(2) Price-setting (Bertrand) duopoly

In a price-setting duopoly, the equilibrium is for both firms to set a price equal to marginal cost, which here is $P^* = \$2$. Substituting into the demand equation

$\$2 = 20 - (Q/10)$ gives $Q^* = 180$. Since $P^* = \text{marginal cost} = \text{average cost}$, **profit = 0**.

Version B

I. Multiple choice

(1)e. (2)b. (3)b. (4)d. (5)a. (6)a. (7)b. (8)c.

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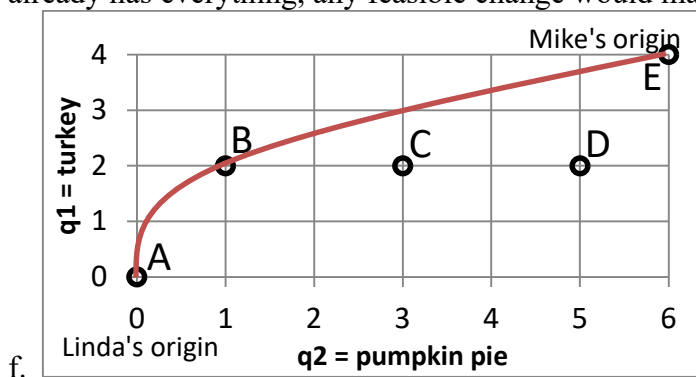
II. Short answer

- (1) a. 4 units of other goods. b. 1/4 energy. c. slope = -4.
d. $P_{\text{energy}} = \$32$, because slope of each consumer's budget line = $-P_{\text{energy}}/P_{\text{other}} = -4$.
- (2) a. $\$1.25 = 3.75 - (0.25 \times 10)$. b. decrease. c. $\$0.75 = 2.00 - 1.25$.
- (3) a. $P_A = MC / (1 + [1/\epsilon_A]) = \36 .
b. $P_C = MC / (1 + [1/\epsilon_C]) = \14 .

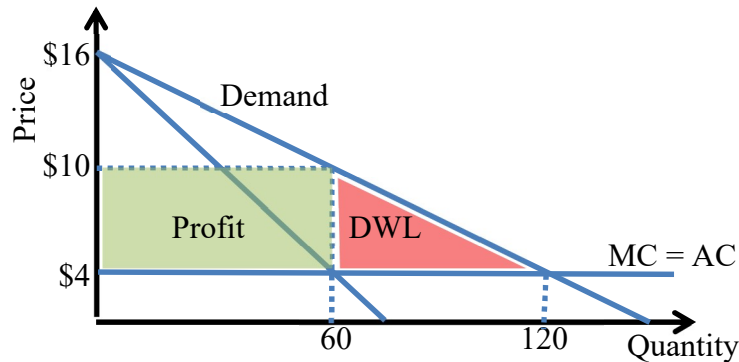
- (4) a. $L = 1/|\epsilon| = 1/5$. b. $L = 1/(n|\epsilon|) = 1/10$.
- (5) a. \$4. b. 9 thousand. c. \$0 because $P=MC$.
d. $MR = 13 - 2Q$ ("Same intercept, twice the slope as demand").
e. Plot MR as a straight line with P-intercept = \$13, slope = $-2/\text{thousand}$.
f. \$8. g. 5 thousand. h. \$10 thousand.
- (6) Note: this game is similar to a "Battle of the Sexes."
a. Pareto optimal: yes, no, no, yes.
b. Dominant-strategy equilibria: no, no, no, no.
c. Nash equilibria in pure strategies: yes, no, no, yes.

III. Problems

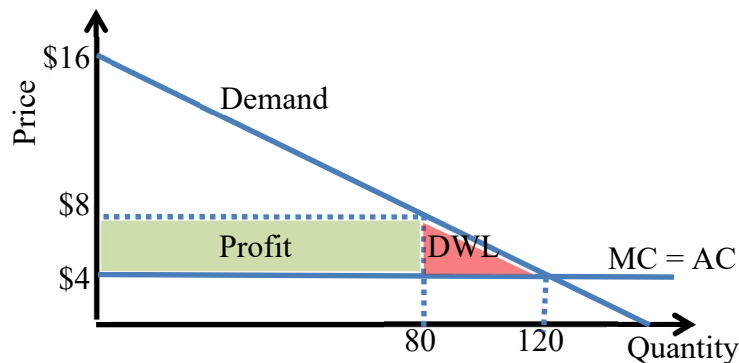
- (1) [Exchange efficiency] Note that Linda's $MRS_L = q_1/q_2$ and Mike's $MRS_M = 5q_1/q_2$.
- a. **Yes**, A is Pareto-efficient, because no one can be made better off without someone else being made worse off. Mike already has everything, so she cannot be made better off. Linda has nothing, so he cannot be made better off without taking some of Mike's turkey or pumpkin pie, which would make Mike worse off. Put simply, since Mike already has everything, any feasible change would make Mike worse off.
- b. **Yes**, B is Pareto-efficient, because $MRS_L = 2 = MRS_M = 2$.
- c. **No**, C is not Pareto-efficient, because $MRS_L = 2/3 \neq MRS_M = 10/3$.
- d. **No**, D is not Pareto-efficient, because $MRS_L = 2/5 \neq MRS_M = 10$.
- e. **Yes**, E is Pareto-efficient, because no one can be made better off without someone else being made worse off. Linda already has everything, so he cannot be made better off. Mike has nothing, so she cannot be made better off without taking some of Linda's turkey or pumpkin pie, which would make Linda worse off. Put simply, since Linda already has everything, any feasible change would make Linda worse off.



- (2) [Monopoly, profit maximization]
- a. $TR = P \times Q = 16Q - (Q^2/10)$.
- b. $MR = dTR/dQ = 16 - (Q/5)$.
- c. Set $MC = 4 = MR$ and solve to get $Q_M = 60$.
- d. Substitute into demand function: $P_M = 16 - (60/10) = \$10$.
- e. Profit = $TR - TC = (60 \times 10) - (60 \times 4) = 60 \times (10 - 4) = \360 (see green rectangle below.)
- f. The efficient level of output lies where marginal cost intersects demand ("marginal cost pricing"). Find this quantity by setting $P = MC$, or $16 - (Q/10) = 4$, which yields $Q=120$. Then evaluate DWL as the area of a triangle: **\$180** (see red triangle below).



- (3) [Cournot duopoly]
- $TR_1 = P q_1 = 16q_1 - (q_1^2/10) - (q_1q_2/10)$.
 - $MR_1 = \partial TR_1(q_1, q_2) / \partial q_1 = 16 - (2q_1/10) - (q_2/10)$.
 - Set $MR_1 = MC = \$4$ and solve to get $q_1^* = 60 - (q_2/2)$.
 - Since $q_1^* = q_2^*$, $q_1^* = 60 - (q_1^*/2)$. Solving yields $q_1^* = 40 = q_2^*$.
 - $Q^* = q_1^* + q_2^* = 80$. Substituting into demand equation: $P^* = 16 - (80/10) = \$8$.
 - Profit $= (P^* \times Q^*) - (AC \times Q^*) = (P^* - AC) \times Q^* = (8 - 4) \times 80 = \320 (see green rectangle below.).
 - The efficient level of output lies where marginal cost intersects demand ("marginal cost pricing"). Find this quantity by setting $P = MC$, or $P = 16 - (Q/10) = 4$ and solving to get $Q = 120$. Deadweight loss is the area between demand and marginal cost, from the Cournot equilibrium quantity $Q^* = 80$ to the efficient quantity $= 120$ (see below). This is the area of a triangle, equal to **\$180** (see red triangle below).



IV. Critical thinking

- (1) Perfect price discrimination.
- Profit-maximizing quantity: Now the firm can charge a different price for every unit sold, so it will set price equal to willingness-to-pay for that unit. To maximize profit, it will sell all units where willingness-to-pay is at least equal to marginal cost. So total quantity is at intersection of demand and marginal cost, which was found in problem (2) above as $Q = 120$.
- Revenue: The highest price charged anyone is \$16, the intercept of the demand curve. The lowest price is \$4, at the intersection of demand and marginal cost. So total revenue

is the area of a trapezoid whose parallel sides are \$16 and \$4, and whose height is 120:

$$TR = (16+4)/2 \times 120 = \mathbf{\$1200}.$$

Profit: Total cost is $TC = Q \times AC = 120 \times 4 = \480 . Profit = $TR - TC = \mathbf{\$720}$. Note that profit is considerably higher with perfect price discrimination than with single-price monopoly.

(2) Price-setting (Bertrand) duopoly

In a price-setting duopoly, the equilibrium is for both firms to set a price equal to marginal cost, which here is $P^* = \mathbf{\$4}$. Substituting into the demand equation

$\$4 = 16 - (Q/10)$ gives $Q^* = \mathbf{120}$. Since $P^* = \text{marginal cost} = \text{average cost}$, **profit = 0**.

[end of answer key]