

EXAMINATION 3 ANSWER KEY “Firms and Competition”

Version A

I. Multiple choice

(1)b. (2)b. (3)c. (4)b. (5)c. (6)a. (7)c. (8)d. (9)a. (10)b.
(11)b. (12)b. (13)d. (14)a. (15)c. (16)b.

II. Short answer

(1) a. 7 percent. b. 1 percent.
(2) a. 4 units. b. 6 units. c. \$240.
d. 12 units. c. \$300.
(3) a. 0 thousand (because price is below shutdown price).
b. 10 thousand (using rule $P=MC$ to find q).
c. 7 thousand (using rule $P=MC$ to find q).
d. \$8 (because breakeven price = $\min(SATC)$).
e. \$2 (because shutdown price = $\min(SAVC)$).
(4) a. import. b. 4 million. c. increase.
d. \$7 million. e. decrease. f. \$5 million.
g. increase. h. \$2 million.

III. Problems

(1) [Input substitution; Returns to scale]
a. $MP_1 = 2 x_1^{-1/3} x_w^{2/3}$. YES, there are diminishing returns to input 1, because as x_1 increases (and x_2 is held constant), MP_1 decreases.
b. $MRSP = \frac{MP_2}{MP_1} = \frac{2 x_1^{2/3} x_2^{-1/3}}{2 x_1^{-1/3} x_2^{2/3}} = \frac{x_1}{x_2}$. YES, this function has diminishing MRSP, because as x_1 decreases and x_2 increases, the numerator decreases and the denominator increases. Therefore, MRSP decreases.
c. Check returns to scale:
$$f(ax_1, ax_2) = 3 (a x_1)^{2/3} (a x_2)^{2/3} = a^{2/3} a^{2/3} 3 x_1^{2/3} x_2^{2/3}$$
$$= a^{4/3} q > a q, \text{ for all } a > 1.$$

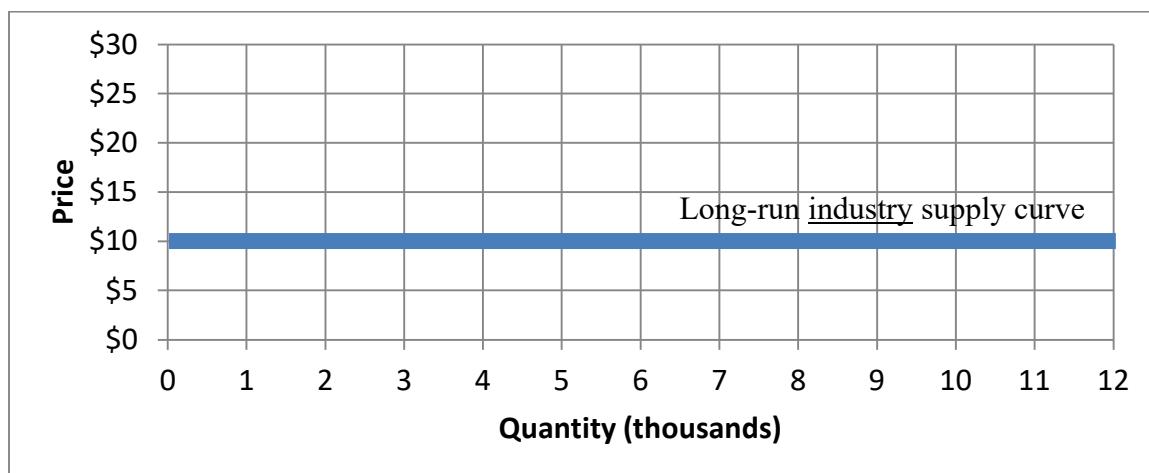
Thus, multiplying all inputs by the same factor (a) causes output to increase by a larger factor. So, this production function has INCREASING returns to scale.

(2) [Cost minimization]

- Equation for isoquant: $30 = 2 x_1^{1/2} x_2^{1/2}$ or $15 = x_1^{1/2} x_2^{1/2}$ or $225 = x_1 x_2$.
- $MRSP = MP_2/MP_1 = \frac{x_1^{1/2} x_2^{-1/2}}{x_1^{-1/2} x_2^{1/2}} = \frac{x_1}{x_2}$.
- Set $MRSP = w_2/w_1 = \$10/\90 (the tangency condition) and solve jointly with $15 = x_1^{1/2} x_2^{1/2}$ (the isoquant) to get $x_1^* = 5$ and $x_2^* = 45$.
- $TC(50) = 5 \times \$90 + 45 \times \$10 = \$900$.

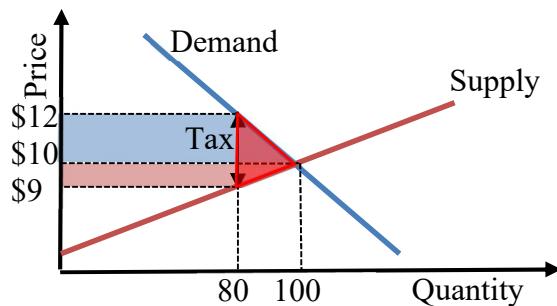
(3) [Cost curves; Long-run market equilibrium]

- $AC = TC/q = 0.01 q^2 - 0.8 q + 26$.
 Set $0 = dAC/dq = 0.02 q - 0.8$ and solve to get $q_{ES} = 40$.
- Breakeven price = minimum $AC = AC(q_{ES}) = \$10$.
- A firm's supply curve shows how much the firm will produce for any given price. If $P >$ minimum average cost, the profit-maximizing firm will choose an output level where $P = MC(q)$, and if $P <$ minimum average cost, it will produce nothing. So, this firm's supply curve is given by the following equations.
 If $P \geq \$10$, $P = MC(q) = dTC/dq = 0.03 q^2 - 1.6 q + 26$.
 If $P \leq \$10$, $q = 0$ (firm shuts down).
- The long-run industry supply curve is a horizontal line at minimum AC :



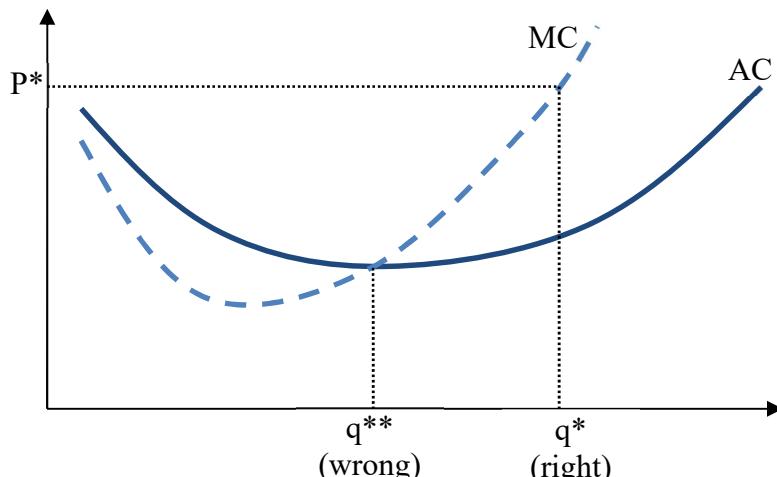
(4) [Welfare effects of tax or subsidy]

- Set $P_D = P_S$ and solve to get $P^* = \$10$ and $Q^* = 100$.
- With an excise tax of \$3, $P_D = P_S + 3$. Substituting and solving gives $Q = 80$. It is useful to also compute the new total price paid by buyers, including the tax ($P_D = \$12$), and the new net price received by sellers, excluding the tax ($P_S = \$9$).
- Consumer surplus decreases by \$180, the area of the trapezoid between \$12 and \$10.
- Producer surplus decreases by \$90, the area of the trapezoid between \$10 and \$9.
- Although the government collects $\$3 \times 80 = \240 in tax revenue, this is less than the combined decreases of consumer and producer surplus. The loss to society as a whole (also called "deadweight loss" or "excess burden of the tax") is \$30, the area of the red triangle in the graph below.

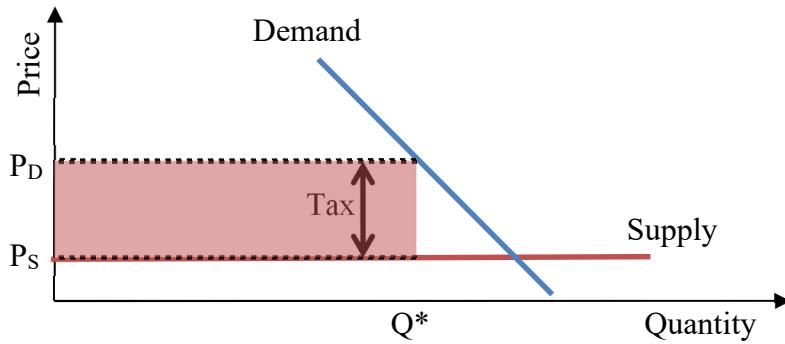


IV. Critical thinking

(1) [The following answer uses a long-run framework, where there are no fixed costs. A similar answer in a short-run framework would be acceptable.] Profit is maximized when the firm operates at the output level where *market price equals marginal cost*, provided price is greater than minimum average cost. (If price is less than average cost, total profit is maximized when the output level is zero.) Thus, the total-profit-maximizing level of output *depends on the market price*. In general, the two output levels will be different, as shown in the graph below. Here, P^* is the market price, q^* is the output level that maximizes total profit, and q^{**} is the output level where average cost is lowest.



(2) To find the burden of a tax, first find the quantity traded. After a tax is imposed, the quantity traded is where the demand curve is higher than the supply curve by the amount of the tax. If supply is *perfectly elastic* (horizontal) then there is no change in the price received by the sellers (P_S) as a result of the tax. Instead, the buyers pay all of the tax. That is, the tax causes the buyers' price (P_D) to rise by the amount of the tax. Similarly, producer surplus is zero both before and after the tax, but there is a loss of consumer surplus. So, in this special case, buyers bear the entire burden of the tax, as shown in the graph below.



Version B

I. Multiple choice

(1)d. (2)c. (3)a. (4)d. (5)d. (6)c. (7)a. (8)b. (9)b. (10)a.
 (11)b. (12)a. (13)a. (14)b. (15)d. (16)a.

II. Short answer

(1)	a. 5 percent.	b. 3 percent.	
(2)	a. 4 units.	b. 6 units.	c. \$240.
	d. 3 units.	c. \$300.	
(3)	a. 0 thousand (because price is below shutdown price).	b. 10 thousand (using rule $P=MC$ to find q).	
	c. 11 thousand (using rule $P=MC$ to find q).	d. \$6 (because breakeven price = $\min(SATC)$).	
	e. \$3 (because shutdown price = $\min(SAVC)$).		
(4)	a. export.	b. 8 million.	c. decrease.
	d. \$8 million.	e. increase.	f. \$16 million.
	g. increase.	h. \$8 million.	

III. Problems

(1) [Input substitution; returns to scale]

a. $MP_1 = 2x_1^{-1/3}$. YES, there are diminishing returns to input 1, because as x_1 increases (and x_2 is held constant), MP_1 decreases.

b. $MRSP = \frac{MP_2}{MP_1} = \frac{(2/3)x_2^{-1/3}x_2^{-1/4}}{2x_1^{-1/3}} = \frac{x_1^{1/3}}{3x_2^{1/3}}$. YES, this function has diminishing MRSP, because as x_1 decreases and x_2 increases, the numerator decreases and the denominator increases. Therefore, MRSP decreases.

c. Check returns to scale:

$$f(ax_1, ax_2) = 3(ax_1)^{2/3} + (ax_2)^{2/3} = 3a^{2/3}x_1^{2/3} + a^{2/3}x_2^{2/3}$$

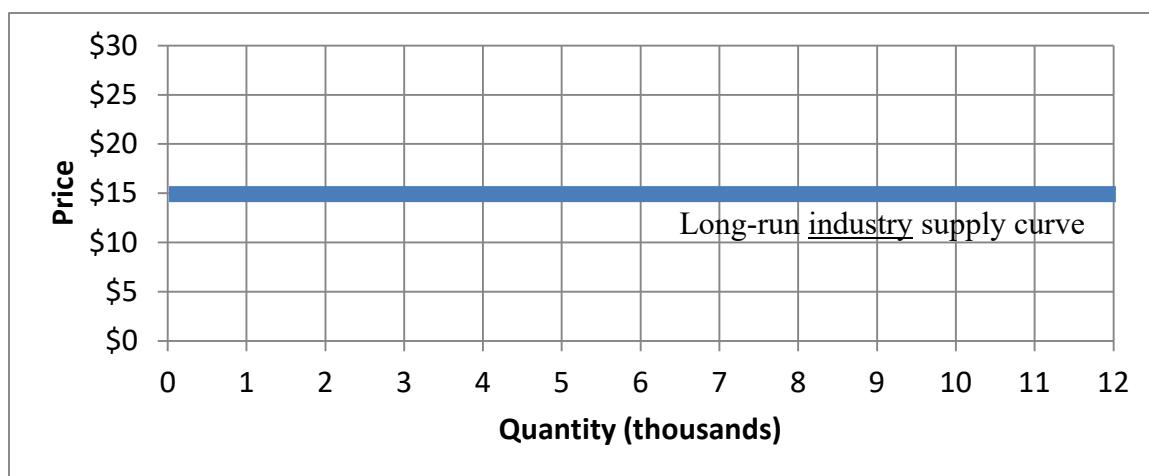
$$= a^{2/3}(3x_1^{2/3} + x_2^{2/3}) = a^{2/3}q < aq$$
, for all $a > 1$.
 Thus, multiplying all inputs by the same factor (a) causes output to increase by a smaller factor. So, this production function has DECREASING returns to scale.

(2) [Cost minimization]

- Equation for isoquant: $100 = 10 x_1^{1/2} x_2^{1/2}$ or $10 = x_1^{1/2} x_2^{1/2}$ or $100 = x_1 x_2$.
- $MRSP = MP_2/MP_1 = \frac{5 x_1^{1/2} x_2^{-1/2}}{5 x_1^{-1/2} x_2^{1/2}} = x_1/x_2$.
- Set $MRSP = w_2/w_1 = \$10/\40 (the tangency condition) and solve jointly with equation for isoquant to get $x_1^* = 5$ and $x_2^* = 20$.
- $TC(100) = 5 \times \$40 + 20 \times \$10 = \$400$.

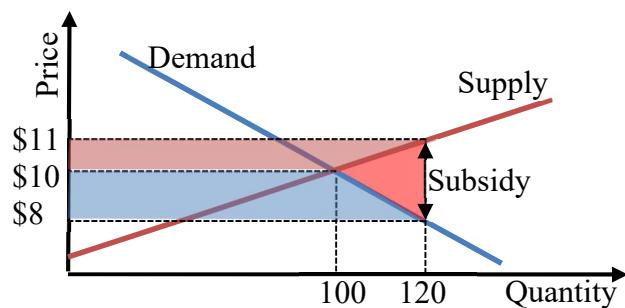
(3) [Cost curves; Long-run market equilibrium]

- $AC = TC/q = 0.01 q^2 - q + 40$.
 Set $0 = dAC/dq = 0.02 q - 1$ and solve to get $q_{ES} = 50$.
- Breakeven price = minimum $AC = AC(q_{ES}) = \$15$.
- A firm's supply curve shows how much the firm will produce for any given price. If $P >$ minimum average cost, the profit-maximizing firm will choose an output level where $P = MC(q)$, and if $P <$ minimum average cost, it will produce nothing. So, this firm's supply curve is given by the following equations.
 If $P \geq \$15$, $P = MC(q) = dTC/dq = 0.15 q^2 - 2 q + 20$.
 If $P \leq \$15$, $q = 0$ (firm shuts down).
- The long-run industry supply curve is a horizontal line at minimum AC :



(4) [Welfare effects of tax or subsidy]

- Set $P_D = P_S$ and solve to get $P^* = \$10$ and $Q^* = 100$.
- With subsidy of \$3, $P_D + 3 = P_S$. Substituting and solving gives $Q = 120$. It is useful to also compute the new net price paid by buyers, excluding the subsidy ($P_D = \$8$), and the new total price received by sellers, including the subsidy ($P_S = \$11$).
- Consumer surplus increases by \$220, the area of the trapezoid between \$10 and \$8.
- Producer surplus increases by \$110, the area of the trapezoid between \$11 and \$10.
- The government pays $\$3 \times 120 = \360 in direct cost, but this is greater than the combined increases of consumer and producer surplus. The loss to society as a whole (also called "deadweight loss") is \$30, the area of the red triangle in the graph below.



IV. Critical thinking
(Same as Version A above.)

[end of answer key]