

## EXAMINATION 2 ANSWER KEY “Consumers and Demand”

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### Version A

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#### I. Multiple choice

(1)b . (2)d. (3)a. (4)c. (5)b. (6)b. (7)a. (8)c. (9)c. (10)b.

#### II. Short answer

(1) a. elastic. b. decrease. c. 6 %.  
d. decrease. e. 2 %.

(2) a. necessary good. b. increase. c. 2 %.  
d. decrease. e. 8 %.

(3) Note: This graph is based on Slutsky's approach to income and substitution effects, not Hicks's approach.  
a. \$4. b. 10 units energy. c. \$10.  
d. 3 units energy. e. -5 units energy. f. -2 units energy.

(4) a.  $\varepsilon^{\text{comp}} = -1.02$ . b. decrease. c. 11 % (using  $\varepsilon$ ).  
d. decrease e. 10.02 % (using  $\varepsilon^{\text{comp}}$ ).

(5) a. Laspeyres = 140. b. Paasche = 130. c.  $\sqrt{140 \times 130}$  .

#### III. Problems

(1) [Budgets and choice]  
a. Equation for budget line (income=spending):  $160 = 5 q_1 + 4 q_2$  .  
b.  $\text{MRSC} = \text{MU}_2/\text{MU}_1 = (q_1-8) / q_2$  .  
c. Solve the tangency condition [ $\text{MRSC} = p_2/p_1$  or  $(q_1-8) / q_2 = 4/5$ ] jointly with equation for budget line (see part a) to get  $q_1^* = 20$ ,  $q_2^* = 15$ .

(2) [Properties of individual demand functions]  
a. Homogeneity of degree zero means that if income and all prices are multiplied by the same factor, the quantity demanded never changes. Check homogeneity of the demand function by replacing  $I$  by  $(aI)$ , replacing  $p_1$  by  $(ap_1)$ , and replacing  $p_2$  by  $(ap_2)$ , where  $a$  is an arbitrary factor:

$$\begin{aligned} & 2 (aI)^{1.5} (ap_1)^{-0.5} (ap_2)^{0.1} \\ &= a^{1.5-0.5+0.1} 2 I^{1.5} p_1^{-0.5} p_2^{0.1} \\ &= a^{1.1} 2 I^{1.5} p_1^{-0.5} p_2^{0.1} \end{aligned}$$

Note that the “ $a$ ” factor does *not* cancel. So multiplying income and prices by some arbitrary positive factor  $a$  changes the quantity demanded. This function is not homogeneous of degree zero in income and prices.

b.  $\frac{\partial q_1^*}{\partial p_1} = 2 I^{1.5} (-0.5)p_1^{-1.5} p_2^{0.1}$ . This expression is negative, so good #1 is an ordinary good, not a Giffen good.

c.  $\frac{\partial q_1^*}{\partial I} = 2 (1.5)I^{0.5} p_1^{-0.5} p_2^{0.1}$ . This expression is positive, so good #1 is a normal good, not an inferior good.

d.  $\frac{\partial q_1^*}{\partial p_2} = 2 I^{1.5} p_1^{-0.5} (0.1)p_2^{-0.9}$ . This expression is positive, so goods #1 and #2 are substitutes, not complements.

(3) [Finding individual demand functions]

a.  $MRS = MU_2/MU_1 = (q_1^2 5q_2^4) / (2 q_1 q_2^5) = (5q_1) / (2q_2)$ .

b. Solve  $MRS = (5q_1) / (2q_2) = p_2/p_1$  (the tangency condition) jointly with

$I = p_1 q_1 + p_2 q_2$  (the budget constraint) to get  $q_1^* = \frac{2I}{7 p_1}$ , and

c.  $q_2^* = \frac{5I}{7 p_2}$ .

[There are many ways to solve the equations jointly. One way to begin is to cross-multiply the tangency condition to get  $(5/2) p_1 q_1 = p_2 q_2$ . Then substitute into the budget constraint:

$$I = p_1 q_1 + (5/2) p_1 q_1 = (7/2) p_1 q_1.$$

Then solve for  $q_1^*$  as a function of  $p_1$  and  $I$ . Finally, substitute your expression for  $q_1^*$  into either the budget constraint or the tangency condition and solve for  $q_2^*$  as a function of  $p_2$  and  $I$ . These particular demand functions depend on income and the own price of the good—they do not depend on the price of the other good. This always happens with Cobb-Douglas utility functions, so you would not want to use a Cobb-Douglas utility function to model substitutes and complements.]

#### IV. Critical thinking

(1) Demand for this good is **inelastic** because spending rises when the price increases. To compute the price elasticity of demand, recall that for small percent changes,

$$\% \text{ change spending} = \% \text{ change price} + \% \text{ change quantity}$$

Substituting the numbers given in the problem:

$$7\% = 10\% + \% \text{ change quantity}$$

So the percent change in quantity is -3%. The price elasticity of demand by definition equals percent change in quantity divided by percent change in price =  $-3\% / 10\% = -0.3$ .

(2) This good must be a **luxury (or superior) good** because the spending share rises when income rises. To compute the income elasticity of demand, recall that for small percent changes,

$$\% \text{ change spending share} = \% \text{ change quantity} - \% \text{ change income}$$

Substituting the numbers given in the problem:

$$2\% = \% \text{ change quantity} - 5\%$$

So the percent change in quantity is 7%. The income elasticity of demand by definition equals percent change in quantity divided by percent change in income =  $7\% / 5\% = 1.4$ .

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## Version B

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### I. Multiple choice

(1)c. (2)b. (3)e. (4)b. (5)d. (6)c. (7)b. (8)d. (9)c. (10)d.

### II. Short answer

(1) a. inelastic. b. decrease. c. 2 %.  
 d. increase. e. 3 %.

(2) a. luxury or superior good. b. increase. c. 6 %.  
 d. increase. e. 1 %.

(3) Note: This graph is based on Slutsky's approach to income and substitution effects, not Hicks's approach.  
 a. \$2. b. 10 units of food. c. \$8.  
 d. 2 units of food. e. -6 units of food. f. -2 units of food.

(4) a.  $\varepsilon^{\text{comp}} = -0.53$ . b. decrease. c. 8 % (using  $\varepsilon$ ).  
 d. decrease e. 5.3 % (using  $\varepsilon^{\text{comp}}$ ).

(5) a. Laspeyres = 160. b. Paasche = 130. c.  $\sqrt{160 \times 130}$ .

### III. Problems

(1) [Budgets and choice]

a. Equation for budget line (income=spending):  $150 = 5 q_1 + 6 q_2$ .  
 b.  $\text{MRSC} = \text{MU}_2/\text{MU}_1 = q_1 / (q_2 - 5)$ .  
 c. Solve the tangency condition [ $\text{MRSC} = p_2/p_1$  or  $q_1 / (q_2 - 5) = 6/5$ ] jointly with equation for budget line (see part a) to get  $q_1^* = 12$ ,  $q_2^* = 15$ .

(2) [Properties of individual demand functions]

a. Homogeneity of degree zero means that if income and all prices are multiplied by the same factor, the quantity demanded never changes. Check homogeneity of the demand function by replacing  $I$  by  $(aI)$ , replacing  $p_1$  by  $(ap_1)$ , and replacing  $p_2$  by  $(ap_2)$ , where  $a$  is an arbitrary factor:

$$\begin{aligned} & (aI - 2ap_2)(ap_1)^{-1} + 3 \\ &= a a^{-1} (I - 2p_2)(p_1)^{-1} + 3 \\ &= (I - 2p_2)(p_1)^{-1} + 3 \end{aligned}$$

Note that the “ $a$ ” factor cancels. So multiplying income and prices by some arbitrary positive factor  $a$  *does not* change the quantity demanded. The function IS homogeneous of degree zero in income and prices.

b.  $\frac{\partial q_1^*}{\partial p_1} = (I - 2p_2)(-1)(p_1)^{-2}$ . This expression is negative, so good #1 is an ordinary good, not a Giffen good.

c.  $\frac{\partial q_1^*}{\partial I} = (p_1)^{-1}$ . This expression is positive, so good #1 is a normal good, not an inferior good.

d.  $\frac{\partial q_1^*}{\partial p_2} = -2(p_1)^{-1}$  . This expression is negative, so goods #1 and #2 are complements, not substitutes.

(3) [Finding individual demand functions]

a.  $MRS = MU_2/MU_1 = (4 q_1^3 q_2^3) / (3 q_1^2 q_2^4) = (4q_1) / (3q_2)$ .

b. Solve  $MRS = (4q_1) / (3q_2) = p_2/p_1$  (the tangency condition) jointly with  $I = p_1 q_1 + p_2 q_2$  (the budget constraint) to get  $q_1^* = \frac{3I}{7p_1}$ , and

c.  $q_2^* = \frac{4I}{7p_2}$ .

[There are many ways to solve the equations jointly. One way to begin is to cross-multiply the tangency condition to get  $(4/3) p_1 q_1 = p_2 q_2$ . Then substitute into the budget constraint:

$$I = p_1 q_1 + (4/3) p_1 q_1 = (7/3) p_1 q_1.$$

Then solve for  $q_1^*$  as a function of  $p_1$  and  $I$ . Finally, substitute your expression for  $q_1^*$  into either the budget constraint or the tangency condition and solve for  $q_2^*$  as a function of  $p_2$  and  $I$ . These particular demand functions depend on income and the own price of the good—they do not depend on the price of the other good. This always happens with Cobb-Douglas utility functions, so you would not want to use a Cobb-Douglas utility function to model substitutes and complements.]

#### IV. Critical thinking

Same as Version A.

[end of answer key]