ECON 173 - Intermediate Microeconomic Analysis
Drake University, Fall 2025
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EXAMINATION 1 ANSWER KEY
"Mathematical Tools"

Version A

I. Multiple choice

(1)c. (2)b. (3)c. (4)d. (5)d. (6)c. (7)d. (8)b. (9)b. (10)e.
(11)c. (12)a. (13)a. (14)c. (15)d.

II. Short answer

(1) a. increase b. 6 units, using derivative since change is given in units.
(2) a. increase b. 14 percent, using elasticities since changes are given in percent.
(3) a. increase b. 3 percent, using approximation rule for multiplication.
(4) a. increase b. 3 percent, using approximation rule for division.
(5) a. increase b. 20 units c. decrease d. 10 units.
(6) a. down b. slope = $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{12}{3} = -4$.

III. Problems

(1) a. dy/dx = 4 x - 40. b. Set dy/dx=0 and solve to get x* = 10.
c. The function slopes up if dy/dx = 4 x - 40> 0, which implies x > 10.
The function slopes down if dy/dx = 4 x - 40< 0, which implies x < 10.
d. y* = f(x*) = f(10) = -100.

(2) a. $\varepsilon_1 = \frac{\partial y}{\partial x_1}\frac{x_1}{y} = 3\,x_1^2\,(x_2+5)^2 \;\; \frac{x_1}{x_1^3\,(x_2+5)^2} = 3$.
b. $\varepsilon_2 = \frac{\partial y}{\partial x_2}\frac{x_2}{y} = \; x_1^3\;2\;(x_2+5) \;\; \frac{x_2}{x_1^3\;2\;(x_2+5)^2} = \frac{2\,x_2}{x_2+5}$.

(3) a. $\frac{\partial y}{\partial x_1} = 2\,x_1^{-1/2}$ b. $\frac{\partial y}{\partial x_2} = \; x_2^{-1/2}$
c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{x_2^{-1/2}}{2\,x_1^{-1/2}} = \; \left(\frac{1}{2}\right)\left(\frac{x_1}{x_2}\right)^{1/2}$

(4) a. $\frac{\partial y}{\partial x_1} = 3(x_1-2)^2(x_2-4)$ b. $\frac{\partial y}{\partial x_2} = (x_1-2)^3$
c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{(x_1-2)^3}{3\,(x_1-2)^2\,(x_2-4)} = \frac{(x_1-2)}{3\,(x_2-4)}$.

IV. Critical thinking

(1) We are given $P \times Q = a$, so $Q = a/P = a\,P^{-1}$.
This is a power function, so the elasticity of Q with respect to P is -1 and does not depend on the value of a.

(2) The percent change in output is given by
$5\% \times \varepsilon_K + 5\% \times \varepsilon_L$
$= 5\% \times (\varepsilon_K + \varepsilon_L)$
$= 5\% \times 1 = 5\%$, an increase.

Version B

I. Multiple choice

(1)b. (2)c. (3)c. (4)c. (5)b. (6)d. (7)a. (8)d. (9)c. (10)a.
(11)d. (12)d. (13)b. (14)b. (15)e.

II. Short answer

(1) a. increase b. 10 percent, using elasticity since change is given in percent.
(2) a. increase b. 40 units, using partial derivatives since changes given in units.
(3) a. increase b. 2 percent, using approximation rule for division.
(4) a. decrease b. 2 percent, using approximation rule for multiplication.
(5) a. increase b. 24 units c. decrease d. 6 units.
(6) a. down b. slope = $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{5}{10} = -\frac{1}{2}$.

III. Problems

(1) a. dy/dx = -10 x + 20. b. Set dy/dx=0 and solve to get x* = 2.
c. The function slopes up if dy/dx = -10 x + 20> 0, which implies 2 > x.
The function slopes down if dy/dx = -10 x + 20< 0, which implies 2 < x.
d. y* = f(x*) = f(2) = 10.

(2) a. $\varepsilon_1 = \frac{\partial y}{\partial x_1}\frac{x_1}{y} = 4\,(x_1+2)^3\,x_2\;\;\frac{x_1}{(x_1+2)^4\,x_2} = \frac{4\,x_1}{x_2+2}$.
b. $\varepsilon_2 = \frac{\partial y}{\partial x_2}\frac{x_2}{y} = \;(x_1+2)^4\;\;\frac{x_2}{(x_1+2)^4\,x_2} = 1$.

(3) a. $\frac{\partial y}{\partial x_1} = (x_2-1)^2$ b. $\frac{\partial y}{\partial x_2} = \;(x_1-5)\;\;2\,(x_2-1)$
c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{(x_1-5)\;\;2\,(x_2-1)}{(x_2-1)^2} = \frac{2(x_1-5)}{(x_2-1)}$.

(4) a. $\frac{\partial y}{\partial x_1} = 2\,x_1^{-2}$ b. $\frac{\partial y}{\partial x_2} = \;3\,x_2^{-2}$
c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{3\,x_2^{-2}}{2\,x_1^{-2}} = \;\left(\frac{3}{2}\right)\left(\frac{x_1}{x_2}\right)^2$

IV. Critical thinking

(Same as version A.)

[end of answer key]