

## EXAMINATION #1 ANSWER KEY “Mathematical Tools”

### Version A

#### I. Multiple choice

- (1)c. (2)c. (3)e. (4)c. (5)b. (6)c. (7)c. (8)c. (9)c. (10)d.  
 (11)d. (12)f. (13)b. (14)a. (15)a.

#### II. Short answer

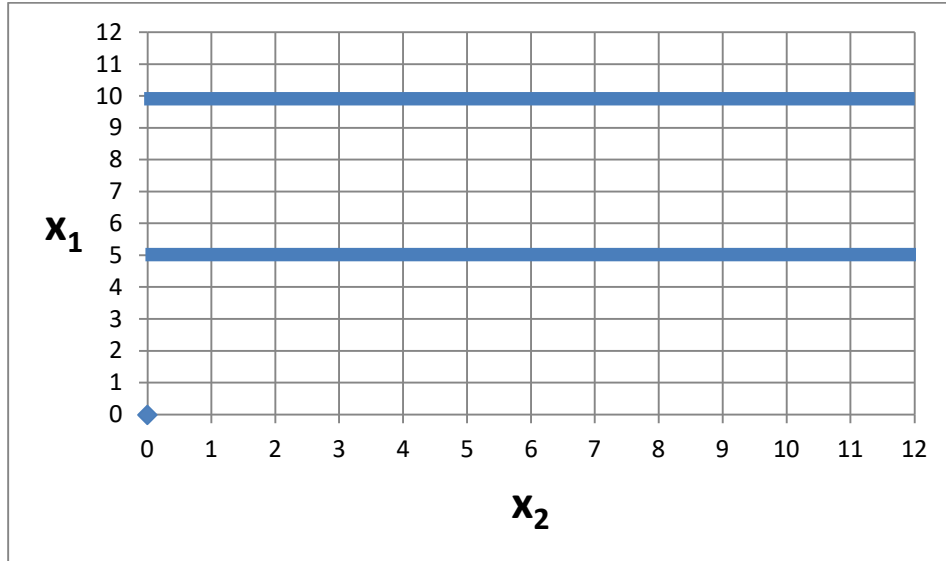
- |     |             |   |
|-----|-------------|---|
| (1) | a. increase | b. 15 units, using derivative since change is given in units.                               |
| (2) | a. increase | b. 8 percent, using elasticities since changes are given in percent.                        |
| (3) | a. decrease | b. 3 percent, using approximation rule for division.  |
| (4) | a. increase | b. 2 percent, using approximation rule for multiplication.                                  |
| (5) | a. increase | b. 12 units                      c. decrease                      d. 4 units.               |
| (6) | a. down     | b. slope = $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{4}{2} = -2$ . |

#### III. Problems

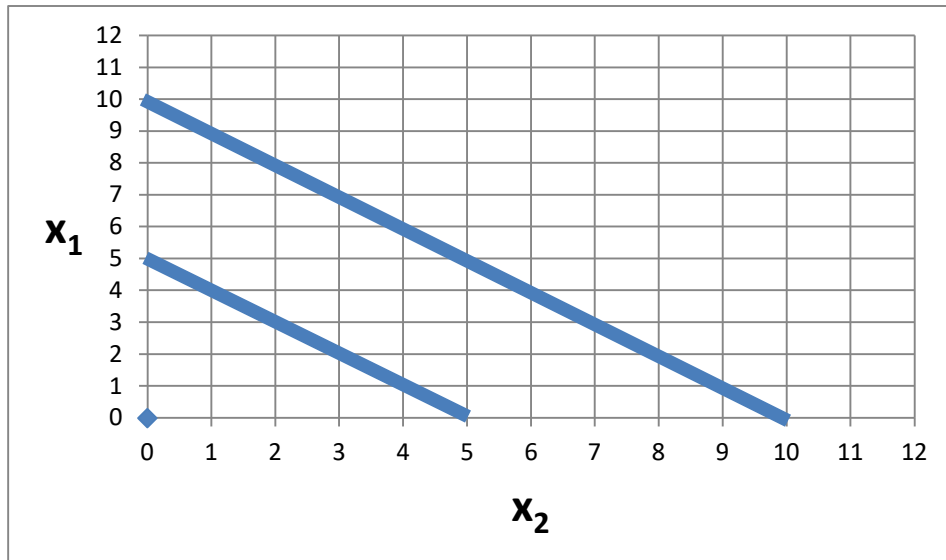
- (1) a.  $dy/dx = 10x - 30$ .    b. Set  $dy/dx=0$  and solve to get  $x^* = 3$ .  
 c. The function slopes up if  $dy/dx = 10x - 30 > 0$ , which implies  $x > 3$ .  
 The function slopes down if  $dy/dx = 10x - 30 < 0$ , which implies  $x < 3$ .  
 d.  $y^* = f(x^*) = f(3) = 15$ .
- (2) a.  $\varepsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} = 5x_1^4 (x_2 + 1)^2 \frac{x_1}{x_1^5 (x_2 + 1)^2} = 5$ .  
 b.  $\varepsilon_2 = \frac{\partial y}{\partial x_2} \frac{x_2}{y} = x_1^5 2(x_2 + 1) \frac{x_2}{x_1^5 (x_2 + 1)^2} = \frac{2x_2}{x_2 + 1}$ .
- (3) a.  $\frac{\partial y}{\partial x_1} = (x_2 - 4)^2$     b.  $\frac{\partial y}{\partial x_2} = (x_1 - 10)^2 2(x_2 - 4)$   
 c.  $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{(x_1 - 10)^2 2(x_2 - 4)}{(x_2 - 4)^2} = \frac{2(x_1 - 10)}{(x_2 - 4)}$ .
- (4) a.  $\frac{\partial y}{\partial x_1} = 3x_1^{-2}$                       b.  $\frac{\partial y}{\partial x_2} = 2x_2^{-2}$   
 c.  $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{2x_2^{-2}}{3x_1^{-2}} = \left(\frac{2}{3}\right) \left(\frac{x_1}{x_2}\right)^2$

#### IV. Critical thinking

- (1) The slope of the level curves is given by  $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{0}{\partial y/\partial x_1} = 0$ . So the level curves are necessarily horizontal straight lines. Two representative level curves are shown below.



- (2) The slope of the level curves is given by  $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{\partial y/\partial x_1}{\partial y/\partial x_1} = -1$ . So the level curves are necessarily downward-sloping straight lines with slope = -1. Two representative level curves are shown below.



## Version B

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### I. Multiple choice

- (1)c. (2)c. (3)e. (4)c. (5)b. (6)c. (7)c. (8)c. (9)c. (10)d.  
(11)d. (12)f. (13)b. (14)a. (15)a.

### II. Short answer

- (1) a. increase                      b. 15 units, using derivative since change is given in units.  
(2) a. increase                      b. 8 percent, using elasticities since changes are given in percent.  
(3) a. decrease                      b. 3 percent, using approximation rule for division.  
(4) a. increase                      b. 2 percent, using approximation rule for multiplication.  
(5) a. increase                      b. 12 units                      c. decrease                      d. 4 units.  
(6) a. down                      b. slope =  $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{4}{2} = -2$ .

### III. Problems

- (1) a.  $dy/dx = 10x - 30$ .    b. Set  $dy/dx=0$  and solve to get  $x^* = 3$ .  
c. The function slopes up if  $dy/dx = 10x - 30 > 0$ , which implies  $x > 3$ .  
The function slopes down if  $dy/dx = 10x - 30 < 0$ , which implies  $x < 3$ .  
d.  $y^* = f(x^*) = f(3) = 15$ .
- (2) a.  $\varepsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} = 3(x_1 + 2)^2 x_2^5 \frac{x_1}{(x_1+2)^3 x_2^5} = \frac{3x_1}{x_1+2}$ .  
b.  $\varepsilon_2 = \frac{\partial y}{\partial x_2} \frac{x_2}{y} = (x_1 + 2)^3 5x_2^4 \frac{x_2}{(x_1+2)^3 x_2^5} = 5$ .
- (3) a.  $\frac{\partial y}{\partial x_1} = x_1^{-1/2}$                       b.  $\frac{\partial y}{\partial x_2} = 3x_2^{-1/2}$   
c.  $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{3x_2^{-1/2}}{x_1^{-1/2}} = 3\left(\frac{x_1}{x_2}\right)^{1/2}$ .
- (4) a.  $\frac{\partial y}{\partial x_1} = 2(x_1 - 5)(x_2 - 3)$                       b.  $\frac{\partial y}{\partial x_2} = (x_1 - 5)^2$   
c.  $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{(x_1-5)^2}{2(x_1-5)(x_2-3)} = \frac{(x_1-5)}{2(x_2-3)}$ .

### IV. Critical thinking

(Same as version A.)

[end of answer key]