EXAMINATION #1 ANSWER KEY
"Mathematical Tools"

Version A

I. Multiple choice
(1)d. (2)b. (3)d. (4)d. (5)c. (6)b. (7)e. (8)d. (9)a. (10)b.
(11)e. (12)b. (13)a. (14)b. (15)e.
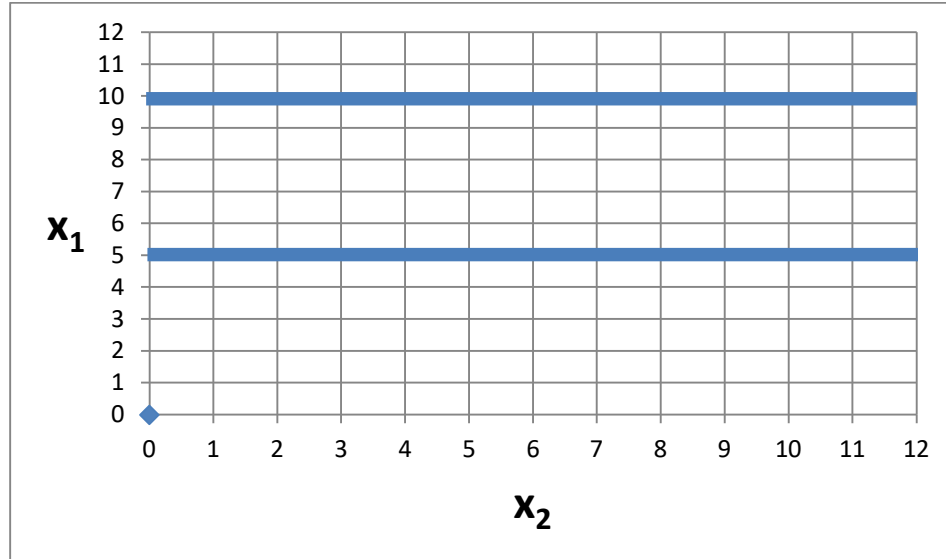
II. Short answer
(1) a. increase   b. 15 units, using derivative since change is given in units.
(2) a. increase   b. 8 percent, using elasticities since changes are given in percent.
(3) a. decrease   b. 3 percent, using approximation rule for division.
(4) a. increase   b. 2 percent, using approximation rule for multiplication.
(5) a. increase   b. 12 units   c. decrease   d. 4 units.
(6) a. down   b. slope $= -\dfrac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\dfrac{4}{2} = -2$.

III. Problems
(1) a. $dy/dx = 10\,x - 30$.   b. Set $dy/dx = 0$ and solve to get $x^* = 3$.
   c. The function slopes up if $dy/dx = 10\,x - 30 > 0$, which implies $x > 3$.
      The function slopes down if $dy/dx = 10\,x - 30 < 0$, which implies $x < 3$.
   d. $y^* = f(x^*) = f(3) = 15$.

(2) a. $\varepsilon_1 = \dfrac{\partial y}{\partial x_1}\dfrac{x_1}{y} = 5\,x_1^4\,(x_2+1)^2\,\dfrac{x_1}{x_1^5\,(x_2+1)^2} = 5$.

   b. $\varepsilon_2 = \dfrac{\partial y}{\partial x_2}\dfrac{x_2}{y} = x_1^5\,2(x_2+1)\,\dfrac{x_2}{x_1^5\,(x_2+1)^2} = \dfrac{2x_2}{x_2+1}$.

(3) a. $\dfrac{\partial y}{\partial x_1} = (x_2-4)^2$ b. $\dfrac{\partial y}{\partial x_2} = (x_1-10)\,2\,(x_2-4)$

   c. $MRS = \dfrac{\partial y/\partial x_2}{\partial y/\partial x_1} = \dfrac{(x_1-10)\,2\,(x_2-4)}{(x_2-4)^2} = \dfrac{2(x_1-10)}{(x_2-4)}$.

(4) a. $\dfrac{\partial y}{\partial x_1} = 3\,x_1^{-2}$ b. $\dfrac{\partial y}{\partial x_2} = 2\,x_2^{-2}$

   c. $MRS = \dfrac{\partial y/\partial x_2}{\partial y/\partial x_1} = \dfrac{2\,x_2^{-2}}{3\,x_1^{-2}} = \left(\dfrac{2}{3}\right)\left(\dfrac{x_1}{x_2}\right)^2$

IV. Critical thinking

(1)   The slope of the level curves is given by $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{0}{\partial y/\partial_1} = 0$. So the level curves are necessarily horizontal straight lines. Two representative level curves are shown below.

(2)   The slope of the level curves is given by $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{\partial y/\partial x_1}{\partial y/\partial x_1} = -1$. So the level curves are necessarily downward-sloping straight lines with slope = -1. Two representative level curves are shown below
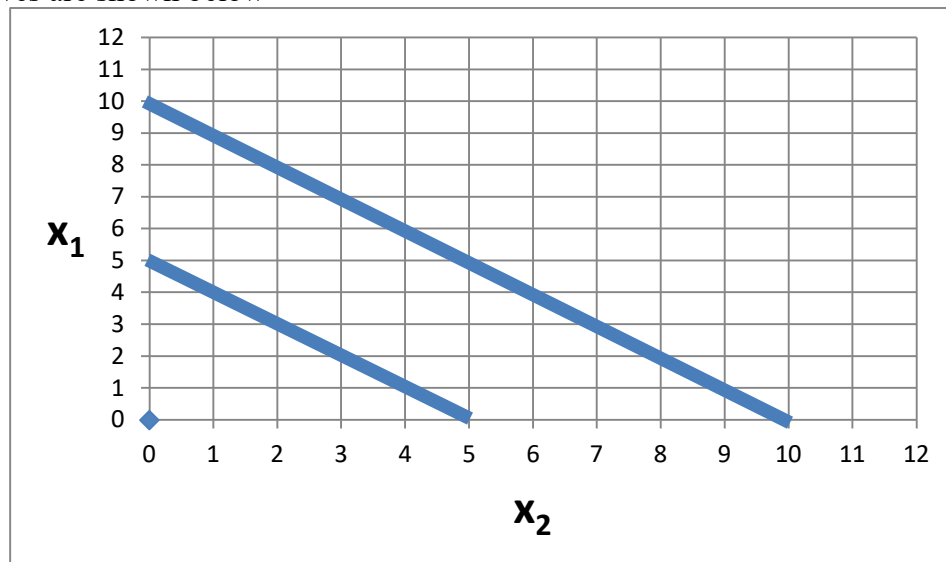
Version B

I. Multiple choice
(1)c.　(2)c.　(3)e.　(4)c.　(5)b.　(6)c.　(7)c.　(8)c.　(9)c.　(10)d.
(11)d.　(12)f.　(13)b.　(14)a.　(15)a.

II. Short answer
(1)　a. increase　　　b. 15 units, using derivative since change is given in units.
(2)　a. increase　　　b. 8 percent, using elasticities since changes are given in percent.
(3)　a. decrease　　　b. 1 percent, using approximation rule for multiplication.
(4)　a. increase　　　b. 2 percent, using approximation rule for division.
(5)　a. increase　　　b. 12 units　　　c. decrease　　　d. 4 units.
(6)　a. down　　　b. slope $= -\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{4}{2} = -2$.

III. Problems
(1)　a. $dy/dx = 10x - 30$.　b. Set $dy/dx=0$ and solve to get $x^* = 3$.
　　c. The function slopes up if $dy/dx = 10x - 30 > 0$, which implies $x > 3$.
　　　The function slopes down if $dy/dx = 10x - 30 < 0$, which implies $x < 3$.
　　d. $y^* = f(x^*) = f(3) = 15$.

(2)　a. $\varepsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} = 3(x_1 + 2)^2 x_2^5 \frac{x_1}{(x_1+2)^3 x_2^5} = \frac{3 x_1}{x_1+2}$.

　　b. $\varepsilon_2 = \frac{\partial y}{\partial x_2} \frac{x_2}{y} = (x_1 + 2)^3 5 x_2^4 \frac{x_2}{(x_1+2)^3 x_2^5} = 5$.

(3)　a. $\frac{\partial y}{\partial x_1} = x_1^{-1/2}$ 　　　　　b. $\frac{\partial y}{\partial x_2} = 3 x_2^{-1/2}$

　　c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{3 x_2^{-1/2}}{x_1^{-1/2}} = 3\left(\frac{x_1}{x_2}\right)^{1/2}$.

(4)　a. $\frac{\partial y}{\partial x_1} = 2(x_1 - 5)(x_2 - 3)$ 　　　b. $\frac{\partial y}{\partial x_2} = (x_1 - 5)^2$

　　c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{(x_1-5)^2}{2(x_1-5)(x_2-3)} = \frac{(x_1-5)}{2(x_2-3)}$.

IV. Critical thinking
(Same as version A.)

[end of answer key]