ECON 173 - Intermediate Microeconomic Analysis
Drake University, Fall 2024
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Signature: _____

Printed name: _____

EXAMINATION 1 VERSION A
"Mathematical Tools"
September 3, 2024

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [2 pts each—30 pts total]

(1) If the derivative of a function is negative, then
a. the value of the function is positive.
b. the value of the function is negative.
c. the graph of the function slopes up.
d. the graph of the function slopes down.

(2) Suppose $y = 4 + (5/x)$. Then the derivative of y with respect to x is given by the formula
a. $dy/dx = -1$.
b. $dy/dx = -5/x^2$.
c. $dy/dx = 4/x^2$.
d. $dy/dx = 5x$.
e. none of the above.

(3) Suppose $y = 2x^2 + 3x + 4$. Then the derivative of y with respect to x is
a. $dy/dx = 2x$.
b. $dy/dx = 9x$.
c. $dy/dx = 3$.
d. $dy/dx = 4x + 3$.
e. $dy/dx = 2x + 3$.
f. $dy/dx = 2x^2 + 3x + 4$.

(4) Suppose $y = (x+2)^4$. Then the derivative of y with respect to x is given by
a. $dy/dx = 5$.
b. $dy/dx = 4x$.
c. $dy/dx = 4(x+2)$.
d. $dy/dx = 4(x+2)^3$.
e. none of the above.

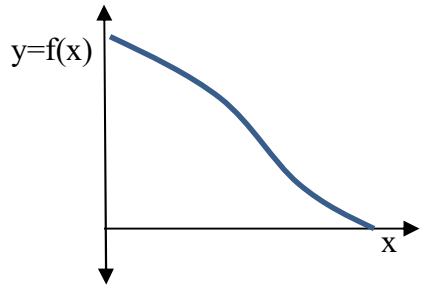
(5) Suppose $y = x^{2/3}$. Then the derivative of y with respect to x is given by
a. $dy/dx = (2/3)\, x$.
b. $dy/dx = x^{-1/3}$.
c. $dy/dx = (2/3)\, x^{-1/3}$.
d. $dy/dx = x^{1/3}$.
e. none of the above.

(6) Suppose $y = 5\,(4x+3)^2$. Then the derivative of y with respect to x is
a. $dy/dx = 5\,(4x+3)$.
b. $dy/dx = 40\,(4x+3)$.
c. $dy/dx = 10\,(4x+3)^2$.
d. $dy/dx = 4\,(x+3)$.
e. $dy/dx = 5\,(7)^2$.

(7) Which of the following functions has constant slope (or derivative)?
a. $y = 3 + 5x + (5/6)\, x^6$.
b. $y = 3\, x^{-6}$.
c. $y = \ln(6x)$.
d. $y = \exp(5x)$.
e. $y = 3 + 6\, x$.
f. $y = 3 + (5/x)$.

(8) If x increases by 5 percent, then ln(x) increases by about
a. 5 percent.
b. 0.05 percent.
c. ln(5), or about 1.609 units.
d. 0.05 units.
e. 5 units.

The next question refers to the following graph of $y = f(x)$.



(9) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at
a. no point on the graph.
b. one point on the graph.
c. two points on the graph.
d. three points on the graph.
e. four points on the graph.
f. more than four points on the graph.

(10) Suppose we have a function $y = f(x)$, which is continuously differentiable. At this function's maximum value,
a. $f(x) = 0$.
b. $df/dx = 0$.
c. df/dx is as large as possible.
d. $f(x) = 1$.
e. $df/dx = 1$

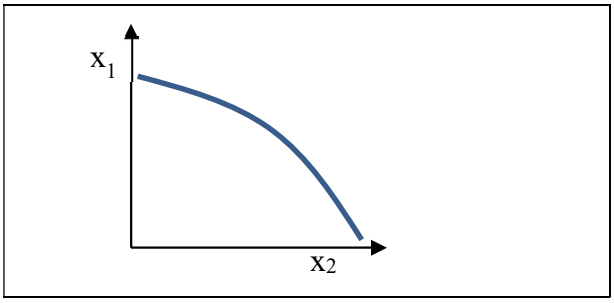
(11) If y is proportional to x, (that is, if $y = ax$, where a is an unknown constant) then the elasticity of y with respect to x equals
a. x.
b. a.
c. zero.
d. one-half.
e. one.

(12) Consider the following functions. Which has constant elasticity?
a. $y = 3 + 5x + (5/6)\, x^6$.
b. $y = 3\, x^{-6}$.
c. $y = \ln(6x)$.
d. $y = \exp(5x)$.
e. $y = 3 + 6\, x$.
f. $y = 3 + (5/x)$.

(13) A straight line has constant
a. slope.
b. elasticity.
c. both of the above.
d. none of the above.

(14) Suppose y depends on both x_1 and x_2, so that $y = f(x_1, x_2)$. By definition, $\partial y/\partial x_1$, the partial derivative of y with respect to x_1, is the ratio of the change in y to the change in x_1 when
a. x_1 is held constant.
b. x_2 is held constant.
c. y is held constant.
d. x_1 is held equal to x_2.

The next question refers to the following graph of a level curve, or contour, of the function $y = f(x_1,x_2)$.



(15) By definition, at all points along the curve in this graph,
a. the marginal rate of substitution is constant.
b. the value of x_1 is constant.
c. the value of x_2 is constant.
d. the values of both x_1, and x_2 are constant.
e. the value of y is constant.
f. all of the above.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 4 at a particular value of x. Moreover, the elasticity of y with respect to x equals 3 . Further suppose that x increases by 2 *percent*. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

b. By about how much?

percent

(2) [4 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 2$, and $\partial y/\partial x_2 = 3$, and that the partial elasticities are $\varepsilon_1 = 5$ and $\varepsilon_2 = 4$. Further suppose that x_1 increases by 4 *units* and simultaneously x_2 increases by 2 *units*. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

b. By about how much?

units

(3) [4 pts] GDP per capita equals GDP divided by population. Suppose GDP increases by 5 percent and population increases by 2 percent.

a. Will GDP per capita *increase* or *decrease*?

b. By about how much? percent

(4) [4 pts] A firm's revenue equals quantity sold times price. Suppose quantity increases by 3 percent and price decreases by 2 percent.

a. Will revenue *increase* or *decrease*?

b. By about how much? percent

(5) [8 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 5$, and $\partial y/\partial x_2 = 2$. First, suppose that x_1 increases by 4 units but x_2 does not change.

a. Will y *increase* or *decrease*?

b. By about how much? units

Now suppose that x_1 increases by 4 units but we want y to remain constant. To keep y constant, we must change the value of x_2.

c. Must x_2 *increase* or *decrease*?

d. By about how much? units

(6) [4 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 6$, and $\partial y/\partial x_2 = 2$. Now consider a graph of the level curve of this function, with x_1 on the vertical axis and x_2 on the horizontal axis.

a. Does the level curve of the function slope *up* or *down* at that point?

b. Give the slope of the level curve (dx_1/dx_2) at this point.

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = -x^2 + 10x - 15$.

a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

b. Compute the value x* that maximizes this function.

c. For what range of values of x does this function slope up? For what range of values does it slope down?

d. Find the maximum value, y*, of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = x_1^5 (x_2 + 1)^2$.

a. Find an expression for ε_1 , the partial elasticity of y with respect to x_1. The variable y should *not* appear in your answer. Simplify if possible.

b. Find an expression for ε_2 , the partial elasticity of y with respect to x_2. The variable y should *not* appear in your answer. Simplify if possible.

(3) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = (x_1 - 10)(x_2 - 4)^2$. The arguments x_1 and x_2 are greater than 10 and 4, respectively.

a. Find an expression for the partial derivative of y with respect to x_1.

b. Find an expression for the partial derivative of y with respect to x_2.

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = -3\,x_1^{-1} - 2\,x_2^{-1}$. The arguments x_1 and x_2 are nonnegative.

a. Find an expression for the partial derivative of y with respect to x_1.

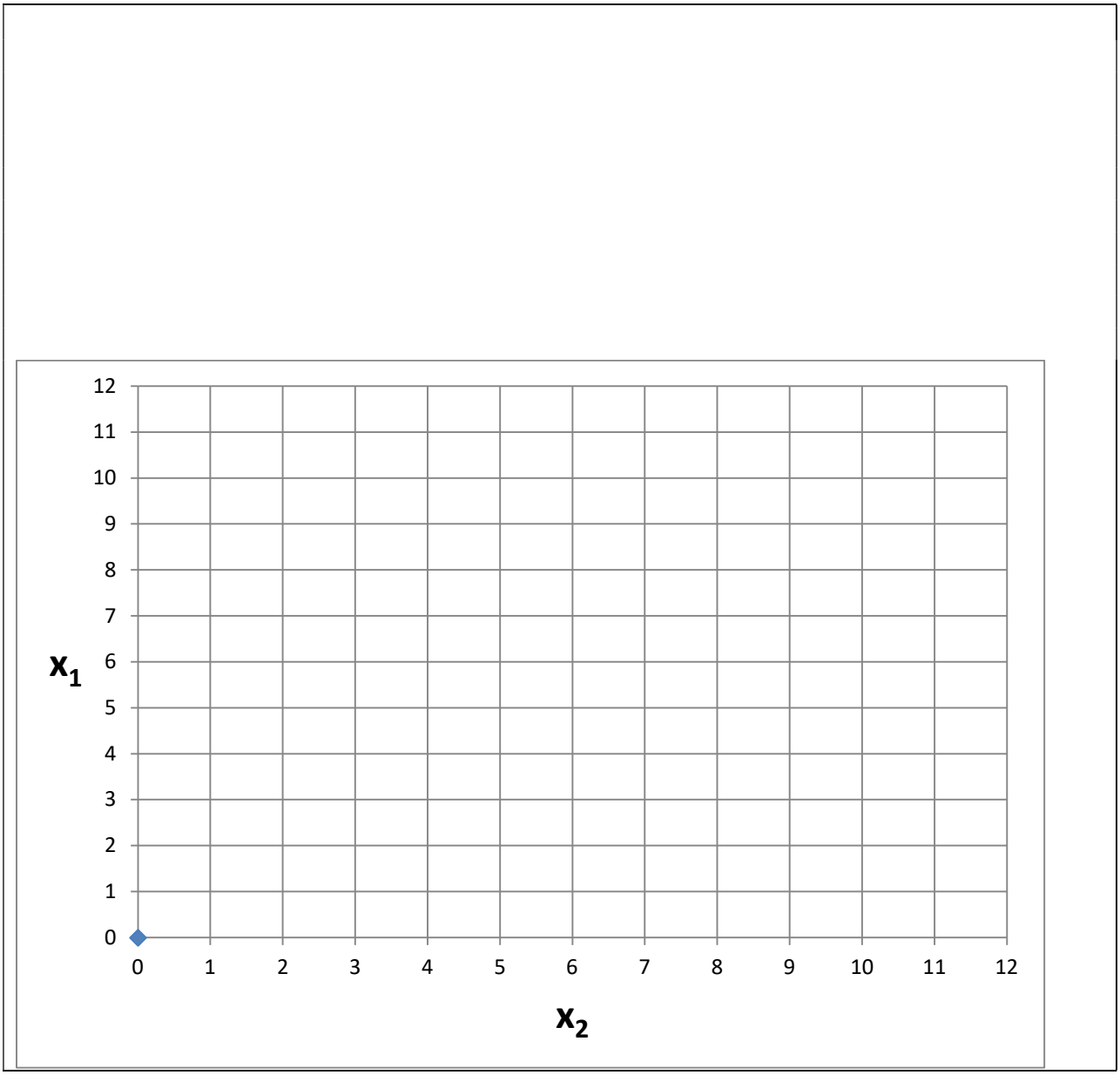
b. Find an expression for the partial derivative of y with respect to x_2.

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

IV. CRITICAL THINKING: [4 pts] Answer *one* question below (your choice). Circle the question you are answering. Justify your answer and show your work.

(1) Suppose $y = f(x_1,x_2)$. Further suppose $\partial y/\partial x_1$ is nonzero, but $\partial y/\partial x_2$ is always equal to zero. Do the level curves slope up or down? Are they curved or straight? Justify your answers. Sketch two level curves of this function in the graph below.

(2) Suppose $y = f(x_1,x_2)$. Further suppose $\partial y/\partial x_1 = \partial y/\partial x_2$ exactly for all values of x_1 and x_2. Do the level curves slope up or down? Are they curved or straight? Justify your answers. Sketch two level curves of this function in the graph below.



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