

EXAMINATION #4 ANSWER KEY

Version A

I. Multiple choice

(1)a. (2)e. (3)d. (4)d. (5)b. (6)b. (7)c. (8)c. (9)b. (10)c.

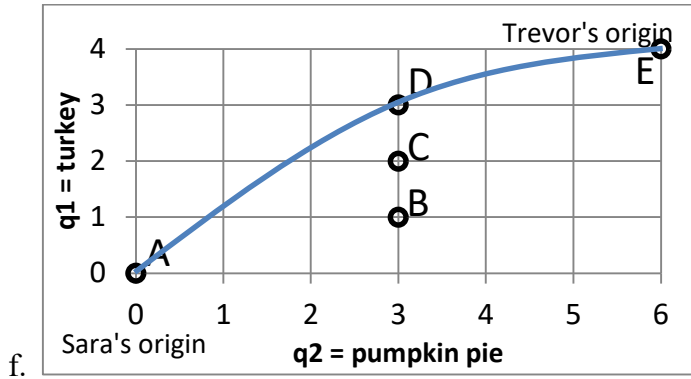
II. Short answer

- (1) a. 3 units of food. b. 1/3 units of clothing. c. slope = -3.
 d. $P_{\text{clothing}} = \$18$, because slope of each consumer's budget line = $-P_{\text{clothing}}/P_{\text{food}} = -3$.
- (2) a. $P_A = MC / (1 + [1/\epsilon_A]) = \40 .
 b. $P_C = MC / (1 + [1/\epsilon_C]) = \25 .
- (3)

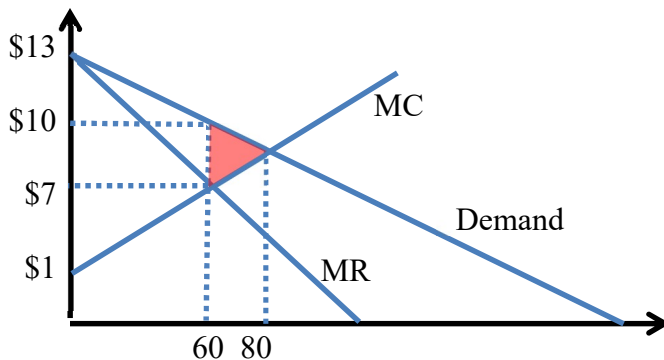
	$P=MC$	$P=AC$
a. Perfect competition:	yes	yes
b. Monopoly:	no	no
c. Monopolistic competition:	no	yes.
- (4) a. \$8. b. 6 thousand. c. \$0 because $P=MC$.
 d. $MR = 14 - 2Q$ ("Same intercept, twice the slope as demand").
 e. Plot MR as a straight line with P-intercept = \$14, slope = $-2/\text{thousand}$.
 f. \$10. g. 4 thousand. h. \$4 thousand.
- (5) a. Pareto optimal: no, yes, yes, yes.
 b. Dominant-strategy equilibria: yes, no, no, no.
 c. Nash equilibria in pure strategies: yes, no, no, no.

III. Problems

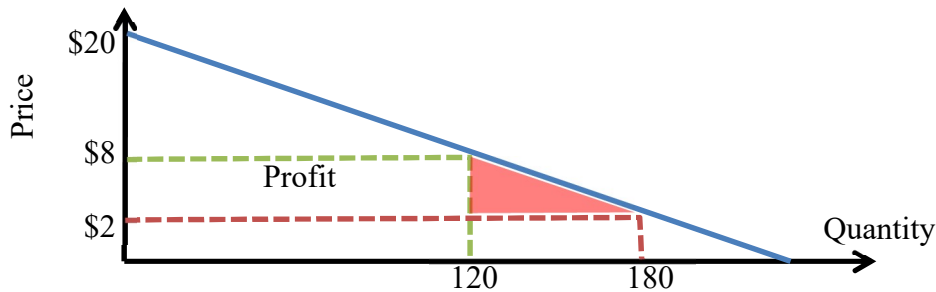
- (1) [Exchange efficiency] Note that Sara's $MRS_S = q_1/q_2$ and Trevor's $MRS_T = 3q_1/q_2$.
- a. **Yes**, A is Pareto-efficient, because no one can be made better off without someone else being made worse off. Trevor has everything, so he cannot be made better off. Sara has nothing, so she cannot be made better off without taking some of Trevor's turkey or pumpkin pie, which would make Trevor worse off. Put simply, since Trevor already has everything, any feasible change would make Trevor worse off.
- b. **No**, B is not Pareto-efficient, because $MRS_S = 1/3 \neq MRS_T = 3$.
- c. **No**, C is not Pareto-efficient, because $MRS_S = 2/3 \neq MRS_T = 2$.
- d. **Yes**, D is Pareto-efficient, because $MRS_S = 1 = MRS_T = 1$.
- e. **Yes**, E is Pareto-efficient, because no one can be made better off without someone else being made worse off. Sara has everything, so she cannot be made better off. Trevor has nothing, so he cannot be made better off without taking some of Sara's turkey or pumpkin pie, which would make Sara worse off. Put simply, since Sara already has everything, any feasible change would make Sara worse off.



- (2) [Monopoly, profit maximization]
- $MC = dTC/dQ = 1 + (Q/10)$.
 - $AC = TC/Q = 1 + (Q/20)$.
 - First find total revenue = $P \times Q = 13Q - (Q^2/20)$. So $MR = dTR/dQ = 13 - (Q/10)$.
 - Set $MC = MR$ and solve to get $Q_M = 60$.
 - Substitute into demand function: $P_M = 13 - (60/20) = \$10$.
 - Profit = $TR - TC = (60 \times 10) - (60 + 60^2/20) = \360 .
 - The efficient level of output lies where marginal cost intersects demand (“marginal cost pricing”). Find this quantity by setting $13 - (Q/20) = 1 + (Q/10)$, which yields $Q=80$. Then find $MC(80) = 1 + (80/10) = \$9$. Then evaluate DWL as the area of a triangle: **\$30** (see red triangle below).



- (3) [Cournot duopoly]
- $TR_1 = P q_1 = 20q_1 - (q_1^2/10) - (q_1q_2/10)$.
 - $MR_1 = \partial TR_1(q_1, q_2) / \partial q_1 = 10 - (2q_1/10) - (q_2/10)$.
 - Set $MR_1 = MC = \$2$ and solve to get $q_1^* = 90 - (q_2/2)$.
 - Since $q_1^* = q_2^*$, $q_1^* = 90 - q_1^*/2$. Solving yields $q_1^* = 60 = q_2^*$.
 - $Q^* = q_1^* + q_2^* = 120$. Substituting into demand equation: $P^* = 20 - (120/10) = \$8$.
 - Profit = $(P^* \times Q^*) - (AC \times Q^*) = (P^* - AC) \times Q^* = (8 - 2) \times 120 = \720 .
 - The efficient level of output lies where marginal cost intersects demand (“marginal cost pricing”). Find this quantity by setting $MC = \$2 = P = 20 - (Q/10)$ and solving to get $Q = 180$. Deadweight loss is the area between demand and marginal cost, from the Cournot equilibrium quantity $Q^*=120$ to the efficient quantity = 180 (see below). This is the area of a triangle, equal to **\$80** (see red triangle below).



IV. Critical thinking

(1) Perfect price discrimination.

The monopolist can charge a different price for every unit sold, so will set price equal to willingness-to-pay (the height of the demand curve) for that unit. To maximize profit, sell all units where willingness-to-pay is at least equal to marginal cost. So, quantity is at intersection of demand and marginal cost, which was found in problem (2a) to be $MC = 1 + (Q/10)$.

So set demand equal to marginal cost: $P = 13 - (Q/20) = 1 + (Q/10)$ and solve to get $Q^* = 80$.

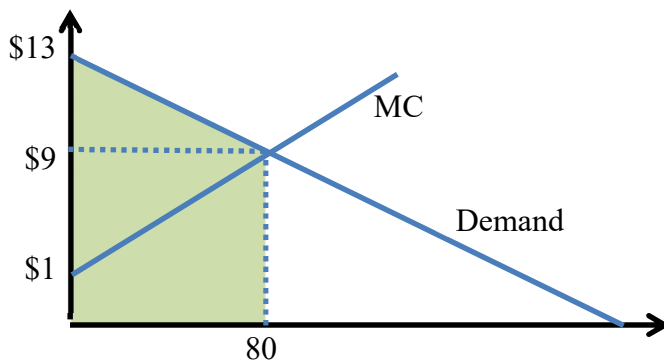
The highest price charged anyone is \$13, the intercept of the demand curve. The lowest price is \$9, at the intersection of demand and marginal cost. So total revenue is the area of a trapezoid shown in green below, whose parallel sides are \$13 and \$9, and whose height (measured sideways) is 80:

$$TR = (13+9)/2 \times 80 = \$880.$$

The total cost function is given as $Q + (Q^2/20)$, so $TC = 80 + (80^2/20) = \$400$.

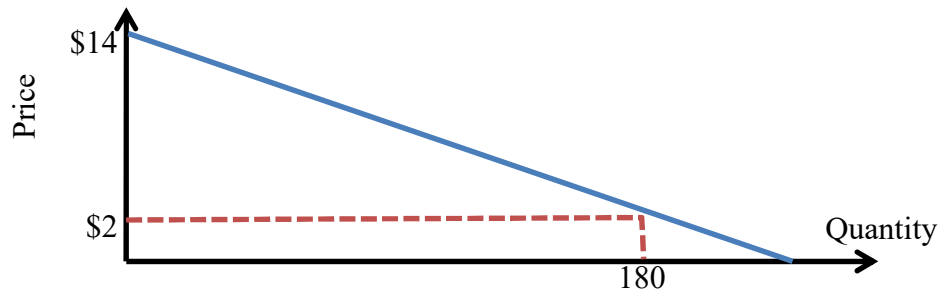
(Alternatively, TC can be computed as $AC \times 80$.) So,

profit = TR – TC = \$480. Note that profit is considerably higher with perfect price discrimination than with single-price monopoly.



(2) Price-setting (“Bertrand”) duopoly

In a price-setting duopoly, the Nash equilibrium is for both firms to set a price equal to marginal cost, which here is $P^* = \$2$. Substituting into the demand equation $\$2 = 20 - (Q/10)$ gives $Q^* = 180$. Since $P^* =$ marginal cost, **deadweight loss = 0**.



Version B

I. Multiple choice

(1)e. (2)b. (3)b. (4)b. (5)e. (6)d. (7)d. (8)e. (9)c. (10)d.

II. Short answer

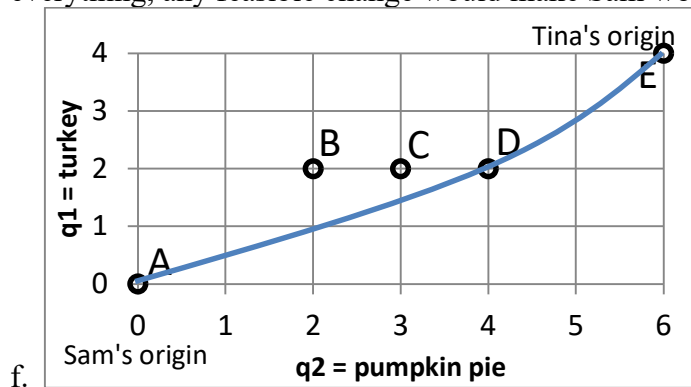
- (1) a. 2 units of food. b. 1/2 units of clothing. c. slope = -2.
 d. $P_{\text{clothing}} = \$12$, because slope of each consumer's budget line = $-P_{\text{clothing}}/P_{\text{food}} = -2$.
- (2) a. $P_A = MC / (1 + [1/\epsilon_A]) = \30 .
 b. $P_C = MC / (1 + [1/\epsilon_C]) = \12 .
- (3)

	<u>P=AC</u>	<u>P=MC</u>
b. Monopoly:	no	no
a. Perfect competition:	yes	yes
c. Monopolistic competition:	yes	no.
- (4) a. \$3. b. 9 thousand. c. \$0 because $P=MC$.
 d. $MR = 12 - 2Q$ ("Same intercept, twice the slope as demand").
 e. Plot MR as a straight line with P-intercept = \$12, slope = -2/thousand.
 f. \$7. g. 5 thousand. h. \$10 thousand.
- (5) a. Pareto optimal: yes, no, no, yes.
 b. Dominant-strategy equilibria: no, no, no, no.
 c. Nash equilibria in pure strategies: yes, no, no, yes.

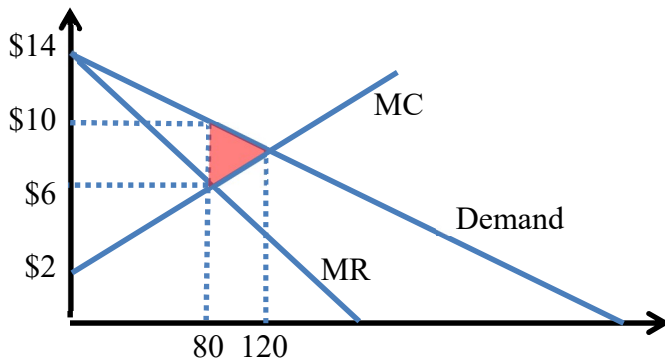
III. Problems

- (1) [Exchange efficiency] Note that Sam's $MRS_S = 2q_1/q_2$ and Tina's $MRS_T = q_1/q_2$.
- a. **Yes**, A is Pareto-efficient, because no one can be made better off without someone else being made worse off. Tina has everything, so she cannot be made better off. Sam has nothing, so he cannot be made better off without taking some of Tina's turkey or pumpkin pie, which would make Tina worse off. Put simply, since Tina already has everything, any feasible change would make Tina worse off.
- b. **No**, B is not Pareto-efficient, because $MRS_S = 2 \neq MRS_T = 1/2$.
- c. **No**, C is not Pareto-efficient, because $MRS_S = 4/3 \neq MRS_T = 2/2$.
- d. **Yes**, D is Pareto-efficient, because $MRS_S = 1 = MRS_T = 1$.
- e. **Yes**, E is Pareto-efficient, because no one can be made better off without someone else being made worse off. Sam has everything, so he cannot be made better off. Tina has

nothing, so she cannot be made better off without taking some of Sam's turkey or pumpkin pie, which would make Sam worse off. Put simply, since Sam already has everything, any feasible change would make Sam worse off.

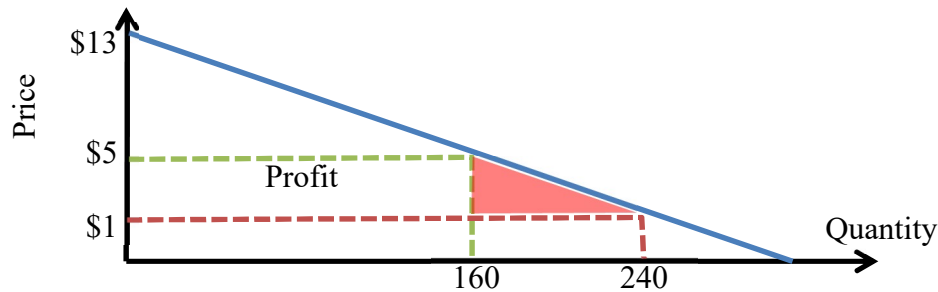


- (2) [Monopoly, profit maximization]
- $MC = dTC/dQ = 2 + (Q/20)$.
 - $AC = TC/Q = 2 + (Q/40)$.
 - First find total revenue $= P \times Q = 14Q - (Q^2/20)$. So $MR = dTR/dQ = 14 - (Q/10)$.
 - Set $MC = MR$ and solve to get $Q_M = 80$.
 - Substitute into demand function: $P_M = 14 - (80/20) = \$10$.
 - Profit $= TR - TC = (80 \times 10) - (2 \times 80 + 80^2/40) = \480 .
 - The efficient level of output lies where marginal cost intersects demand ("marginal cost pricing"). Find this quantity by setting $14 - (Q/20) = 2 + (Q/20)$, which yields $Q=120$. Then find $MC(80) = 2 + (80/20) = \$7$. Then evaluate DWL as the area of a triangle: **\$80** (see red triangle below).



- (3) [Cournot duopoly]
- $TR_1 = P q_1 = 13q_1 - (q_1^2/20) - (q_1q_2/20)$.
 - $MR_1 = \partial TR_1(q_1, q_2) / \partial q_1 = 13 - (2q_1/20) - (q_2/20)$.
 - Set $MR_1 = MC = \$2$ and solve to get $q_1^* = 120 - (q_2/2)$.
 - Since $q_1^* = q_2^*$, $q_1^* = 120 - q_1^*/2$. Solving yields $q_1^* = 80 = q_2^*$.
 - $Q^* = q_1^* + q_2^* = 160$. Substituting into demand equation: $P^* = 13 - (160/20) = \$5$.
 - Profit $= (P^* \times Q^*) - (AC \times Q^*) = (P^* - AC) \times Q^* = (5 - 1) \times 160 = \640 .

g. The efficient level of output lies where marginal cost intersects demand (“marginal cost pricing”). Find this quantity by setting $MC = \$1 = P = 13 - (Q/20)$ and solving to get $Q = 240$. Deadweight loss is the area between demand and marginal cost, from the Cournot equilibrium quantity $Q^*=160$ to the efficient quantity $= 240$ (see below). This is the area of a triangle, equal to **\$160** (see red triangle below).



IV. Critical thinking

(1) Perfect price discrimination.

The monopolist can charge a different price for every unit sold, so will set price equal to willingness-to-pay (the height of the demand curve) for that unit. To maximize profit, sell all units where willingness-to-pay is at least equal to marginal cost. So, quantity is at intersection of demand and marginal cost, which was found in problem (2a) to be $MC = 2 + (Q/20)$.

So set demand equal to marginal cost: $P = 14 - (Q/20) = 2 + (Q/20)$ and solve to get **$Q^* = 120$** .

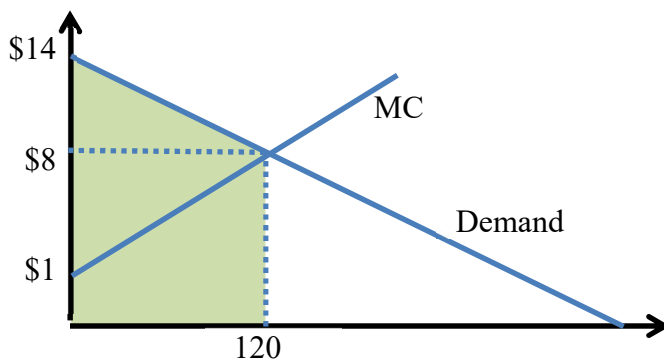
The highest price charged anyone is \$14, the intercept of the demand curve. The lowest price is \$8, at the intersection of demand and marginal cost. So total revenue is the area of a trapezoid shown in green below, whose parallel sides are \$14 and \$8, and whose height (measured sideways) is 120:

$$TR = (14+8)/2 \times 120 = \$1320.$$

The total cost function is given as $2Q + (Q^2/40)$, so $TC = 120 + (120^2/40) = \320 .

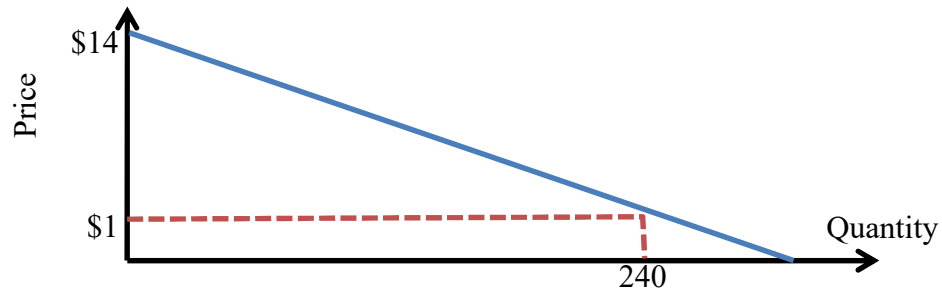
(Alternatively, TC can be computed as $AC \times 120$.) So,

profit = $TR - TC = \$720$. Note that profit is considerably higher with perfect price discrimination than with single-price monopoly.



(2) Price-setting (“Bertrand”) duopoly

In a price-setting duopoly, the Nash equilibrium is for both firms to set a price equal to marginal cost, which here is $P^* = \$1$. Substituting into the demand equation $\$1 = 13 - (Q/20)$ gives $Q^* = 240$. Since $P^* =$ marginal cost, **deadweight loss = 0**.



[end of answer key]