ECON 173 - Intermediate Microeconomic Analysis
Drake University, Fall 2023
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EXAMINATION #2 ANSWER KEY
"Consumers and Demand"

Version A

I. Multiple choice

(1)b . (2)d. (3)b. (4)e. (5)b. (6)d. (7)b. (8)a. (9)c. (10)b.

II. Short answer

(1) a. elastic. b. decrease. c. 6 %.
d. decrease. e. 2 %.

(2) a. luxury or superior good. b. increase. c. 7 %.
d. increase. e. 2 %.

(3) Note: This graph is based on Slutsky's approach to income and substitution effects, not Hicks's approach.
a. $4. b. 10 units. c. $10.
d. 3 units. e. – 5 units. f. – 2 units.

(4) a. $\varepsilon^{comp} = -0.34$. b. decrease. c. 4 % (using ε).
d. decrease e. 3.4 % (using ε^{comp}).

(5) a. Laspeyres = 140. b. Paasche = 130. c. $\sqrt{140 \times 130}$.

III. Problems

(1) [Budgets and choice]
a. Equation for budget line (income=spending): $100 = 5\, q_1 + 4\, q_2$.
b. $MRSC = MU_2/MU_1 = q_1 / (q_2 - 5)$.
c. Solve the tangency condition ($MRSC = p_2/p_1 = 4/5$) jointly with equation for budget line (see part a) to get $q_1^* = 12$, $q_2^* = 10$.

(2) [Properties of individual demand functions]
a. Homogeneity of degree zero means that if income and all prices are multiplied by the same factor, the quantity demanded never changes. Check homogeneity of the demand function by replacing I by (aI), replacing p_1 by (ap_1), and replacing p_2 by (ap_2), where a is an arbitrary factor:

$$10\,(a\,I)^{0.9}\,(a\,p_1)^{-0.3}\,(a\,p_2)^{0.1}$$
$$=\ a^{0.9}\,a^{-0.3}\,a^{0.1}\,(10\,I^{0.9}\,p_1^{-0.3}\,p_2^{0.1})$$
$$=\ a^{0.7}\,(10\,I^{0.9}\,p_1^{-0.3}\,p_2^{0.1})$$

Note that the "a" factor does *not* cancel. So multiplying income and prices by some arbitrary positive factor a *does* change the quantity demanded. The function is NOT homogeneous of degree zero in income and prices.

b. $\frac{\partial q_1^*}{\partial p_1} = -0.3\ (10\ I^{0.9}\ p_1^{-1.3}\ p_2^{0.1})$. This expression is negative, so good #1 is an ordinary good, not a Giffen good.
c. $\frac{\partial q_1^*}{\partial I} = \ 0.9\ (10\ I^{0.9}\ p_1^{-0.3}\ p_2^{0.1})$. This expression is positive, so good #1 is a normal good, not an inferior good.
d. $\frac{\partial q_1^*}{\partial p_2} = \ 0.1\ (10\ I^{0.9}\ p_1^{-0.3}\ p_2^{-0.9})$. This expression is positive, so goods #1 and #2 are substitutes, not complements.

(3) [Finding individual demand functions]
a. MRS = $MU_2/MU_1 = (5\ q_1^2\ q_2^4) / (2\ q_1\ q_2^5) = (5q_1) / (2q_2)$.
b. Solve MRS = $(5q_1) / (2q_2) = p_2/p_1$ (the tangency condition) jointly with $I = p_1q_1 + p_2q_2$ (the budget constraint) to get $q_1^* = \frac{2I}{7\ p_1}$, and
c. $q_2^* = \frac{5I}{7\ p_2}$.
[There are many ways to solve the equations jointly. One way to begin is to cross-multiply the tangency condition to get $(5/2)p_1\ q_1 = p_2\ q_2$. Then substitute into the budget constraint: $I = p_1\ q_1 + (5/2)p_1\ q_1$. Then solve for q_1^* as a function of p_1 and I. Finally, substitute your expression for q_1^* into either the budget constraint or the tangency condition and solve for q_2^* as a function of p_2 and I.]

IV. Critical thinking

(1) a. Target indifference curve: $600 = q_1\ q_2$.
b. Tangency condition: MRSC = p_2/p_1 , which here is $q_1/q_2 = 2/3$.
c. Solving the two equations above jointly gives $q_1^* = 20$ and $q_2^* = 30$.
d. Minimum income required is $p_1\ q_1^* + p_2\ q_2^* = 3\times20 + 2\times30 =$ $\boxed{\$120.}$

(2) Demand is given by $q_1^* = 15\ p_1^{\varepsilon}\ p_2^{\alpha}\ I^{\eta}$. Homogeneity of degree zero means that if income and prices are all multiplied by some factor (a) then that factor cancels out, leaving the original demand function. In other words, quantity demanded is unaffected by perfectly balanced inflation. So we check homogeneity of the demand function by replacing p_1 by (ap_1), replacing p_2 by (ap_2), and replacing I by (aI), and simplifying as follows:

$$q_1^* = 15\ (ap_1)^{\varepsilon}(ap_2)^{\alpha}(aI)^{\eta}$$
$$q_1^* = 15\ \ a^{\varepsilon}\ p_1^{\varepsilon}\ \ a^{\alpha}\ p_2^{\alpha}\ \ a^{\eta}\ I^{\eta}$$
$$q_1^* = a^{\varepsilon}a^{\alpha}a^{\eta}\ \ \ 15\ p_1^{\varepsilon}\ p_2^{\alpha}\ I^{\eta}$$
$$q_1^* = a^{\varepsilon+\alpha+\eta}\ \ \ 15\ p_1^{\varepsilon}\ p_2^{\alpha}\ I^{\eta}$$

Now the factor (a) cancels out if and only if $a^{\varepsilon+\alpha+\eta} = 1$, which is true if and only if the exponent equals zero: $\boxed{\boldsymbol{\varepsilon + \alpha + \eta = 0},}$ answer.

Version B

I. Multiple choice

(1)c . (2)d. (3)d. (4)e. (5)e. (6)b. (7)c. (8)b. (9)c. (10)d.

II. Short answer

(1) a. inelastic. b. increase. c. 3 %.
d. decrease. e. 1 %.

(2) a. necessary good. b. increase. c. 3 %.
d. decrease. e. 2 %.

(3) Note: This graph is based on Slutsky's approach to income and substitution effects, not Hicks's approach.
a. \$2. b. 10 units. c. \$8.
d. 2 units. e. -6 units. f. -2 units.

(4) a. $\varepsilon^{comp} = -0.56$. b. decrease. c. 6 % (using ε).
d. decrease e. 5.6 % (using ε^{comp}).

(5) a. Laspeyres = 160. b. Paasche = 130. c. $\sqrt{160 \times 130}$.

III. Problems

(1) [Budgets and choice]
a. Equation for budget line (income=spending): $100 = 5\, q_1 + 4\, q_2$.
b. MRSC = $MU_2/MU_1 = \frac{q_1+4}{q_2}$.
c. Solve the tangency condition (MRSC = p_2/p_1 = 4/5) jointly with equation for budget line (see part a) to get $q_1^* = 8$, $q_2^* = 15$.

(2) [Properties of individual demand functions]
a. Homogeneity of degree zero means that if income and all prices are multiplied by the same factor, the quantity demanded never changes. Check homogeneity of the demand function by replacing I by (aI), replacing p_1 by (ap_1), and replacing p_2 by (ap_2) , where a is an arbitrary factor:

$$\big((aI) + 2(ap_2)\big)(ap_1)^{-1} + 7$$
$$= a\big((I) + 2(p_2)\big)a^{-1}(p_1)^{-1} + 7$$
$$= (I + 2p_2)p_1^{-1} + 7.$$

Note that the "a" factor cancels. So multiplying income and prices by some arbitrary positive factor a *does not* change the quantity demanded. The function is homogeneous of degree zero in income and prices.

b. $\frac{\partial q_1^*}{\partial p_1} = (I + 2p_2)(-1)p_1^{-2}$. This expression is negative, so good #1 is an ordinary good, not a Giffen good.

c. $\frac{\partial q_1^*}{\partial I} = p_1^{-1}$. This expression is positive, so good #1 is a normal good, not an inferior good.

d. $\frac{\partial q_1^*}{\partial p_2} = 2\, p_1^{-1}$. This expression is positive, so goods #1 and #2 are substitutes, not complements.

(3) [Finding individual demand functions]

a. MRS = $MU_2/MU_1 = (4\, q_1^3\, q_2^3) / (3\, q_1^2\, q_2^4) = (4q_1) / (3q_2)$.

b. Solve MRS = $(4q_1) / (3q_2) = p_2/p_1$ (the tangency condition) jointly with $I = p_1q_1 + p_2q_2$ (the budget constraint) to get $q_1^* = \frac{3\,I}{7\,p_1}$, and

c. $q_2^* = \frac{4I}{7\,p_2}$.

[There are many ways to solve the equations jointly. One way to begin is to cross-multiply the tangency condition to get $(4/3)\, p_1\, q_1 = p_2\, q_2$. Then substitute into the budget constraint: $I = I = p_1q_1 + (4/3)\, p_1\, q_1$. Then solve for q_1^* as a function of p_1 and I. Finally, substitute your expression for q_1^* into either the budget constraint or the tangency condition and solve for q_2^*.]

IV. Critical thinking

Same as Version A.

[end of answer key]