ECON 173 - Intermediate Microeconomic Analysis
Drake University, Fall 2023
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EXAMINATION #1 ANSWER KEY
"Mathematical Tools"

Version A

I. Multiple choice

(1)b. (2)c. (3)d. (4)d. (5)a. (6)c. (7)e. (8)c. (9)b. (10)b.
(11)c. (12)b. (13)a. (14)c. (15)b.

II. Short answer

(1) a. increase b. 10 units, using derivative since change is given in units.
(2) a. increase b. 10 percent, using elasticities since changes are given in percent.
(3) a. increase b. 4 percent, using approximation rule for products.
(4) a. increase b. 2 percent, using approximation rule for ratios.
(5) a. increase b. 36 units c. decrease d. 12 units.
(6) a. down b. slope = $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{2}{3}$.

III. Problems

(1) a. dy/dx = -4x + 20. b. Set dy/dx=0 and solve to get x* = 5.
c. The function slopes up if dy/dx = -4x + 20 > 0, which implies x < 5.
The function slopes down if dy/dx = -4x + 20 < 0, which implies x > 5.
d. y* = f(x*) = f(5) = 53.

(2) a. $\varepsilon_1 = \frac{\partial y}{\partial x_1}\frac{x_1}{y} = 2\,(x_1+5)\,x_2^4\;\frac{x_1}{(x_1+5)^2\,x_2^4} = \frac{2x_1}{x_1+5}$.

b. $\varepsilon_2 = \frac{\partial y}{\partial x_2}\frac{x_2}{y} = (x_1+5)\,4\,x_2^4\;\frac{x_2}{(x_1+5)^2\,x_2^4} = 4$.

(3) a. $\frac{\partial y}{\partial x_1} = 7\,x_1^{-2}$ b. $\frac{\partial y}{\partial x_2} = 5\,x_2^{-2}$

c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{5\,x_2^{-2}}{7\,x_1^{-2}} = \left(\frac{5}{7}\right)\left(\frac{x_1}{x_2}\right)^2$

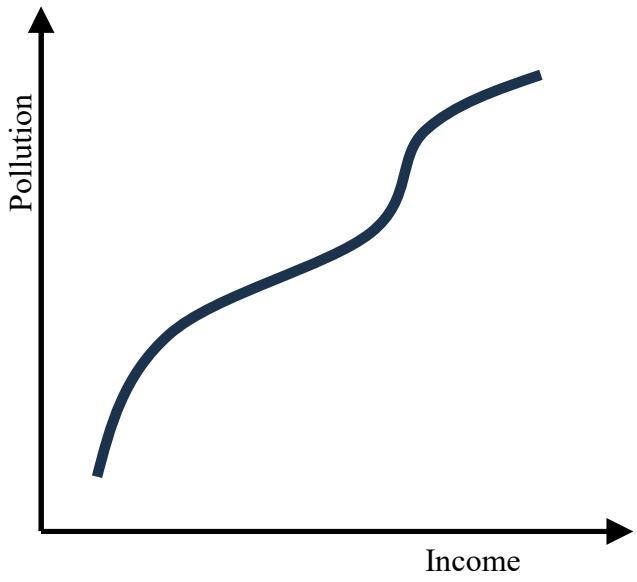
(4) a. $\frac{\partial y}{\partial x_1} = 3\,(x_1+4)^2\,(x_2+2)$ b. $\frac{\partial y}{\partial x_2} = (x_1+4)^3$

c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{(x_1+4)^3}{3\,(x_1+4)^2\,(x_2+2)} = \frac{(x_1+4)}{3\,(x_2+2)}$.

IV. Critical thinking

(1) When f(x) is maximized, $0 = \frac{df}{dq} = \frac{dr}{dq} - \frac{dc}{dq}$. So $\frac{dr}{dq} = \frac{dc}{dq}$. In words, marginal revenue equals marginal cost.

(2) The slope of the level curves of any function is the negative of the ratio of partial derivatives. Here, the function is U = f (I,P) so the slope of the level curves (with I on the horizontal axis and P on the vertical axis) is $-\frac{\partial U/\partial I}{\partial U/\partial P}$. We are given that ∂U/∂I is positive and ∂U/∂P is negative. Therefore, the slope of the level curves is positive and they slope up. A typical level curve is sketched below.



Version B

I. Multiple choice

(1)a. (2)d. (3)e. (4)a. (5)d. (6)e. (7)a. (8)c. (9)d. (10)d.
(11)d. (12)d. (13)b. (14)a. (15)a.

II. Short answer

(1) a. increase b. 3 percent, using elasticities since changes are given in percent.
(2) a. increase b. 17 units, using derivative since change is given in units.
(3) a. increase b. 6 percent, using approximation rule for products.
(4) a. increase b. 3 percent, using approximation rule for ratios.
(5) a. increase b. 20 units c. decrease d. 2 units.
(6) a. up b. slope = $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{-10}{5} = 2$.

III. Problems

(1) a. dy/dx = x - 4. b. Set dy/dx=0 and solve to get x* = 4.
c. The function slopes up if dy/dx = x – 4 > 0, which implies x>4.
The function slopes down if dy/dx = x – 4 < 0, which implies x<4.
d. y* = f(x*) = f(4) = -1.

(2) a. $\varepsilon_1 = \frac{\partial y}{\partial x_1}\frac{x_1}{y} = 3x_1^2\,(x_2-1)^2\,\frac{x_1}{x_1^3\,(x_2-1)^2} = 3$.

b. $\varepsilon_2 = \frac{\partial y}{\partial x_2}\frac{x_2}{y} = x_1^3\,2\,(x_2-1)\,\frac{x_2}{x_1^3\,(x_2-1)^2} = \frac{2\,x_2}{x_2-1}$.

(3) a. $\frac{\partial y}{\partial x_1} = 2\,(x_1-1)(x_2-3)^4$ b. $\frac{\partial y}{\partial x_2} = (x_1-1)^2\,4(x_2-3)^3$

c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{(x_1-1)^2\,4(x_2-3)^3}{2(x_1-1)(x_2-3)^4} = \frac{2\,(x_1-1)}{(x_2-3)}$.

(4) a. $\frac{\partial y}{\partial x_1} = 2x_1^{-1/2}$ b. $\frac{\partial y}{\partial x_2} = 5\,x_2^{-1/2}$

c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{5\,x_2^{-1/2}}{2\,x_1^{-1/2}} = \left(\frac{5}{2}\right)\left(\frac{x_1}{x_2}\right)^{1/2}$.

IV. Critical thinking

(Same as version A.)

[end of answer key]