EXAMINATION #1 ANSWER KEY "Mathematical Tools"

Version A

I. Multiple choice

(4)c.(1)c. (2)c. (3)e.(5)b.(6)c. (7)c.(8)c.(9)c.(10)d.

(11)d. (12)f. (13)b. (14)a. (15)a.

II. Short answer

(1) a. decrease b. 2 percent, using elasticities since changes are given in percent.

b. 38 units, using derivative since change is given in units. (2) a. increase

b. 8 percent, using approximation rule for products. (3) a. increase

(4) a. increase b. 7 percent, using approximation rule for ratios.

b. 12 units (5) a. increase c. decrease d. 4 units.

b. slope = $-\frac{\partial y/\partial x_2}{\partial y/\partial x_3} = -\frac{6}{-2} = 3$. (6) a. up

III. Problems

b. Set dy/dx=0 and solve to get $x^* = 5$. (1) a. dy/dx = 4x - 20.

c. The function slopes up if dy/dx = 4x - 20 > 0, which implies x > 5.

The function slopes down if dy/dx = 4x - 20 < 0, which implies x<5.

d.
$$y^* = f(x^*) = f(5) = -47$$
.
(2) a. $\varepsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} = \frac{2x_1}{x_1 - 3}$.
b. $\varepsilon_2 = \frac{\partial y}{\partial x_2} \frac{x_2}{y} = 3$.

(3) a.
$$\frac{\partial y}{\partial x_1} = 2(x_1 - 5) (x_2 - 4)^3$$
 b. $\frac{\partial y}{\partial x_2} = (x_1 - 5)^2 3(x_2 - 4)^2$

c.
$$MRS = \frac{\partial y/\partial z}{\partial y/\partial x_1} = \frac{(x_1 - 5)^2 3(x_2 - 4)^2}{2(x_1 - 5) (x_2 - 4)^3} = \frac{3(x_1 - 5)}{2(x_2 - 4)}$$
.

(4) a.
$$\frac{\partial y}{\partial x_1} = 7x_1^{-2}$$
 b. $\frac{\partial y}{\partial x_2} = 5x_2^{-2}$ c. $MRS = \frac{\partial y/\partial z}{\partial y/\partial x_1} = \frac{5x_2^{-2}}{7x_1^{-2}} = \left(\frac{5}{7}\right)\left(\frac{x_1}{x_2}\right)^2$.

IV. Critical thinking

- When f(x) is maximized, $0 = \frac{df}{dx} = \frac{db}{dx} \frac{dc}{dx}$. So $\frac{db}{dx} = \frac{dc}{dx}$. In words, marginal benefit (1) equals marginal cost.
- We are given that $P \times Q = c$, where c is some constant. Therefore $Q = c/P = c \ P^{-1}$. So (2) the price elasticity of Q with respect to P ("price elasticity of demand") is found as

$$\varepsilon = \left(\frac{dQ}{dP}\right) \left(\frac{P}{Q}\right) = (c(-1)P^{-2}) \left(\frac{P}{CP^{-1}}\right) = -1.$$

The elasticity does *not* depend on the value of c.

Version B

I. Multiple choice

(1)d. (2)b. (3)d.(4)d. (5)c.(6)b. (7)e. (8)d. (9)a. (10)b.

(11)e. (12)b. (13)a. (14)b. (15)e.

II. Short answer

b. 6 units, using derivatives since changes are given in units. (2) a. increase

(1) a. increase b. 5 percent, using elasticity since change is given in percent.

a. decrease b. 4 percent, using approximation rule for products. (3)

b. 2 percent, using approximation rule for ratios. **(4)** a. decrease

b. 24 units a. increase c. decrease (5) d. 8 units.

b. slope = $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -\frac{3}{5}$. (6) a. down

III. Problems

a. dy/dx = -x - 10. b. Set dy/dx=0 and solve to get $x^* = -10$. (1)

> c. The function slopes up if dy/dx = -x - 10 > 0, which implies -10 > x. The function slopes down if dy/dx = -x - 10 < 0, which implies -10 < x.

d. $y^* = f(x^*) = f(10) = 57$.

(2) a.
$$\varepsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} = 5x_1^4 (x_2 + 1)^4 \frac{x_1}{x_1^5 (x_2 + 1)^4} = 5$$
. b. $x_1^5 4(x_2 + 1)^3 \frac{x_2}{x_1^5 (x_2 + 1)^4} = \frac{4x_2}{x_2 + 1}$.
(3) a. $\frac{\partial y}{\partial x_1} = 6\left(\frac{1}{2}\right) x_1^{-1/2} = 3x_1^{-1/2}$ b. $\frac{\partial y}{\partial x_2} = 2\left(\frac{1}{2}\right) x_2^{-1/2} = x_2^{-1/2}$

(3) a.
$$\frac{\partial y}{\partial x_1} = 6\left(\frac{1}{2}\right)x_1^{-1/2} = 3x_1^{-1/2}$$
 b. $\frac{\partial y}{\partial x_2} = 2\left(\frac{1}{2}\right)x_2^{-1/2} = x_2^{-1/2}$

c.
$$MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{x_2^{-1/2}}{3x_1^{-1/2}} = \left(\frac{1}{3}\right) \left(\frac{x_1}{x_2}\right)^{1/2}$$

(4) a.
$$\frac{\partial y}{\partial x_1} = 3(x_1 + 2)^2 (x_2 + 1)^4$$
 b. $\frac{\partial y}{\partial x_2} = (x_1 + 2)^3 4(x_2 + 1)^3$ c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{(x_1 + 2)^3 4(x_2 + 1)^3}{3(x_1 + 2)^2 (x_2 + 1)^4} = \frac{4(x_1 + 2)}{3(x_2 + 1)}$.

IV. Critical thinking

(Same as version A.)

[end of answer key]