ECON 173 - Intermediate Microeconomic Analysis
Drake University, Fall 2022
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# EXAMINATION #1 VERSION A
# "Mathematical Tools"
# September 6, 2022

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

**I. MULTIPLE CHOICE:** Circle the one best answer to each question. Use margins for scratch work. [2 pts each—30 pts total]

(1) If the derivative of a function is positive, then
a. the value of the function is positive.
b. the value of the function is negative.
c. the graph of the function slopes up.
d. the graph of the function slopes down.

(2) Suppose $y = 2 - (3/x)$. Then the derivative of y with respect to x is given by the formula
a. $dy/dx = -3$.
b. $dy/dx = -3/x^2$ .
c. $dy/dx = 3/x^2$ .
d. $dy/dx = 3x$ .
e. none of the above.

(3) Suppose $y = -4x^2 + 3x + 2$. Then the derivative of y with respect to x is
a. $dy/dx = -4$ .
b. $dy/dx = 1$ .
c. $dy/dx = 3$ .
d. $dy/dx = -4x + 3$ .
e. $dy/dx = -8x + 3$ .
f. $dy/dx = -8x^2 + 3x + 2$ .

(4) Suppose $y = (5+x)^3$. Then the derivative of y with respect to x is given by
a. $dy/dx = 5$ .
b. $dy/dx = 3x^2$ .
c. $dy/dx = 3(5+x)^2$ .
d. $dy/dx = 3(5+x)$.
e. none of the above.

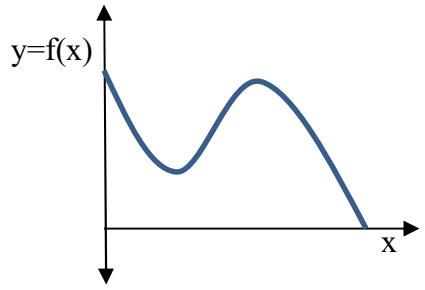
(5) Suppose $y = x^{1/2}$. Then the derivative of y with respect to x is given by
a. $dy/dx = x/2$.
b. $dy/dx = x^{-1/2} / 2$ .
c. $dy/dx = x^{3/2}$ .
d. $dy/dx = (1/2)\, x$ .
e. none of the above.

(6) Suppose $y = 2\,(3x+4)^5$. Then the derivative of y with respect to x is
a. $dy/dx = 5\,(3x+4)^4$.
b. $dy/dx = 10\,(3x+4)^4$ .
c. $dy/dx = 30\,(3x+4)^4$ .
d. $dy/dx = 2\,(3x+4)^4$ .
e. $dy/dx = 2\,(3)^5$ .

(7) Which of the following functions has constant slope (or derivative)?
a. $y = \ln(5x)$ .
b. $y = \exp(3x)$ .
c. $y = 2 + 5x$ .
d. $y = 2 + (3/x)$ .
e. $y = 2 + 3x + (3/5)x^5$ .
f. $y = 2x^{-5}$ .

(8) If x increases by 3 percent, then ln(x) increases by about
a. ln(3), or about 1.099 units.
b. 0.03 percent.
c. 0.03 units.
d. 3 percent.
e. 3 units.

The next question refers to the following graph of $y = f(x)$ .




(9) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at
a. no point on the graph.
b. one point on the graph.
c. two points on the graph.
d. three points on the graph.
e. four points on the graph.
f. more than four points on the graph.

(10) Suppose we have a function $y = f(x)$, which is continuously differentiable. At this function's minimum value,
a. $f(x) = -1$.
b. $df/dx = -1$
c. $f(x) = 0$.
d. $df/dx = 0$.
e. df/dx is as negative as possible.

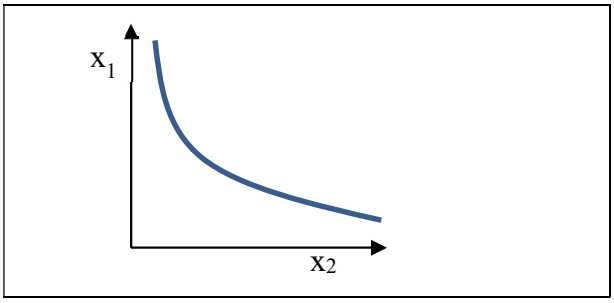
(11) If y is inversely proportional to x (that is, if $y = a/x$ , where a is an unknown constant) then the elasticity of y with respect to x equals
a. zero.
b. one-half.
c. one.
d. minus one.
e. -a.

(12) Consider the following functions. Which has constant elasticity?
a. $y = \ln(5x)$ .
b. $y = \exp(3x)$ .
c. $y = 2 + 5x$ .
d. $y = 2 + (3/x)$ .
e. $y = 2 + 2x + (3/5)x^5$ .
f. $y = 2x^{-5}$ .

(13) A straight line does *not* have constant
a. slope.
b. elasticity.
c. It has neither constant slope nor constant elasticity.
d. It has both constant slope and constant elasticity.

(14) Suppose y depends on both $x_1$ and $x_2$, so that $y = f(x_1, x_2)$. By definition, $\partial y/\partial x_2$, the partial derivative of y with respect to $x_2$, is the ratio of the change in y to the change in $x_2$ when
a. $x_1$ is held constant.
b. $x_2$ is held constant.
c. y is held constant.
d. $x_1$ is held equal to $x_2$.

The next question refers to the following graph of a level curve, or contour, of the function $y = f(x_1,x_2)$.




(15) By definition, all points along the curve in this graph have identical values of
a. y .
b. $x_1$ .
c. $x_2$ .
d. both $x_1$ and $x_2$ .
e. the marginal rate of substitution.
f. all of the above.

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**II. SHORT ANSWER:** Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals -2 at a particular value of x. Moreover, the elasticity of y with respect to x equals -0.5 . Further suppose that x increases by 4 *percent*. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

b. By about how much?

| |
|---|
| percent |

(2) [4 pts] Consider the function $y = f(x_1,x_2)$ . Suppose at a particular point, $\partial y/\partial x_1 = 4$, and $\partial y/\partial x_2 = 6$ , and that the partial elasticities are $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 1.5$. Further suppose that $x_1$ increases by 5 *units* and simultaneously $x_2$ increases by 3 *units*. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

b. By about how much?

| |
|---|
| units |

(3) [4 pts] Consumer spending on gasoline equals price (per gallon) times quantity (number of gallons). Suppose price increases by10 percent and quantity decreases by 2 percent.

a. Will spending on gasoline *increase* or *decrease*?

b. By about how much? percent

(4) [4 pts] Income per capita for a country equals total income divided by total population. Suppose total income increases by 10 percent and total population increases by 3 percent.

a. Will income per capita *increase* or *decrease*?

b. By about how much? percent

(5) [8 pts] Consider the function $y = f(x_1,x_2)$ . Suppose at a particular point, $\partial y/\partial x_1 = 2$, and $\partial y/\partial x_2 = 3$. First, suppose that $x_1$ increases by 6 units but $x_2$ does not change.

a. Will $y$ *increase* or *decrease*?

b. By about how much? units

Now suppose that $x_1$ increases by 6 units but we want $y$ to remain constant. To keep $y$ constant, we must change the value of $x_2$ .

c. Must $x_2$ *increase* or *decrease*?

d. By about how much? units

(6) [4 pts] Consider the function $y = f(x_1,x_2)$ . Suppose at a particular point, $\partial y/\partial x_1 = -2$, and $\partial y/\partial x_2 = 6$. Now consider a graph of the level curve of this function, with $x_1$ on the vertical axis and $x_2$ on the horizontal axis.

a. Does the level curve of the function slope *up* or *down* at that point?

b. Give the slope of the level curve at this point.

**III. PROBLEMS:** Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = 2x^2 - 20x + 3$.

a. Find an expression (in terms of x ) for the derivative of y with respect to x ( dy/dx ).

b. Compute the value x* that minimizes this function.

c. For what range of values of x does this function slope up? For what range of values does it slope down?

d. Find the minimum value, y*, of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = (x_1 - 3)^2 x_2^3$.

a. Find an expression for $\varepsilon_1$, the partial elasticity of y with respect to $x_1$. The variable y should *not* appear in your answer. Simplify if possible.

b. Find an expression for $\varepsilon_2$, the partial elasticity of y with respect to $x_2$. The variable y should *not* appear in your answer. Simplify if possible.

(3) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = (x_1 - 5)^2 (x_2 - 4)^3$. The arguments $x_1$ and $x_2$ are strictly positive.

a. Find an expression for the partial derivative of $y$ with respect to $x_1$.

b. Find an expression for the partial derivative of $y$ with respect to $x_2$.

c. Find an expression for the marginal rate of substitution of $x_2$ for $x_1$ (that is, the formula for the |slope| of the level curves of $y$, with $x_1$ on the vertical axis and $x_2$ on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = -7\,x_1^{-1} - 5\,x_2^{-1}$. The arguments $x_1$ and $x_2$ are strictly positive.

a. Find an expression for the partial derivative of $y$ with respect to $x_1$.

b. Find an expression for the partial derivative of $y$ with respect to $x_2$.

c. Find an expression for the marginal rate of substitution of $x_2$ for $x_1$ (that is, the formula for the |slope| of the level curves of $y$, with $x_1$ on the vertical axis and $x_2$ on the horizontal axis). Simplify if possible.

**IV. CRITICAL THINKING:** [4 pts] Answer *one* question below (your choice). Circle the question you are answering. Justify your answer and show your work.

(1) Let the function $f(x)$ represent net benefit to society as a function of some variable $x$, which might represent the amount of output of some good or service. Now $f(x)$ is itself the difference of two other functions $b(x)$ and $c(x)$, representing benefits and costs of $x$, respectively, so that $f(x) = b(x) - c(x)$. When $f(x)$ is maximized, what must be the relationship between the derivatives $db/dx$ and $dc/dx$ ? Why?

(2) Suppose consumer spending on some good is constant, regardless of the price, so that for all values of $P$ and $Q$, $P \times Q = c$, where $c$ is some constant. Solve for $Q$ and find the elasticity of $Q$ with respect to $P$. Does the elasticity depend on the value of $c$ ?

[end of exam]