

EXAMINATION #4 ANSWER KEY

Version A

I. Multiple choice

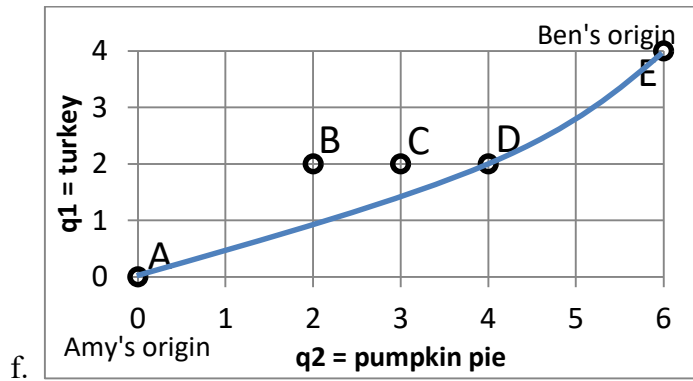
- (1)a. (2)e. (3)b. (4)a. (5)b. (6)b. (7)b. (8)c.

II. Short answer

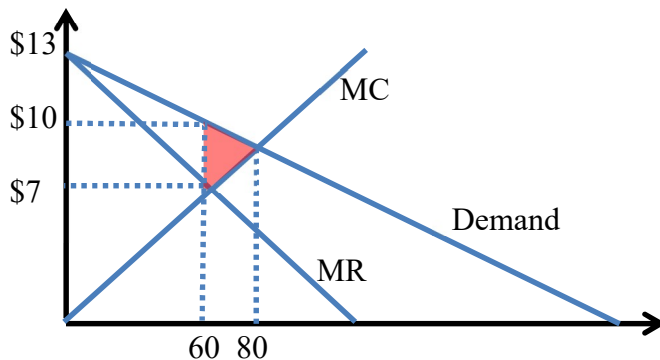
- (1) a. 1/2 units of clothing. b. 2 units of food. c. slope = -1/2.
d. $P_{\text{food}} = \$3$, because slope of each consumer's budget line = $-P_{\text{food}}/P_{\text{clothing}} = -1/3$.
- (2) a. $L = 1/(|\epsilon| n) = 1/20 = 0.05$ b. $L = 1/6$. c. $L = 1/2$.
- (3) a. Monopoly: no, no.
b. Perfect competition: yes, yes.
c. Monopolistic competition: yes, no.
- (4) a. \$5. b. 7 thousand. c. \$0.
d. $MR = 12 - 2Q$
e. Plot MR as a straight line with P-intercept = \$12, slope = -2/thousand.
f. \$8. g. 4 thousand. h. \$6 thousand.
- (5) a. Pareto optimal: yes, no, no, yes.
b. Dominant-strategy equilibria: no, no, no, no.
c. Nash equilibria in pure strategies: yes, no, no, yes.

III. Problems

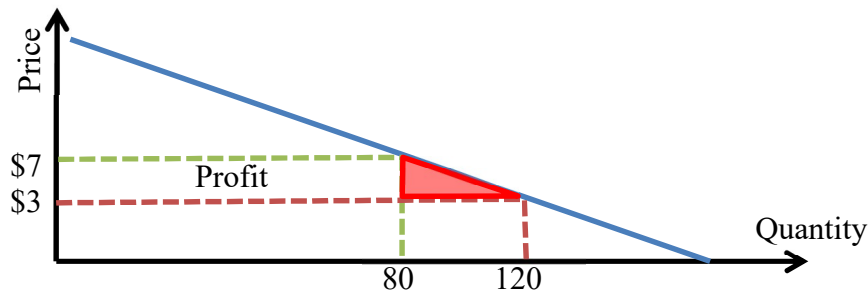
- (1) [Exchange efficiency] Note that Amy's $MRS_A = 2q_1/q_2$ and Ben's $MRS_B = q_1/q_2$.
- a. **Yes**, A is Pareto-efficient, because no one can be made better off without someone else being made worse off. Ben has everything, so he cannot be made better off. Amy has nothing, so she cannot be made better off without taking some of Ben's turkey or pumpkin pie, which would make Ben worse off. Put simply, since Ben already has everything, any feasible change would make Ben worse off.
- b. **No**, B is not Pareto-efficient, because $MRS_A = 2 \neq MRS_B = 1/2$.
- c. **No**, C is not Pareto-efficient, because $MRS_A = 4/3 \neq MRS_B = 2/3$.
- d. **Yes**, D is Pareto-efficient, because $MRS_A = 1 = MRS_B = 1$.
- e. **Yes**, E is Pareto-efficient, because no one can be made better off without someone else being made worse off. Amy has everything, so she cannot be made better off. Ben has nothing, so he cannot be made better off without taking some of Amy's turkey or pumpkin pie, which would make Amy worse off. Put simply, since Amy already has everything, any feasible change would make Amy worse off.



- (2) [Monopoly, profit maximization]
- $MC = dTC/dQ = 1 + (Q/10)$.
 - $AC = TC/Q = 1 + (Q/20)$.
 - First find total revenue = $P \times Q = 13Q - (Q^2/20)$. So $MR = dTR/dQ = 13 - (Q/10)$.
 - Set $MC = MR$ and solve to get $Q_M = 60$.
 - Substitute into demand function: $P_M = 13 - (60/20) = \$10$.
 - Profit = $TR - TC = (60 \times 10) - (60 + 60^2/20) = \360 .
 - The efficient level of output lies where marginal cost intersects demand (“marginal cost pricing”). Find this quantity by setting $13 - (Q/20) = 1 + (Q/10)$, which yields $Q=80$. Then find $MC(80) = 1 + (80/20) = \$5$. Then evaluate DWL as the area of a triangle: **\$30**.



- (3) [Cournot duopoly]
- $TR_1 = P q_1 = 15q_1 - (q_1^2/10) - (q_1q_2/10)$.
 - $MR_1 = \partial TR_1(q_1, q_2) / \partial q_1 = 15 - 2q_1/10 - q_2/10$.
 - Set $MR_1 = MC = \$3$ and solve to get $q_1^* = 60 - q_2/2$.
 - Since $q_1^* = q_2^*$, $q_1^* = 60 - q_1^*/2$. Solving yields $q_1^* = 40 = q_2^*$.
 - $Q^* = q_1^* + q_2^* = 80$. Substituting into demand equation: $P^* = 15 - (80/10) = \$7$.
 - Profit = $(P^* \times Q^*) - (AC \times Q^*) = (P^* - AC) \times Q^* = (7 - 3) \times 80 = \320 .
 - The efficient level of output lies where marginal cost intersects demand (“marginal cost pricing”). Find this quantity by setting $MC = \$3 = P = 15 - (Q/10)$ and solving to get $Q = 120$. Deadweight loss is the area between demand and marginal cost, from the Cournot equilibrium quantity $Q^*=80$ to the efficient quantity = 120 (see below). This is the area of a triangle, equal to **\$80**.



IV. Critical thinking

(1) Perfect price discrimination.

Firm can charge a different price for every unit sold, so will set price equal to willingness-to-pay for that unit. To maximize profit, sell all units where willingness-to-pay is at least equal to marginal cost. So quantity is at intersection of demand and marginal cost, which was found in problem (2) as $MC = 1 + (Q/10)$.

So set $P = 13 - (Q/20) = 1 + (Q/10)$ and solve to get $Q^* = 80$.

The highest price charged anyone is \$13, the intercept of the demand curve. The lowest price is \$9, at the intersection of demand and marginal cost. So total revenue is the area of a trapezoid whose parallel sides are \$13 and \$9, and whose height is 80:

$$TR = (13+9)/2 \times 80 = \$880.$$

The total cost function is given as $Q + (Q^2/20)$, so $TC = 80 + (80^2/20) = \$400$, so

profit = $TR - TC = \$480$. Note that profit is considerably higher with perfect price discrimination than with single-price monopoly.

(2) Price-setting (Bertrand) duopoly

In a price-setting duopoly, the equilibrium is for both firms to set a price equal to marginal cost, which here is $P^* = \$3$. Substituting into the demand equation

$\$3 = 15 - (Q/10)$ gives $Q^* = 120$. Since $P^* = \text{marginal cost} = \text{average cost}$, **profit = 0**.

Version B

I. Multiple choice

(1)e. (2)b. (3)c. (4)b. (5)c. (6)b. (7)c. (8)a.

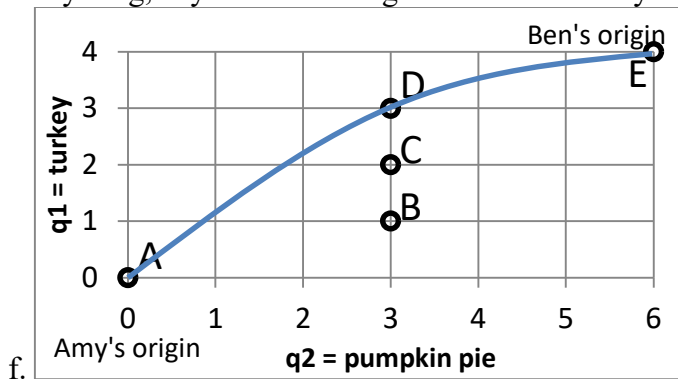
II. Short answer

- (1) a. 3 units of clothing b. 1/3 units of food. c. slope = -3.
 d. $P_{\text{food}} = \$18$, because slope of each consumer's budget line = $-P_{\text{food}}/P_{\text{clothing}} = -3$.
- (2) a. $L = 1/(|\epsilon|n) = 1/20 = 0.05$ b. $L = 1/10 = 0.10$. c. $L = 1/5$.
- (3) a. Perfect competition: yes, yes.
 b. Monopoly: no, no.
 c. Monopolistic competition: no, yes.
- (4) a. \$3. b. 9 thousand. c. \$0.
 d. $MR = 12 - 2Q$

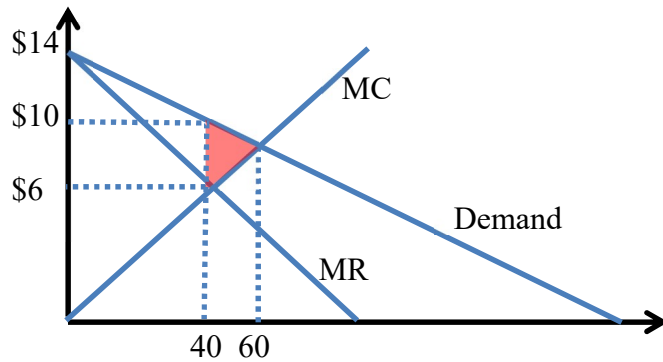
- e. Plot MR as a straight line with P-intercept = \$12, slope = -2/thousand.
 f. \$7. g. 5 thousand. h. \$10 thousand.
- (5) a. Pareto optimal: no, yes, yes, yes.
 b. Dominant-strategy equilibria: yes, no, no, no.
 c. Nash equilibria in pure strategies: yes, no, no, no.

III. Problems

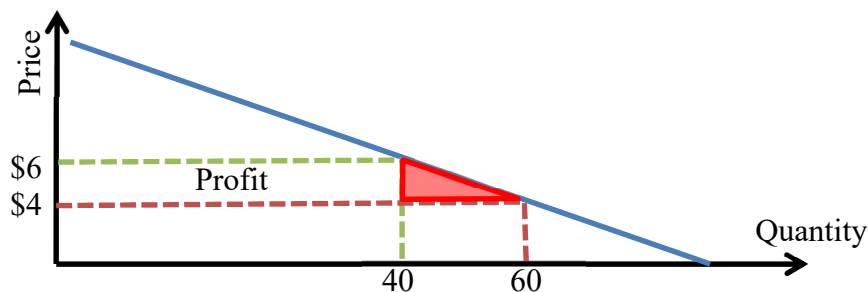
- (1) [Exchange efficiency] Note that Amy's $MRS_A = q_1/q_2$ and Ben's $MRS_B = 3q_1/q_2$.
- a. **Yes**, A is Pareto-efficient, because no one can be made better off without someone else being made worse off. Ben has everything, so he cannot be made better off. Amy has nothing, so she cannot be made better off without taking some of Ben's turkey or pumpkin pie, which would make Ben worse off. Put simply, since Ben already has everything, any feasible change would make Ben worse off.
- b. **No**, B is not Pareto-efficient, because $MRS_A = 1/3 \neq MRS_B = 3$.
- c. **No**, C is not Pareto-efficient, because $MRS_A = 2/3 \neq MRS_B = 2$.
- d. **Yes**, D is Pareto-efficient, because $MRS_A = 1 = MRS_B = 1$.
- e. **Yes**, E is Pareto-efficient, because no one can be made better off without someone else being made worse off. Amy has everything, so she cannot be made better off. Ben has nothing, so he cannot be made better off without taking some of Amy's turkey or pumpkin pie, which would make Amy worse off. Put simply, since Amy already has everything, any feasible change would make Amy worse off.



- (2) [Monopoly, profit maximization]
- a. $MC = dTC/dQ = 2 + (Q/10)$.
- b. $AC = TC/Q = 2 + (Q/20)$.
- c. First find total revenue = $P \times Q = 14Q - (Q^2/10)$. So $MR = dTR/dQ = 14 - (2Q/10)$.
- d. Set $MC = MR$ and solve to get $Q_M = 40$.
- e. Substitute into demand function: $P_M = 14 - (40/10) = \$10$.
- f. Profit = $TR - TC = (40 \times 10) - (2 \times 40 + 40^2/20) = \240 .
- g. The efficient level of output lies where marginal cost intersects demand ("marginal cost pricing"). Find this quantity by setting $14 - (Q/10) = 2 + (Q/10)$, which yields $Q=60$. Then find $MC(40) = 2 + (40/10) = \$6$. Then evaluate DWL as the area of a triangle: **\$40**.



- (3) [Cournot duopoly]
- $TR_1 = P q_1 = 10q_1 - (q_1^2/10) - (q_1q_2/10)$.
 - $MR_1 = \partial TR_1(q_1, q_2) / \partial q_1 = 10 - 2q_1/10 - q_2/10$.
 - Set $MR_1 = MC = \$4$ and solve to get $q_1^* = 30 - q_2/2$.
 - Since $q_1^* = q_2^*$, $q_1^* = 30 - q_1^*/2$. Solving yields $q_1^* = 20 = q_2^*$.
 - $Q^* = q_1^* + q_2^* = 40$. Substituting into demand equation: $P^* = 10 - (40/10) = \$6$.
 - Profit = $(P^* \times Q^*) - (AC \times Q^*) = (P^* - AC) \times Q^* = (6 - 4) \times 40 = \80 .
 - The efficient level of output lies where marginal cost intersects demand ("marginal cost pricing"). Find this quantity by setting $MC = \$4 = P = 10 - (Q/10)$ and solving to get $Q = 60$. Deadweight loss is the area between demand and marginal cost, from the Cournot equilibrium quantity $Q^* = 40$ to the efficient quantity = 60 (see below). This is the area of a triangle, equal to \$20.



IV. Critical thinking

- (1) Perfect price discrimination.
 Firm can charge a different price for every unit sold, so will set price equal to willingness-to-pay for that unit. To maximize profit, sell all units where willingness-to-pay is at least equal to marginal cost. So quantity is at intersection of demand and marginal cost, which was found in problem (2) as $MC = 2 + (Q/10)$.
 So set $P = 14 - (Q/10) = 2 + (Q/10)$ and solve to get $Q^* = 60$.
 The highest price charged anyone is \$14, the intercept of the demand curve. The lowest price is \$8, at the intersection of demand and marginal cost. So total revenue is the area of a trapezoid whose parallel sides are \$14 and \$8, and whose height is 60:
 $TR = (14+8)/2 \times 60 = \660 .

The total cost function is given as $2Q + (Q^2/20)$, so $TC = 2 \times 60 + (60^2/20) = \300 , so **profit = TR – TC = \$360**. Note that profit is considerably higher with perfect price discrimination than with single-price monopoly.

(2) Price-setting (Bertrand) duopoly

In a price-setting duopoly, the equilibrium is for both firms to set a price equal to marginal cost, which here is $P^* = \$4$. Substituting into the demand equation $\$4 = 10 - (Q/10)$ gives $Q^* = 60$. Since $P^* = \text{marginal cost} = \text{average cost}$, **profit = 0**.

[end of answer key]