

**EXAMINATION #3 ANSWER KEY**  
**“Firms and Competition”**

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**Version A**

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**I. Multiple choice**

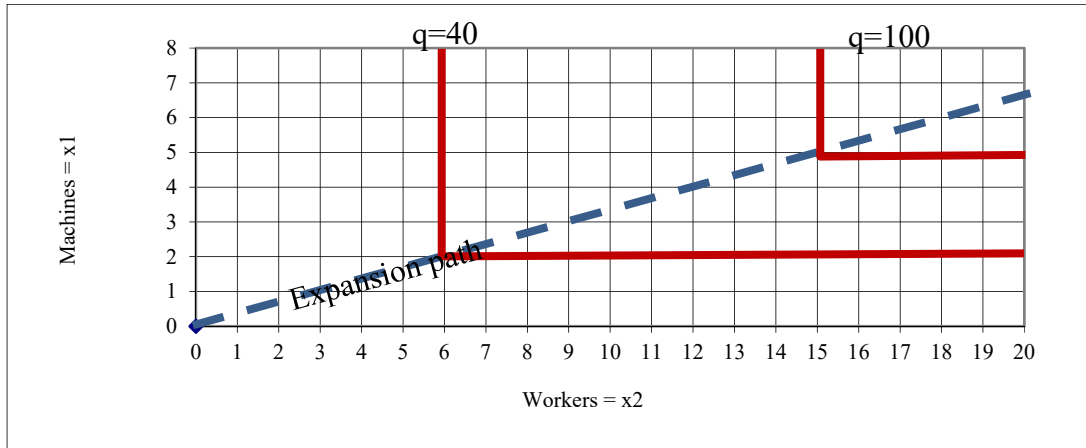
- (1)c. (2)b. (3)d. (4)a. (5)d. (6)c. (7)a. (8)b. (9)b. (10)b.  
(11)b. (12)b. (13)d. (14)c. (15)c.

**II. Short answer**

- (1) a. 4%. b. 3%.  
(2) a. 0 thousand (because price is below shutdown price).  
b. 11 thousand (using rule  $P=MC$  to find  $q$ ).  
c. 8 thousand (using rule  $P=MC$  to find  $q$ ).  
d. \$6 (because breakeven price =  $\min(\text{SATC})$ ).  
e. \$2 (because shutdown price =  $\min(\text{SAVC})$ ).  
(3) a. import. b. 6 million pounds. c. increase.  
d. \$14 million. e. decrease. f. \$8 million.  
g. increase. h. \$6 million.

**III. Problems**

- (1) [Production functions]  
a.  $MP_1 = 6x_1^{-1/4}$ . Yes, there are diminishing returns to input 1, because as  $x_1$  increases (and  $x_2$  is held constant),  $MP_1$  decreases.  
b.  $MRSP = MP_2/MP_1 = \frac{(3/4)x_2^{-1/4}}{6x_1^{-1/4}} = \frac{1}{8} \left(\frac{x_1}{x_2}\right)^{1/4}$ . Yes, this function does have diminishing  $MRSP$ , because as  $x_1$  decreases and  $x_2$  increases,  $MRSP$  diminishes.  
c. Check returns to scale:  
 $f(ax_1, ax_2) = 8(ax_1)^{3/4} + (ax_2)^{3/4} = 8a^{3/4}x_1^{3/4} + a^{3/4}x_2^{3/4}$   
 $= a^{3/4}(8x_1^{3/4} + x_2^{3/4}) = a^{3/4}q < aq$ , for all  $a > 1$ .  
Thus, multiplying all inputs by the same factor ( $a$ ) causes output to increase by a smaller factor. So this production function has DECREASING returns to scale.  
(2) [Fixed-proportions technology]  
a.  $x_1 = (x_2/3)$ . b.  $q = 20x_1$ . c.  $q = (20/3)x_2$ .  
d.  $q = \min\{20x_1, (20/3)x_2\}$ .  
e.



(3) [Cost minimization]

a. Equation for isoquant:  $60 = 4 x_1^{1/2} x_2^{1/2}$  or  $15 = x_1^{1/2} x_2^{1/2}$  or  $225 = x_1 x_2$ .

b.  $MRSP = MP_2/MP_1 = \frac{2 x_1^{1/2} x_2^{-1/2}}{2 x_1^{-1/2} x_2^{1/2}} = x_1/x_2$ .

c. Set  $MRSP = \$18/\$2$  and solve jointly with  $60 = 4 x_1^{1/2} x_2^{1/2}$ , to get  $x_1^*=45$  and  $x_2^*=5$ .

d.  $TC(60) = 45 \times \$2 + 5 \times \$18 = \$180$ .

(4) [Long-run profit maximization and supply]

a.  $AC = TC/q = 0.01 q^2 - q + 35$ .

Set  $0 = dAC/dq = 0.02 q - 1$  and solve to get  $q_{ES} = 50$ .

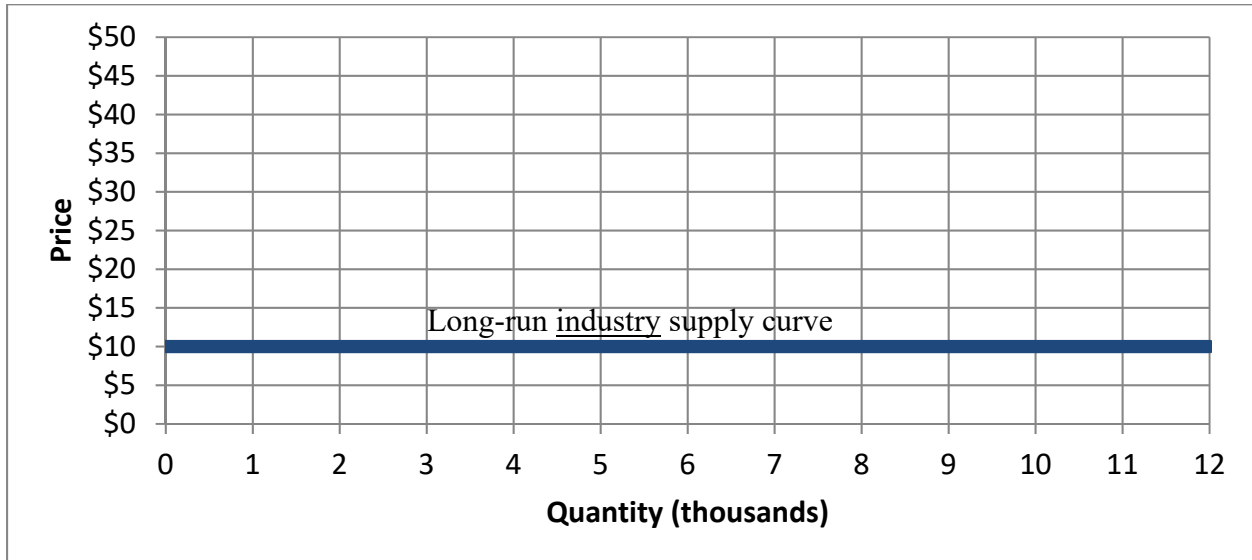
b. Breakeven price = minimum  $AC = AC(q_{ES}) = \$10$ .

c. A supply curve shows how much will be produced for any given price. If  $P >$  minimum average cost, the profit-maximizing firm will choose an output level where  $P = MC(q)$ , and if  $P <$  minimum average cost, it will produce nothing. So the firm's supply curve is given by the following equations.

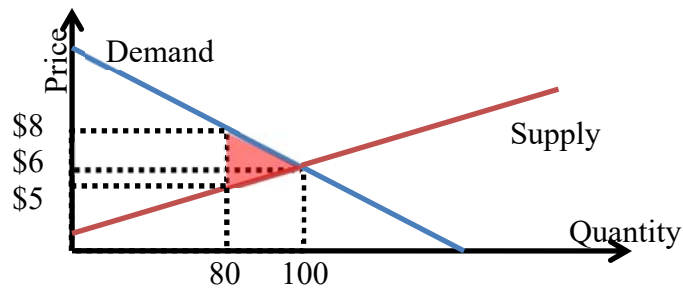
If  $P \geq \$10$ ,  $P = MC(q) = dTC/dq = 0.03 q^2 - 2 q + 35$ .

If  $P \leq \$10$ ,  $q=0$  (firm shuts down).

d. The long-run industry supply curve is a horizontal line at minimum  $AC$ :



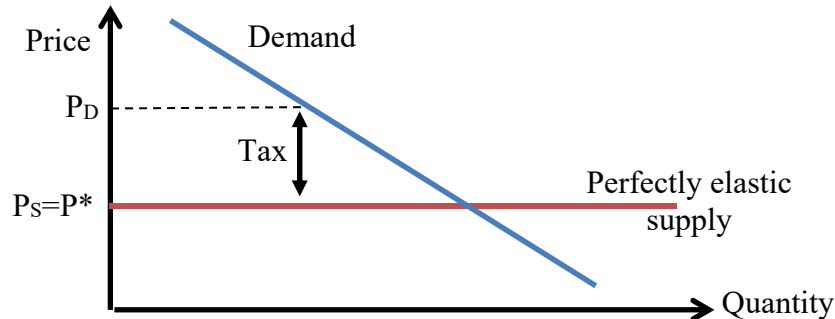
- (5) [Welfare effects of tax or subsidy]
- Set  $P_D = P_S$  and solve to get  $P^* = \$6$  and  $Q^* = 100$ .
  - With an excise tax of \$3,  $P_D = P_S + 3$ . Substituting and solving gives  $Q = 80$ . It is useful to also compute the new total price paid by buyers, including the tax ( $P_D = \$8$ ), and the new net price received by sellers, excluding the tax ( $P_S = \$5$ ).



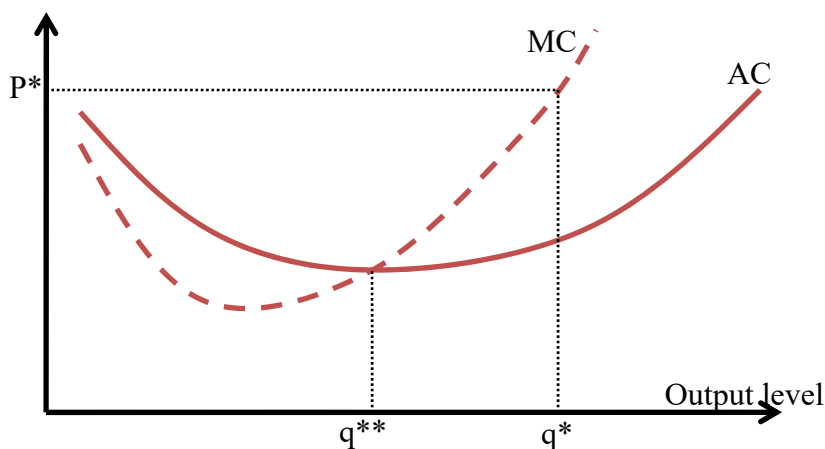
- Consumer surplus decreases by \$180, the area of the trapezoid between \$6 and \$8.
- Producer surplus decreases by \$90, the area of the trapezoid between \$6 and \$5.
- Although the government collects  $\$3 \times 80 = \$240$  in tax revenue, this is less than the combined decreases of consumer and producer surplus. The loss to society as a whole (also called “deadweight loss” or “excess burden of the tax”) is \$30.

**IV. Critical thinking**

(1) If supply is perfectly elastic (horizontal) then only buyers bear the burden of the tax. The tax inserts a wedge between demand and supply, but only the buyers' price ( $P_D$ ) moves. The sellers' price ( $P_S$ ) remains the same as before the tax.



(2) [The following answer uses a long-run framework, where there are no fixed costs. A similar answer in a short-run framework would be acceptable.] It is true that *per-unit profit* is maximized when the firm operates at the output level where average cost is lowest. However *total profit* is maximized when the firm operates at the output level where *market price equals marginal cost*, provided price is greater than minimum average cost. (If price is less than average cost, total profit is maximized when the output level is zero.) Thus, the total-profit-maximizing level of output depends on the market price. In general, the two output levels will be different, as shown in the graph below. Here,  $P^*$  is the market price,  $q^*$  is the output level that maximizes total profit, and  $q^{**}$  is the output level where average cost is lowest.



## Version B

### I. Multiple choice

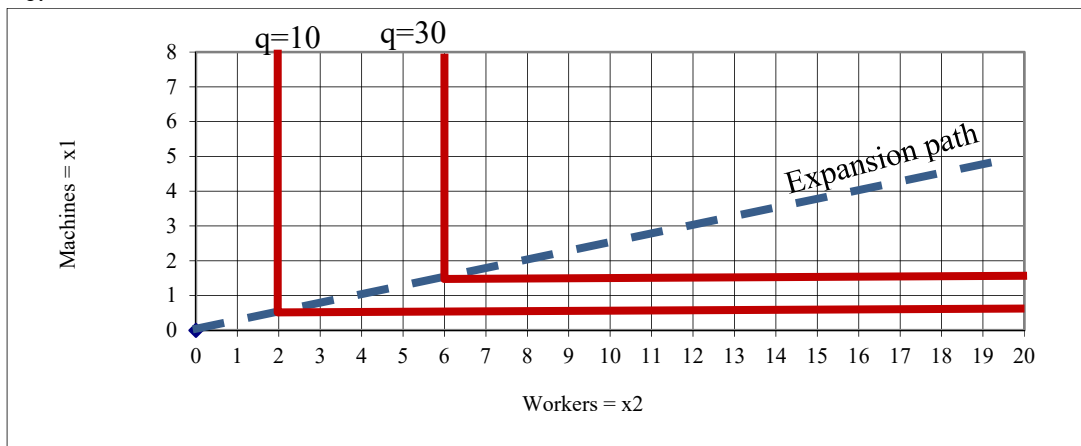
- (1)d. (2)c. (3)b. (4)c. (5)b. (6)d. (7)b. (8)a. (9)a. (10)f.  
 (11)d. (12)a. (13)b. (14)d. (15)e.

### II. Short answer

- (1) a. 3%. b. 4%.  
 (2) a. 7 thousand (using rule  $P=MC$  to find  $q$ ).  
 b. 9 thousand (using rule  $P=MC$  to find  $q$ ).  
 c. 0 thousand (because price is below shutdown price).  
 d. \$7 (because breakeven price =  $\min(\text{SATC})$ ).  
 e. \$4 (because shutdown price =  $\min(\text{SAVC})$ ).  
 (3) a. export. b. 6 million pounds. c. decrease.  
 d. \$10 million. e. increase. f. \$16 million.  
 g. increase. h. \$6 million.

### III. Problems

- (1) [Production functions]  
 a.  $MP_1 = 6 x_1^{-1/4} x_2^{3/4}$ . Yes, there are diminishing returns to input 1, because as  $x_1$  increases (and  $x_2$  is held constant),  $MP_1$  decreases.  
 b.  $MRSP = MP_2/MP_1 = \frac{6 x_1^{3/4} x_2^{-1/4}}{6 x_1^{-1/4} x_2^{3/4}} = \frac{x_1}{x_2}$ . Yes, this function does have diminishing MRSP, because as  $x_1$  decreases and  $x_2$  increases, MRSP diminishes.  
 c. Check returns to scale:  
 $f(ax_1, ax_2) = 8 (ax_1)^{3/4} (ax_2)^{3/4} = 8 a^{3/4} x_1^{3/4} a^{3/4} x_2^{3/4}$   
 $= a^{3/4} a^{3/4} (8 x_1^{3/4} x_2^{3/4}) = a^{3/2} q > aq$ , for all  $a > 1$ .  
 Thus, multiplying all inputs by the same factor ( $a$ ) causes output to increase by a larger factor. So this production function has INCREASING returns to scale.  
 (2) [Fixed-proportions technology]  
 a.  $x_1 = (x_2/4)$ . b.  $q = 20 x_1$ . c.  $q = (20/4) x_2$ .  
 d.  $q = \min\{20 x_1, (20/4) x_2\} = \min\{20 x_1, 5 x_2\}$ .  
 e.



(3) [Cost minimization]

a. Equation for isoquant:  $30 = 5 x_1^{1/2} x_2^{1/2}$  or  $6 = x_1^{1/2} x_2^{1/2}$  or  $36 = x_1 x_2$ .

b.  $MRSP = MP_2/MP_1 = \frac{(5/2) x_1^{1/2} x_2^{-1/2}}{(5/2) x_1^{-1/2} x_2^{1/2}} = x_1/x_2$ .

c. Set  $MRSP = \$20/\$5$  and solve jointly with  $30 = 5 x_1^{1/2} x_2^{1/2}$ , to get  $x_1^*=12$  and  $x_2^*=3$ .

d.  $TC(30) = 12 \times \$5 + 3 \times \$20 = \$120$ .

(4) [Long-run profit maximization and supply]

a.  $AC = TC/q = 0.01 q^2 - 0.8 q + 22$ .

Set  $0 = dAC/dq = 0.02 q - 0.8$  and solve to get  $q_{ES} = 40$ .

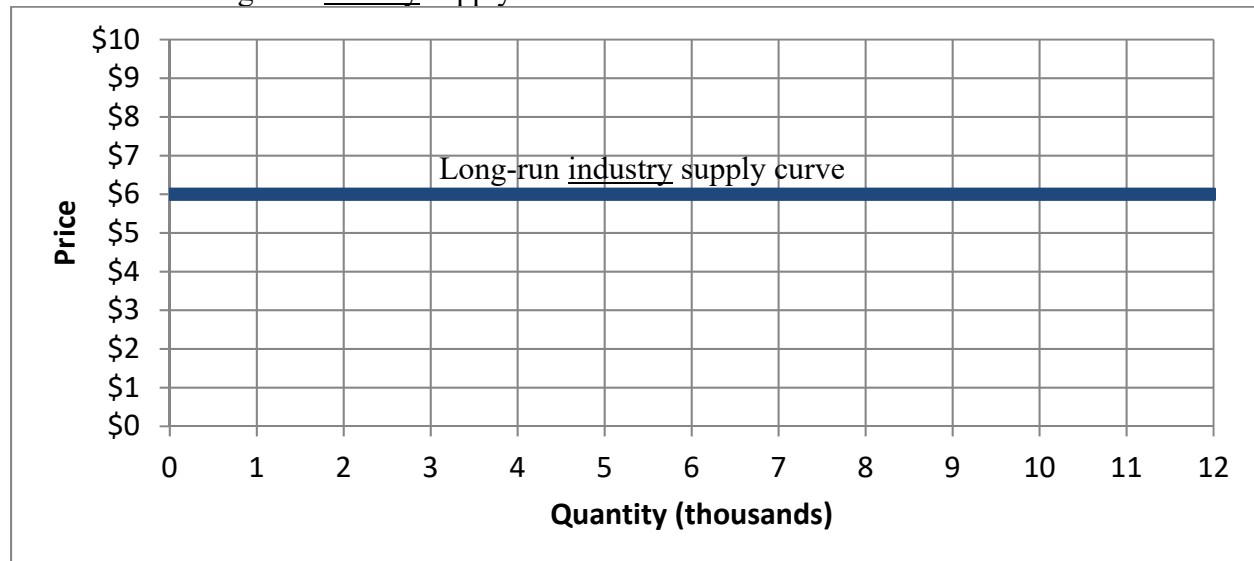
b. Breakeven price = minimum  $AC = AC(q_{ES}) = \$6$ .

c. A supply curve shows how much will be produced for any given price. If  $P >$  minimum average cost, the profit-maximizing firm will choose an output level where  $P = MC(q)$ , and if  $P <$  minimum average cost, it will produce nothing. So the firm's supply curve is given by the following equations.

If  $P \geq \$6$ ,  $P = MC(q) = dTC/dq = 0.03 q^2 - 1.6 q + 22$ .

If  $P \leq \$6$ ,  $q = 0$  (firm shuts down).

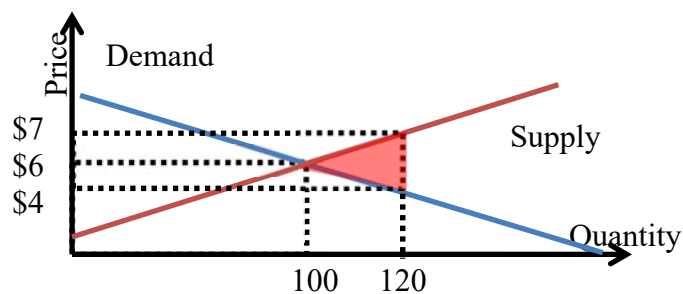
d. The long-run industry supply curve is a horizontal line at minimum  $AC$ :



(5) [Welfare effects of tax or subsidy]

a. Set  $P_D = P_s$  and solve to get  $P^* = \$6$  and  $Q^* = 100$ .

b. With subsidy of \$3,  $P_D + 3 = P_s$ . Substituting and solving gives  $Q = 120$ . It is useful to also compute the new net price paid by buyers, excluding the subsidy ( $P_D = \$4$ ), and the new total price received by sellers, including the subsidy ( $P_s = \$7$ ).



- c. Consumer surplus increases by \$220, the area of the trapezoid between \$6 and \$4.
- d. Producer surplus increases by \$110, the area of the trapezoid between \$6 and \$7.
- e. The government pays  $3 \times 120 = \$360$  in tax revenue, but this is less than the combined increases of consumer and producer surplus. The loss to society as a whole (also called “deadweight loss”) is \$30.

#### IV. Critical thinking

(Same as Version A above.)

[end of answer key]