

EXAMINATION #2 ANSWER KEY
“Consumers and Demand”

Version A

I. Multiple choice

(1)a . (2)c. (3)a. (4)b. (5)c. (6)a. (7)a. (8)c. (9)c. (10)d.

II. Short answer

- (1) a. inelastic. b. decrease. c. 2 %.
 d. increase. e. 6 %.
- (2) a. luxury or superior good. b. increase. c. 5 %.
 d. increase. e. 1 %.
- (4) Note: This graph is based on Slutsky’s approach to income and substitution effects, not Hicks’s approach.
 a. \$3. b. 10 units. c. \$10.
 d. 4 units. e. – 4 units. f. – 2 units.
- (5) a. $\epsilon^{\text{comp}} = -0.19$. b. decrease. c. 2.5 %.
 d. decrease e. 1.9 %.
- (6) a. Laspeyres = 140. b. Paasche = 120. c. $\sqrt{140 \times 120}$.

III. Problems

- (1) [Budgets and choice]
 a. Equation for budget line (income=spending): $60 = 3 q_1 + 2 q_2$.
 b. $\text{MRSC} = \text{MU}_2/\text{MU}_1 = (1/2 q_2^{-1/2}) / (1/2 q_1^{-1/2}) = q_2^{-1/2} / q_1^{-1/2} = (q_1 / q_2)^{1/2}$.
 c. Solve the tangency condition ($\text{MRSC} = p_2/p_1 = 2/3$) jointly with equation for budget line (see part a) to get $q_1^* = 8$, $q_2^* = 18$.
- (2) [Properties of individual demand functions]
 a. Homogeneity of degree zero means that if income and all prices are multiplied by the same factor, the quantity demanded never changes. Check homogeneity of the demand function by replacing I by (aI), replacing p_1 by (ap_1), and replacing p_2 by (ap_2), where a is an arbitrary factor:

$$\begin{aligned} & 5 (a p_1)^{-0.8} (a I)^{1.2} (a p_2)^{0.1} \\ &= a^{-0.8} a^{1.2} a^{0.1} (5 p_1^{-0.8} I^{1.2} p_2^{0.1}) \\ &= a^{0.5} (5 p_1^{-0.8} I^{1.2} p_2^{0.1}) \end{aligned}$$

Note that the “a” factor does *not* cancel. So multiplying income and prices by some arbitrary positive factor a *does* change the quantity demanded. The function is NOT homogeneous of degree zero in income and prices.

- b. $\frac{\partial q_1^*}{\partial p_1} = -0.8 (5 p_1^{-1.8} I^{1.2} p_2^{0.1})$. This expression is negative, so good #1 is an ordinary good, not a Giffen good.
- c. $\frac{\partial q_1^*}{\partial I} = 1.2 (5 p_1^{-0.8} I^{0.2} p_2^{0.1})$. This expression is positive, so good #1 is a normal good, not an inferior good.
- d. $\frac{\partial q_1^*}{\partial p_2} = 0.1 (5 p_1^{-0.8} I^{1.2} p_2^{-0.9})$. This expression is positive, so goods #1 and #2 are substitutes, not complements.

(3) [Finding individual demand functions]

- a. $MRS = MU_2/MU_1 = (2q_1) / q_2$.
- b. Solve $MRS = p_2/p_1$ jointly with $I = p_1q_1 + p_2q_2$ to get
- $q_1^* = \frac{I}{3 p_1}$, and c. $q_2^* = \frac{2I}{3 p_2}$.

III. Critical thinking

(1) The minimum amount of income that Ana must have to attain a target level of utility of $\bar{U} = 300$ utils is the cost of a bundle that is (a) on her target indifference curve and (b) at a tangency point with a budget line. Since Ana's utility function is given as $U = q_1 q_2$, the equation for the target indifference curve is

$$(a) 300 = q_1 q_2 .$$

The general formula for the tangency condition is $MRSC = p_2/p_1$. Since Amanda's utility function is given here as $U = q_1 q_2$, we have $MRSC = MU_2/MU_1 = q_1/q_2$. The price ratio is $p_2/p_1 = \$4/\3 . So the tangency condition is

$$(b) q_1/q_2 = 4/3.$$

Solving (a) and (b) jointly gives $q_1^* = 20$ and $q_2^* = 15$. This bundle will cost $20 \times \$3 + 15 \times \$4 = \boxed{\$120}$, answer.

(2) Because all prices increase at the same rate, all three of Bob's COL indices increase at exactly the *same rate*. *Justification with algebra:*

Bob's Laspeyres COL index

$$\begin{aligned} &= \frac{p_s^{new} q_s^{old} + p_g^{new} q_g^{old}}{p_s^{old} q_s^{old} + p_g^{old} q_g^{old}} \times 100 = \frac{2p_s^{old} q_s^{old} + 2p_g^{old} q_g^{old}}{p_s^{old} q_s^{old} + p_g^{old} q_g^{old}} \times 100 \\ &= 2 \left(\frac{p_s^{old} q_s^{old} + p_g^{old} q_g^{old}}{p_s^{old} q_s^{old} + p_g^{old} q_g^{old}} \right) \times 100 = 2 \times 100 = 200 . \end{aligned}$$

Bob's Paasche COL index

$$\begin{aligned} &= \frac{p_s^{new} q_s^{new} + p_g^{new} q_g^{new}}{p_s^{old} q_s^{new} + p_g^{old} q_g^{new}} \times 100 = \frac{2p_s^{old} q_s^{new} + 2p_g^{old} q_g^{new}}{p_s^{old} q_s^{new} + p_g^{old} q_g^{new}} \times 100 \\ &= 2 \left(\frac{p_s^{old} q_s^{new} + p_g^{old} q_g^{new}}{p_s^{old} q_s^{new} + p_g^{old} q_g^{new}} \right) \times 100 = 2 \times 100 = 200 . \end{aligned}$$

Bob's Fisher COL index

$$= \sqrt{\text{Laspeyres} \times \text{Paasche}} = \sqrt{200 \times 200} = 200 .$$

Version B

I. Multiple choice

(1)b . (2)c. (3)f. (4)a. (5)c. (6)b. (7)b. (8)d. (9)c. (10)b.

II. Short answer

- (1) a. elastic. b. decrease. c. 6 %.
 d. decrease. e. 1 %.
- (2) a. necessary good. b. increase. c. 2 %.
 d. decrease. e. 6 %.
- (4) Note: This graph is based on Slutsky's approach to income and substitution effects, not Hicks's approach.
 a. \$2. b. 10 units. c. \$6.
 d. 2 units. e. - 6 units. f. - 2 units.
- (5) a. $\epsilon^{\text{comp}} = -1.2$. b. decrease. c. 15 %.
 d. decrease e. 12 %.
- (6) a. Laspeyres = 160. b. Paasche = 130. c. $\sqrt{160 \times 130}$.

III. Problems

- (1) [Budgets and choice]
 a. Equation for budget line (income=spending): $60 = 4 q_1 + 9 q_2$.
 b. $\text{MRSC} = \text{MU}_2/\text{MU}_1 = \frac{q_2^{-2}}{q_1^{-2}} = \left(\frac{q_1}{q_2}\right)^2$.
 c. Solve the tangency condition ($\text{MRSC} = p_2/p_1 = 9/4$) jointly with equation for budget line (see part a) to get $q_1^* = 6$, $q_2^* = 4$.
- (2) [Properties of individual demand functions]
 a. Homogeneity of degree zero means that if income and all prices are multiplied by the same factor, the quantity demanded never changes. Check homogeneity of the demand function by replacing I by (aI) , replacing p_1 by (ap_1) , and replacing p_2 by (ap_2) , where a is an arbitrary factor:

$$\frac{0.1(aI)+(ap_2)}{(ap_1)} + 6 = \frac{a(0.1I+p_2)}{a(p_1)} + 6 = \frac{(0.1I+p_2)}{(p_1)} + 6 .$$
 Note that the "a" factor cancels. So multiplying income and prices by some arbitrary positive factor a *does not* change the quantity demanded. The function is homogeneous of degree zero in income and prices.
 b. $\frac{\partial q_1^*}{\partial p_1} = (0.1I + p_2)(-1)p_1^{-2} < 0$. This expression is negative, so good #1 is an ordinary good, not a Giffen good.
 c. $\frac{\partial q_1^*}{\partial I} = \frac{0.1}{p_1} > 0$. This expression is positive, so good #1 is a normal good, not an inferior good.
 d. $\frac{\partial q_1^*}{\partial p_2} = \frac{1}{p_1} > 0$. This expression is positive, so goods #1 and #2 are substitutes, not complements.

- (3) [Finding individual demand functions]
- a. $MRS = MU_2/MU_1 = q_1 / (3q_2)$.
- b. Solve $MRS = p_2/p_1$ jointly with $I = p_1q_1 + p_2q_2$ to get
- $q_1^* = \frac{3I}{4p_1}$, and c. $q_2^* = \frac{I}{4p_2}$.

III. Critical thinking

Same as Version A.

[end of answer key]