

EXAMINATION #1 ANSWER KEY
“Mathematical Tools”

Version A

I. Multiple choice

- (1)c. (2)b. (3)c. (4)c. (5)b. (6)d. (7)a. (8)c. (9)d. (10)b.
 (11)d. (12)d. (13)a. (14)b. (15)c.

II. Short answer

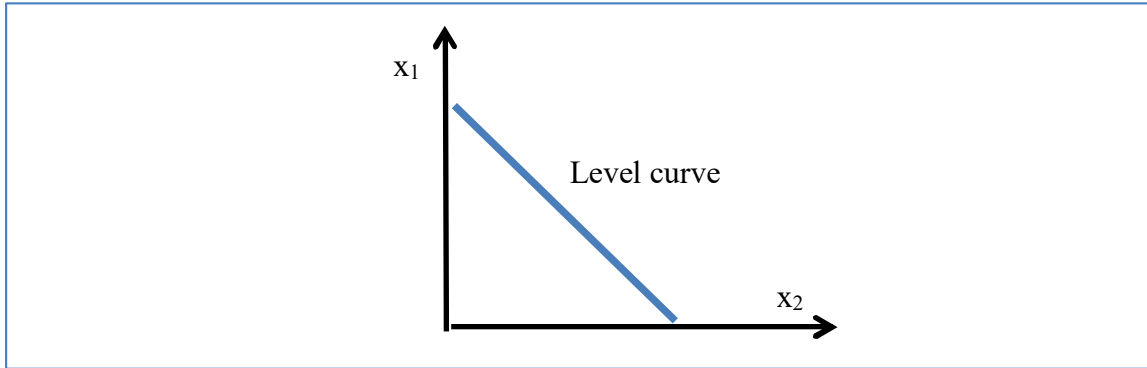
- (1) a. decrease b. 12 units, using derivative since change is given in units.
 (2) a. increase b. 4 percent, using elasticities since changes are given in percent.
 (3) a. increase b. 8 percent, using approximation rule for products.
 (4) a. increase b. 3 percent, using approximation rule for ratios.
 (5) a. increase b. 30 units c. decrease d. 10 units.
 (6) a. down b. slope = $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = -3/4 = -0.75$.

III. Problems

- (1) a. $dy/dx = -6x + 18$. b. Set $dy/dx=0$ and solve to get $x^* = 3$.
 c. The function slopes up if $dy/dx = -6x + 18 > 0$, which implies $x < 3$.
 The function slopes down if $dy/dx = -6x + 18 < 0$, which implies $x > 3$.
 d. $y^* = f(x^*) = f(3) = 32$.
 (2) a. $\varepsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} = 3$. b. $\varepsilon_2 = \frac{\partial y}{\partial x_2} \frac{x_2}{y} = \frac{2x_2}{x_2-1}$.
 (3) a. $\frac{\partial y}{\partial x_1} = 2(x_1 - 3)(x_2 - 5)^3$ b. $\frac{\partial y}{\partial x_2} = (x_1 - 3)^2 3(x_2 - 5)^2$
 c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{3(x_1-3)}{2(x_2-5)}$.
 (4) a. $\frac{\partial y}{\partial x_1} = x_1^{-1/2}$ b. $\frac{\partial y}{\partial x_2} = 2x_2^{-1/2}$ c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{2x_2^{-1/2}}{x_1^{-1/2}} = 2\left(\frac{x_1}{x_2}\right)^{1/2}$.

IV. Critical thinking

- (1) The formula for the slope of a level curve is $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1}$. Here, we are given that $\frac{\partial y}{\partial x_1} = \frac{\partial y}{\partial x_2}$. Therefore, the slope of any level curve equals exactly -1 everywhere. Since the slope is negative, the level curves slope *down*. Since the slope is constant, the level curves are *straight*. Sketch should show a straight level curve with slope = -1.



- (2) We are given that $\varepsilon_K + \varepsilon_L = 1$ and that both labor and capital increase by 5 percent. Using the approximation formula for partial elasticities, percent change in output
- $$\begin{aligned} &= \varepsilon_K (\% \text{ change in capital}) + \varepsilon_L (\% \text{ change in labor}) \\ &= \varepsilon_K (5\%) + \varepsilon_L (5\%) \\ &= (\varepsilon_K + \varepsilon_L) (5\%) \\ &= (1) (5\%) = \mathbf{5\%} . \end{aligned}$$

Version B

I. Multiple choice

- (1)d. (2)c. (3)c. (4)d. (5)c. (6)c. (7)d. (8)e. (9)c. (10)c.
(11)d. (12)a. (13)b. (14)a. (15)a.

II. Short answer

- (1) a. decrease b. 1.5 percent, using elasticity since change is given in percent.
(2) a. increase b. 6 units, using derivatives since changes are given in units.
(3) a. increase b. 9 percent, using approximation rule for products.
(4) a. decrease b. 1 percent, using approximation rule for ratios.
(5) a. increase b. 24 units c. decrease d. 8 units.
(6) a. up b. slope = $-\frac{\partial y/\partial x_2}{\partial y/\partial x_1} = 4$.

III. Problems

- (1) a. $dy/dx = -x + 10$. b. Set $dy/dx=0$ and solve to get $x^* = 10$.
c. The function slopes up if $dy/dx = -x + 10 > 0$, which implies $x < 10$.
The function slopes down if $dy/dx = -x + 10 < 0$, which implies $x > 10$.
d. $y^* = f(x^*) = f(10) = 55$.
- (2) a. $\epsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} = \frac{2x_1}{x_1+5} \cdot 2$. b. $\epsilon_2 = \frac{\partial y}{\partial x_2} \frac{x_2}{y} = 4$.
- (3) a. $\frac{\partial y}{\partial x_1} = 3(x_1 - 10)^2 (x_2 - 6)^2$ b. $\frac{\partial y}{\partial x_2} = (x_1 - 10)^3 \cdot 2(x_2 - 6)$
c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{2(x_1-10)}{3(x_2-6)}$.
- (4) a. $\frac{\partial y}{\partial x_1} = x_1^{-2}$ b. $\frac{\partial y}{\partial x_2} = 2x_2^{-2}$ c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{2x_2^{-2}}{x_1^{-2}} = 2\left(\frac{x_2}{x_1}\right)^2$.

IV. Critical thinking

(Same as version A.)

[end of answer key]